An Estimated Monetary DSGE Model with Unemployment and Staggered Nominal Wage Bargaining* (very preliminary)

Mark Gertler*, Luca Sala** and Antonella Trigari**
*New York University, **Bocconi University, IGIER

June 5, 2007

Abstract

We develop and estimate a medium scale macroeconomic model that allows for unemployment and staggered nominal wage contracting. In contrast to most existing quantitative models, the employment of existing workers is efficient. Wage rigidity, however, affects the hiring of new workers. The former is introduced via the staggered Nash bargaing setup of Gertler and Trigari (2006). A robust finding is that the model with wage rigidity provides a better description of the data than does a flexible wage version. In addition, we are able to quantify the effect of wage rigidity on output and inflation dynamics. More work is necessary, however, to ensure a robust identification of the key labor market parameters.

*Prepared for the JMCB conference in honor of Ernst Baltensberger, Bern, Switzerland, 2007. Gertler thanks the National Science Foundation and the Guggenheim Foundation for financial support.

1 Introduction

This paper develops and estimates a quantitative macroeconomic framework that incorporates labor market frictions. Our starting point is the now conventional monetary DSGE model developed by Christiano, Eichenbaum and Evans (CEE, 2005), Smets and Wouters (SW, 2006) and others. We introduce labor market frictions with a variant of the Mortensen/Pissarides search and matching framework. This variant allows for staggered Nash wage bargaining, as in Gertler and Trigari (GT, 2006).

Our motivation is twofold. First, there are some compelling theoretical considerations. The recent vintage of monetary DSGE models typically has employment adjusting along intensive margin along with staggered nominal wage contracting. The latter feature, further, is important for the quantitative performance of the model: the wage stickiness helps the framework account for the volatility of hours. However, as a consequence, these frameworks are susceptible to Barro's (1977) argument that wages may not be allocational in this kind of environment, given that firm's and workers have an on-going relationship. If wages are not allocational then wage rigidity does not influence model dynamics. By contrast, in the model we present, firms adjust employment along the extensive margin. In this instance, wage rigidity affects employment by influencing the rate at which firms add new workers to their respective labor forces. As emphasized by Hall (2005a), since new workers have yet to form on-going relationships with firms, in this kind of setting the Barro critique does not apply.

Second, within the search and matching a literature, there is a debate over how well the baseline Mortensen and Pissarides framework can account for labor market volatility, or whether it may be necessary to introduce additional features such as wage rigidity, etc. (e.g. Shimer (2005), Hall (2005), Hagedorn and Manovskii (2006), Mortensen and Nagypal (2006)). A typical approach in the literature has been to develop a calibrated model, subject the model to productivity shocks, and then examine model moments against moments of the data, with various features such as wage rigidity or on-the-job search shut on and then shut off. We instead estimate a complete macroeconomic framework using Bayesian methods. Doing so allows us to formally evaluate the significance of different mechanisms such as wage rigidity to overall model performance. In addition, our full information procedure permits us to account for the complete range of shocks that hit the economy.

In section 2 we develop the model. The basic framework follows CEE and SW closely. The only significant difference involves the treatment of the labor market. As in GT, we incorporate a variation of Mortensen/Pissarides that retains the empirically appealing feature of Nash bargaining, but replace the assumption of period-by-period wage negotiations to allow for staggered multi-period wage contracting. Each period, only a subset of firms and workers negotiate a wage contract. Each wage bargain, further, is between a firm and its existing workforce: Workers hired in-between

contract settlements receive the existing wage.[1] We restrict the form of the wage contract to call for a fixed wage per period over an exogenously given horizon. Though it would be undoubtedly preferable to completely endogenize the contract structure, these restrictions are reasonable from an empirical standpoint. The payoff is a simple empirically appealing wage equation that is an intuitive generalization of the standard Nash bargaining outcome. In this instance, the key primitive parameter of the model is the average frequency of wage adjustment. Whereas in GT, we calibrated this parameter to existing evidence on wage contract length, here we are able to estimate it.

Another significant difference from GT, which was a purely real model, is that wage contracting is in nominal terms. However, as in CEE and SW, we allow for indexing of wages to past inflation and estimate the degree of indexing. This consideration is important for the following reasons. As indexing to past inflation becomes complete, nominal wage rigidity begins to approximate real wage rigidity. As Blanchard and Galí (2006) emphasizes, real wage rigidity complicates the short run output/inflation trade-off that the central bank faces, beyond what would arise from simple nominal rigidities.

In section 3 we describe our estimation procedure and then present the model estimates. We present a variety of diagnostics to evaluate the overall model performance and in particular the role of wage rigidity in our framework.

Before proceeding, we emphasize that there have been a number of papers related to ours. Trigari (2004) and Walsh (2005) were among the first to integrate a search and matching setup within a monetary DSGE model with nominal price rigidities. Blanchard and Galí (2006) develop a qualitative version of this model with a simple form of real wage rigidities. Christoffel et al. (2006) have also estimated a monetary DSGE model with labor market frictions and wage rigidity. They employ a setup with right-to-manage bargaining as in Trigari (2006), where ex post hours may be inefficient. Since part our interest is to address the Barro critique, we employ the setup of GT, which has efficient bargaining along with staggered wage setting. Thus, within our setup the employment of existing workers is fully efficient: wage rigidity affects hiring at the extensive margin. In addition, while Christoffel et al. (2006) model wage rigidity by introducing adjustment costs of wage changes for a representative firm, we do so by having staggered contracting. We also differ in the details of the precise model we estimate, as well as the exact estimation procedure and data.

2 The Model

As we discussed, the model is a variant of the conventional monetary DSGE framework. It has the key features that many have found useful for capturing the data. These include habit formation,

[1] In the language of Hall (2005), the existing contract wage provides a wage norm for the workers hired in between contracting periods.

costs of adjusting the flow of investment, variable capital utilization, nominal price and wage rigidities, and so on. The key changes involve the labor market. Rather than having hours vary on the intensive margin, we introduce variation on the extensive margin and unemployment. We do so by introducing search and matching in the spirit of Mortensen and Pissarides and others. Further, to introduce nominal wage rigidity, we use the staggered Nash bargaining approach of Gertler and Trigari (2006).

We note that in an earlier version of this paper we also allowed for variation in hours on the intensive margin. We drop this feature for two reasons. First, the evidence suggest that for the U.S., most of the cyclical variation in hours is on the intensive margin. Second, our estimates confirmed that the intensive margin was completely unimportant to cyclical variation, we estimated a Frisch elasticity close to zero, which is certainly in line with the microeconomic evidence.

There are three types of agents: households, wholesale firms, and retail firms. We use a representative family construct, similar to Merz (1995) in order to introduce complete consumption insurance. Production takes place at wholesale firms. These firms are competitive. They hire workers and negotiate wage contracts with them. Retail firms buy goods from wholesalers and then repackage them as final goods. Retailers are monopolistic competitors and set prices on a staggered basis. We separate retailers from wholesalers to keep the wage bargaining problem tractable.[2]

Finally, following SW, we introduce a number of exogenous shocks that corresponds exactly to the number of data series we consider in our estimation.

2.1 Households

There is a representative household with a continuum of members of measure unity. The number of family members currently employed is n_t. Employment is determined through a search and matching process that we describe shortly.

Accordingly, conditional on n_t, the household chooses consumption c_t, government bonds B_t, capital utilization z_t, investment i_t, and physical capital k_t^p to maximize the utility function

$$E_t \sum_{s=0}^{\infty} \beta^s \varepsilon_{t+s}^b \log\left(c_{t+s} - hc_{t+s-1}\right), \tag{1}$$

where h is the degree of habit persistence in consumption preferences and where ε_t^b is a preference shock with mean unity that obeys

$$\log \varepsilon_t^b = \rho^b \log \varepsilon_{t-1}^b + \varsigma_t^b, \tag{2}$$

and where all primitive innovations, including ς_t^b, are mean zero i.i.d. random variables.

[2] To keep the bargaining problem tractable, it is necessary to have constant returns at the firm level. This is to make the average and marginal worker the same, thus avoid bargaining spillovers among workers. Introducing staggered price setting requires that firms face downward sloping demand curves, implying differences between average and marginal.

Let Π_t be lump sum profits, T_t lump sum transfers, p_t the nominal price level, and r_t the one period nominal interest rate (specifically the central banks policy instrument). Then the household's budget constraint is

$$c_t + i_t + \frac{B_t}{p_t r_t} = w_t n_t + (1 - n_t) b_t + r_t^k z_t k_{t-1}^p + \Pi_t + T_t - a(z_t) k_{t-1}^p + \frac{B_{t-1}}{p_t}. \tag{3}$$

Households own capital and choose the capital utilization rate, z_t, which transforms physical capital into effective capital according to

$$k_t = z_t k_{t-1}^p. \tag{4}$$

Effective capital is rented to the firms at the rate r_t^k. The cost of capital utilization per unit of physical capital is $a(z_t)$. We assume that $z_t = 1$ in the steady state and $a(1) = 0$.

The physical capital accumulation equation is

$$k_t^p = (1 - \delta) k_{t-1}^p + \varepsilon_t^i \left(1 - s\left(\frac{i_t}{i_{t-1}}\right)\right) i_t, \tag{5}$$

where we assume $s(\gamma_a) = s'(\gamma_a) = 0$ and $s''(\gamma_a) > 0$ where γ_a is the economy's steady state growth rate.

ε_t^i is an investment specific technological shock affecting the efficiency with which consumption goods are transformed into capital. We assume ε_t^i follows the exogenous stochastic process

$$\log \varepsilon_t^i = \rho^i \log \varepsilon_{t-1}^i + \varsigma_t^i. \tag{6}$$

The first order necessary conditions yield:

(c_t)

$$\lambda_t = \frac{\varepsilon_t^b}{c_t - hc_{t-1}} - \beta h E_t \frac{\varepsilon_{t+1}^b}{c_{t+1} - hc_t}, \tag{7}$$

(B_t)

$$\lambda_t = r_t \beta E_t \left(\frac{\lambda_{t+1} p_t}{p_{t+1}}\right), \tag{8}$$

(z_t)

$$r_t^k = a'(z_t), \tag{9}$$

(i_t)

$$q_t^k \varepsilon_t^i \left(1 - s\left(\frac{i_t}{i_{t-1}}\right)\right) = q_t^k \varepsilon_t^i s'\left(\frac{i_t}{i_{t-1}}\right) \frac{i_t}{i_{t-1}} - \beta E_t q_{t+1}^k \varepsilon_{t+1}^i \frac{\lambda_{t+1}}{\lambda_t} s'\left(\frac{i_{t+1}}{i_t}\right) \left(\frac{i_{t+1}}{i_t}\right)^2 + 1, \tag{10}$$

(k_t^p)

$$q_t^k = \beta E_t \frac{\lambda_{t+1}}{\lambda_t} \left((1 - \delta) q_{t+1}^k + r_{t+1}^k z_{t+1} - a(z_{t+1})\right), \tag{11}$$

where q_t^k is the value of installed capital in consumption units.

Except for the treatment of labor market, the household sector is conventional.

2.2 Unemployment, Vacancies and Matching

At time t, each firm posts $v_t(i)$ vacancies in order to attract new workers and employs $n_t(i)$ workers. The total number of vacancies and employed workers are $v_t = \int_0^1 v_t(i)di$ and $n_t = \int_0^1 n_t(i)di$. All unemployed workers at t look for jobs. Our timing assumptions are such that unemployed workers who find a match go to work immediately within the period. Accordingly, the pool of unemployed workers searching for a job at t, u_t, is given by the difference between unity (the total population of workers) and the number of employed workers at the end of period one, n_{t-1}:

$$u_t = 1 - n_{t-1}. \tag{12}$$

The number of new hires or "matches", m_t, is a function of searching workers and vacancies, as follows:

$$m_t = \sigma_m u_t^{\sigma} v_t^{1-\sigma}. \tag{13}$$

The probability a firm fills a vacancy in period t, q_t, is given by

$$q_t = \frac{m_t}{v_t}. \tag{14}$$

Similarly, the probability a searching worker finds a job, s_t, is given by

$$s_t = \frac{m_t}{u_t}. \tag{15}$$

Both firms and workers take q_t and s_t as given.

Finally, each period, firms exogenously separate from a fraction $1-\rho$ of their existing workforce $n_{t-1}(i)$. Workers losing their job at time t are not allowed to search until next period. Accordingly, within our framework fluctuations in unemployment are due to cyclical variation in hiring as opposed to separations. Both Hall (2005b,c) and Shimer (2005a,b) present evidence in support of this phenomenon.

2.3 Wholesale Firms

Each period, wholesale firms produce output $y_t(i)$ using capital, $k_t(i)$, and labor, $n_t(i)$:

$$y_t(i) = [k_t(i)]^{\alpha} [a_t n_t(i)]^{1-\alpha}, \tag{16}$$

where a_t is a common labor-augmenting productivity factor. We assume $\varepsilon_t^a = a_t/a_{t-1}$ obeys the exogenous stochastic process

$$\log \varepsilon_t^a = (1-\rho^a)\log \varepsilon^a + \rho^a \log \varepsilon_{t-1}^a + \varsigma_t^a. \tag{17}$$

Note that the steady state value ε^a corresponds to the economy's growth rate γ_a. Thus, following Primiceri, Schaumburg, and Tambalotti (PST, 2006), we are allowing technology be non-stationary in levels, though stationary in growth rates.

For simplicity, we assume that capital is perfectly mobile across firms and that there is a competitive rental market in capital.

It is useful to define the hiring rate $x_t(i)$ as the ratio of new hires $q_t v_t(i)$ to the existing workforce $n_{t-1}(i)$:

$$x_t(i) = \frac{q_t v_t(i)}{n_{t-1}(i)}. \tag{18}$$

Observe that due to the law of large numbers the firm knows $x_t(i)$ with certainty at time t since it knows the likelihood q_t that each vacancy it posts will be filled. The hiring rate is thus effectively the firm's control variable.

The total workforce, in turn, is the sum of the number of surviving workers $\rho n_{t-1}(i)$ and new hires $x_t(i) n_{t-1}(i)$:

$$n_t(i) = \rho n_{t-1}(i) + x_t(i) n_{t-1}(i). \tag{19}$$

Equation (19) reflects the timing assumption that new hires go to work immediately.[3]

Let p_t^w be the relative price of intermediate goods, $w_t(i)$ the hourly wage, z_t, the rental rate of capital, and $\beta E_t \Lambda_{t,t+1}$ be the firm's discount rate, where the parameter β is the household's subjective discount factor and where $\Lambda_{t,t+1} = u'(c_{t+1})/u'(c_t)$. Then, the value of firm $F_t(i)$, may be expressed as:

$$F_t(i) = p_t^w y_t(i) - w_t(i) n_t(i) - \frac{\kappa_t}{2} x_t(i)^2 n_{t-1}(i) - r_t^k k_t(i) + \beta E_t \Lambda_{t,t+1} F_{t+1}(i), \tag{20}$$

with

$$\kappa_t = \kappa a_t. \tag{21}$$

As we noted earlier, wage dispersion is present, we replace the standard assumption of fixed costs of posting a vacancy with quadratic labor adjustment costs, given here by $\frac{\kappa_t}{2} x_t(i)^2 n_{t-1}(i)$. We allow adjustment costs to drift proportionately with productivity in order to maintain a balanced steady state growth path (otherwise adjustment cost become relatively less important as the economy grows.)

At any time, the firm maximizes its value by choosing the hiring rate (by posting vacancies) and its capital stock, given its existing employment stock, the probability of filling a vacancy, the rental rate on capital and the current and expected path of wages. If it is a firm that is able to renegotiate the wage, it bargains with its workforce over a new contract. If it is not renegotiating, it takes as given the wage at the previous period's level, as well the likelihood it will be renegotiating in the future.

We next consider the firm's hiring and capital rental decisions, and defer a bit the description of the wage bargain. The first order condition for capital is simply:

$$r_t^k = p_t^w \alpha \frac{y_t(i)}{k_t(i)} = p_t^w \alpha \frac{y_t}{k_t}. \tag{22}$$

[3] Blanchard and Gali (2006) use a similar timing.

Given Cobb-Douglas technology and perfect capital mobility, all firms choose the same capital/output ratio and in turn, the same capital/labor and labor/output ratios

Firms choose $n_t(i)$ by setting $x_t(i)$ or, equivalently, $v_t(i)$. The firm's hiring decision yields:

$$\kappa_t x_t(i) = p_t^w f_{nt}(i) - w_t(i) + \beta E_t \Lambda_{t,t+1} \partial F_{t+1}(i) / \partial n_t(i), \tag{23}$$

with

$$f_{nt}(i) = (1-\alpha)\frac{y_t(i)}{n_t(i)} = (1-\alpha)\frac{y_t}{n_t} = f_{nt}. \tag{24}$$

By making use of the envelope theorem to obtain $\partial F_t(i) / \partial n_{t-1}(i)$ and combining equations, we obtain

$$\kappa_t x_t(i) = p_t^w f_{nt} - w_t(i) + \beta E_t \Lambda_{t,t+1} \frac{\kappa_{t+1}}{2} x_{t+1}(i)^2 + \rho \beta E_t \Lambda_{t,t+1} \kappa_{t+1} x_{t+1}(i). \tag{25}$$

The hiring rate thus depends on the discounted stream of earnings and the saving on adjustment costs.

Finally, for the purpose of the wage bargain it is useful to define $J_t(i)$, the value to the firm of having another worker at time t after new workers have joined the firm, i.e., after adjustment costs are sunk. Differentiating $F_t(i)$ w.r.t. $n_t(i)$, taking $x_t(i)$ as given yields:

$$J_t(i) = p_t^w f_{nt} - w_t(i) + \beta E_t \Lambda_{t,t+1} \partial F_{t+1}(i) / \partial n_t(i). \tag{26}$$

By making use of the hiring rate condition and the relation for the evolution of the workforce, $J_t(i)$ may be expressed as expected average profits per worker net of the first period adjustment costs, with the discount factor accounting for future changes in workforce size:

$$J_t(i) = p_t^w f_{nt} - w_t(i) - \beta E_t \Lambda_{t,t+1} \frac{\kappa_{t+1}}{2} x_{t+1}(i)^2 + E_t \frac{n_{t+1}}{n_t}(i) \beta \Lambda_{t,t+1} J_{t+1}(i). \tag{27}$$

2.4 Workers

In this sub-section we develop an expression for a worker's surplus from employment, which is a critical determinant of the outcome of the wage bargain.

Let $V_t(i)$ be the value to a worker of employment at firm i and let U_t be the value of unemployment. These values are defined after hiring decisions at time t have been made and are in units of consumption goods. $V_t(i)$ is given by

$$V_t(i) = w_t(i) + \beta E_t \Lambda_{t,t+1} \left[\rho V_{t+1}(i) + (1-\rho) U_{t+1}\right] \tag{28}$$

To construct the value of unemployment, we first define $V_{x,t}$ as the average value of employment conditional on being a new worker at t:

$$V_{x,t} = \int_0^1 V_t(i) \frac{x_t(i) n_{t-1}(i)}{x_t n_{t-1}} di \tag{29}$$

where $x_t(i)n_{t-1}(i)$ is total new workers at firm i and $x_t n_{t-1}$ is total new workers at t.[4] Next, let b_t be the flow value from unemployment, including unemployment benefits, as well as other factors that can be measured in units of consumption goods. As before, let s_t be the probability of finding a job for the subsequent period. Then U_t may be expressed as

$$U_t = b_t + \beta E_t \Lambda_{t,t+1} \left[s_{t+1} V_{x,t+1} + (1 - s_{t+1}) U_{t+1} \right] \tag{30}$$

with

$$b_t = b\overline{k}_t^p$$

and where $\overline{k}_t^p$ is the economy-wide capital stock. We assume that b_t grows proportionately to $\overline{k}_t^p$ in order to maintain a balanced growth; otherwise b_t would become a smaller fraction of labor income as the economy grows. The value of unemployment thus depends on the current flow value b_t and the likelihood of being employed versus unemployed next period. Note that the value of finding a job next period for a worker that is currently unemployed is $V_{x,t+1}$, the average value of working next period conditional on being a new worker. That is, unemployed workers do not have a priori knowledge of which firms might be paying higher wages next period. They instead just randomly flock to firms posting vacancies.[5]

$H_t(i)$ and $H_{x,t}$ are given by:

$$H_t(i) = V_t(i) - U_t \tag{31}$$

and

$$H_{x,t} = V_{x,t} - U_t \tag{32}$$

It follows that:

$$H_t(i) = w_t(i) - b_t + \beta E_t \Lambda_{t,t+1} \left[\rho H_{t+1}(i) - s_{t+1} H_{x,t+1} \right] \tag{33}$$

2.5 Nash Bargaining and Wage Dynamics

As we noted earlier, we introduce staggered Nash wage bargaining, following Gertler and Trigari (2006), but with the following difference:.Because we now have a monetary model, we now allow for nominal wage contracting, though we also allow for the possibility of indexing to past inflation.

We introduce staggered multi-period wage contracting in a way that simplifies aggregation. In particular, each period a firm has a fixed probability $1 - \lambda$ that it may re-negotiate the wage.[6] This adjustment probability is independent of its history, making it unnecessary to keep track of

[4] $V_{x,t}$ is thus distinct from the unconditional average value of employment $V_t = \int_0^1 V_t(i) \frac{n_t(i)}{n_t} di$.

[5] There is accordingly no directed search. Note, however, that wage differentials across firms are only due to the differential timing of contracts, which is transitory. Thus, because a worker who arrives at a firm in the midst of an existing contract may expect a new one reasonably soon, the payoff from directed search may not be large.

[6] This kind of Poisson adjustment process is widely used in macroeconomic models with staggered price setting, beginning with Calvo (1983).

individual firms' wage histories. Thus, while how long an individual wage contract lasts is uncertain, the average duration is fixed at $1/(1-\lambda)$. The coefficient λ is thus a measure of the degree of wage stickiness. In our previous work we calibrated λ. Here we are able to estimate it.

Since we allow for the possibility of indexing to past inflation, π_{t-1}, the fraction λ of firms that cannot renegotiate their contract set their nominal wages $w_t^n(i)$ following the indexation rule:

$$w_t^n(i) = \overline{\gamma} w_{t-1}^n(i) \pi_{t-1}^{\gamma}, \tag{34}$$

where $\pi_t = p_t/p_{t-1}$, $\overline{\gamma} = \gamma_a \pi^{1-\gamma}$ and where $\gamma \epsilon [0,1]$ reflects the degree of indexing to past inflation. In the limiting case of no indexing, $\gamma = 0$, the contract simply calls for a fixed nominal wage over the life of the contract. We also estimate the parameter γ. The term $\overline{\gamma}$ in the indexing rule provides an adjustment for trend productivity growth and trend inflation.

Firms that enter a new wage agreement at t negotiate with the existing workforce, including the recent new hires. Due to constant returns, all workers are the same at the margin. The wage is chosen so that the negotiating firm and the marginal worker share the surplus from the marginal match. Given the symmetry to which we just alluded, all workers employed at the firm receive the same newly-negotiated wage.[7] When firms are not allowed to renegotiate the wage, all existing and newly hired workers employed at the firm receive the existing contracting wage (i.e. last period nominal wage adjusted for possibly indexing.)[8] As we discussed earlier, we appeal to scale economies in bargaining to rule out separate negotiations for worker who arrive in between contracting periods.[9] Of course, the newly hired workers recognize that they will be able to renegotiate the wage at the next round of contracting.

Let w_t^{n*} denote the wage of a firm that renegotiates at t. Given constant returns, all sets of renegotiating firms and workers at time t face the same problem, and thus set the same wage. As we noted earlier, the firm negotiates with the marginal worker over the surplus from the marginal match. We assume Nash bargaining, which implies that the contract wage w_t^{n*} is chosen to solve

$$\max H_t(r)^{\eta_t} J_t(r)^{1-\eta_t}, \tag{35}$$

given the indexing rule for wages, equation (34), with:

[7] To be clear, with constant returns, one could either think of the firm bargaining with each marginal worker individually or bargaining with a union that wishes to maximixe average worker surplus.

[8] The only research we are aware of that studies the relation between the wages of existing employees and new hires is Bewley (1999), who finds such a link. Other studies of the cyclical behavior of wages for new hires (e.g. Bils, 1985) do not examine the link with existing workers wages (due to data limitations) and thus do not speak to our hypothesis. We think that explaining the facts in these studies will require introducing heterogeneity into our framework.

[9] In addition to scale economies in bargaining, there are several complementary justifications for why hires in between contracts receive the existing contract wage. First, as we noted earlier, Bewley (1999) argues that internal equity constrains workers of similar productivity to receive similar wages. Second, Menzio and Moen (2006) show how asymmetric information can link the wages of new and existing workers. Third, consistent with Hall (2005a), one might interpret the existing contract wage as the "wage norm" for workers hired in between contracts.

$$\eta_t = \eta \varepsilon_t^{\eta}. \tag{36}$$

Here, the coefficient η_t reflects the workers' relative bargaining power, where ε_t^{η} is a shock to the bargaining power with mean unity that obeys:

$$\log \varepsilon_t^{\eta} = \rho^{\eta} \log \varepsilon_{t-1}^{\eta} + \varsigma_t^{\eta}. \tag{37}$$

We add ε_t^{η} in order to allow for a disturbance to the wage equation that we describe later.

The first order necessary conditions for the Nash bargaining solution are given by

$$\chi_t(r) J_t(r) = (1 - \chi_t(r)) H_t(r), \tag{38}$$

with

$$\chi_t(r) = \frac{\eta_t}{\eta_t + (1 - \eta_t) \Sigma_t(r) / \Delta_t}, \tag{39}$$

and

$$\Sigma_t(r) = E_t \sum_{s=0}^{\infty} \frac{n_{t+s}}{n_t}(r) (\lambda\beta)^s \Lambda_{t,t+s} \frac{p_t}{p_{t+s}} \left(\prod_{k=1}^{s} \overline{\gamma} \pi_{t+k-1}^{\gamma} \right), \tag{40}$$

$$\Delta_t = E_t \sum_{s=0}^{\infty} (\rho\lambda\beta)^s \Lambda_{t,t+s} \frac{p_t}{p_{t+s}} \left(\prod_{k=1}^{s} \overline{\gamma} \pi_{t+k-1}^{\gamma} \right). \tag{41}$$

One difference between our solution and the period-by-period Nash solution is that the relative weight $\chi_t(r)$ depends not only on the worker's bargaining power η_t, but also on $\Sigma_t(r)$ and Δ_t, the firm and worker's respective cumulative discount factors. This difference arises because the two agents have different horizons: unlike the worker, the firm cares about the implications of the contract for new workers as well as existing ones, whereas the worker only cares about his or her tenure at the job. Observe that each term s in the firm's cumulative discount factor depends on the expectation of the product of three factors: the employment size at firm $t+s$ relative to time t, $\frac{n_{t+s}}{n_t}(r)$, the probability the contract survives to $t+s$, λ^s, and the households' discount factor, $\beta^s \Lambda_{t,t+s}$. It is similar for the worker, except the survival probability ρ^s replaces the relative employment size. Since on average $\frac{n_{t+s}}{n_t}(r)$ exceeds ρ^s, the firm places relatively more weight on the future than does the worker. Since $\Sigma_t(r)/\Delta_t$ exceeds unity on average, $\chi_t(r)$ is less than η_t on average. Intuitively, since movements in the contract wage have a larger impact on discounted firm surplus than on worker surplus, the "horizon effect" works to raise the effective bargaining power of firms from $1-\eta$ to $1-\chi(r)$ on average. On the other hand, while the horizon effect (arising from the difference between η_t and $\chi_t(r)$) may be of interest from a theoretical perspective, it will turns out not to be of quantitative importance in this case.

Following Gertler and Trigari (2006), the solution to the bargaining problem leads to the following difference equation for the real wage $w_t^* = w_t^{n*}/p_t$:

$$\Delta_t w_t^* = w_t^o(r) + \rho\lambda\beta E_t \Lambda_{t,t+1} \Delta_{t+1} w_{t+1}^*, \tag{42}$$

where the forcing variable $w_t^o(r)$ can be thought of as the real "target" wage and is given by

$$\begin{aligned} w_t^o(r) &= \chi\left(p_t^w f_{nt} + \beta E_t \Lambda_{t,t+1} \frac{\kappa_{t+1}}{2} x_{t+1}(r)^2\right) \\ &\quad + (1-\chi)\left(b_t + s_{t+1}\beta E_t \Lambda_{t,t+1} H_{x,t+1}\right) + \Phi_t(r), \end{aligned} \tag{43}$$

with

$$\Phi_t(r) = \Omega_t(r) - \rho\beta E_t \Lambda_{t,t+1} \Omega_{t+1}(r),$$

$$\Omega_t(r) = (\chi_t(r) - \chi)\left[J_t(r) + H_t(r)\right].$$

In contrast to the standard case of period-by-period Nash bargaining, the wage set in the current period may apply in the future. Hence in this instance the contract wage w_t^* depends not only on the current target wage, but on the expected sequence of future target wages (as iterating forward equation (42) suggests.)

Observe that the target compensation, $w_t^o(r)$, has the same form that would emerge under period-by-period Nash bargaining. In particular, it is a convex combination of what a worker contributes to the match and what the worker loses by accepting a job, where the weights depend (indirectly) on the worker's relative bargaining power, η. The worker's contribution is the marginal product of labor plus the saving on adjustment costs. With our quadratic cost formulation, this saving is measured by $\frac{\kappa_t}{2} x_t(r)^2$. The foregone benefit from unemployment, in turn, is the flow value of unemployment, b_t , plus the expected discounted gain of moving from unemployment this period to employment next period, $s_t \beta \Lambda_{t,t+1} H_{x,t+1}$. The additional term $\Phi_t(r)$ reflects the horizon effect. It goes to zero as $\chi_t(r)$ converges to η_t.

In the limiting case of period-by-period wage contracting, $\lambda = 0$, the solution corresponds to the conventional Nash bargaining outcome: $w_t^* = w_t^o(r)$, with $\chi_t(r) = \eta_t$. With multi-period contracting, however, w_t^* depends on a discounted stream of future values of $w_t^o(r)$, where the latter depend on horizon adjusted bargaining weights.

Finally, the average nominal wage across workers is given by

$$w_t^n = \int_0^1 w_t^n(i) \frac{n_t(i))}{n_t} di. \tag{44}$$

By the law of large of numbers:

$$w_t^n = (1-\lambda) w_t^{*n} + \lambda \int_0^1 w_{t-1}^n(i) \overline{\gamma} \pi_{t-1}^{\gamma} \frac{n_t(i))}{n_t} di. \tag{45}$$

2.6 Retailers

There is a continuum of monopolistically competitive retailers indexed by j on the unit interval. Retailers buy intermediate goods from the wholesale firms described in the previous section. They in turn differentiate them with a technology that transforms one unit of intermediate goods into

one unit of retail goods, then re-sell them to the households. In addition, they set prices on a staggered basis.

Let $y_t(j)$ be the quantity of output sold by retailer j and let $p_t(j)$ be the nominal sale price. Final goods, denoted with y_t, are the following composite of individual retail goods:

$$y_t = \left[\int_0^1 y_t(j)^{\frac{1}{\varepsilon_t^p}} dj \right]^{\varepsilon_t^p}, \tag{46}$$

where ε_t^p depends on the elasticity of substitution and corresponds to the desired markup in the flexible price equilibrium.

We allow for shocks to the desired markup, as follows:

$$\log \varepsilon_t^p = (1-\rho^p) \log \varepsilon^p + \rho^p \log \varepsilon_{t-1}^p + \varsigma_t^p. \tag{47}$$

Cost minimization by households then leads to the follow demand curve facing each retailer j:

$$y_t(j) = \left(\frac{p_t(j)}{p_t} \right)^{\frac{\varepsilon_t^p}{1-\varepsilon_t^p}} y_t, \tag{48}$$

where p_t is the aggregate price index:

$$p_t = \left[\int_0^1 p_t(j)^{\frac{1}{1-\varepsilon_t^p}} dj \right]^{1-\varepsilon_t^p}. \tag{49}$$

Note that the aggregator given by equation (46) leads to an exogenous elasticity of demand given by $\frac{\varepsilon_t^p}{1-\varepsilon_t^p}$. In the estimation, we actually allow for a generalization due to Kimball (1995) that permits the elasticity to depend inversely on the firms relative market share. The endogenous elasticity introduces a price complementarity (or real rigidity) that makes it easier for the model to match the micro evidence on price adjustment (see Smets and Wouters, 2006). Since the presentation of the Kimball generalization is quite messy, we defer it to an appendix for now.

We assume "Calvo" staggering applies to price setting and we also allow for indexing. Let $1-\lambda^p$ be the probability a firm adjusts its price. Firms not adjusting their target price obey the following indexing rule:

$$p_t(j) = \overline{\gamma}^p p_{t-1}(j) \pi_{t-1}^{\gamma^p}, \tag{50}$$

where, in analogy to the case of wage setting, both γ^p and λ^p are parameters that we estimate, and where $\overline{\gamma}^p = \pi^{1-\gamma^p}$ is an adjustment for trend inflation. It follows that by the law of large numbers we can express the price index as

$$p_t = \left[(1-\lambda^p)(p_t^*)^{\frac{1}{1-\varepsilon_t^p}} + \lambda^p (\overline{\gamma}^p p_{t-1} \pi_{t-1}^{\gamma})^{\frac{1}{1-\varepsilon_t^p}} \right]^{1-\varepsilon_t^p}.$$

Re-optimizing retailers choose a target price, p_t^*, to maximize the following discounted stream of future profits:

$$E_t \sum_{s=0}^{\infty} (\lambda^p \beta)^s \Lambda_{t,t+s} \left[\frac{p_t^*}{p_{t+s}} \left(\prod_{k=1}^{s} \overline{\gamma}^p \pi_{t+k-1}^{\gamma^p} \right) - p_{t+s}^w \right] y_{t+s}(j), \tag{51}$$

where the sequence of discount factors depend on λ^p, the probability that the price remains fixed in the subsequent period. The first order condition for is given by

$$E_t \sum_{s=0}^{\infty} (\lambda^p \beta)^s \Lambda_{t,t+s} \left[\frac{1}{1-\varepsilon_{t+s}^p} \frac{p_t^*}{p_{t+s}} \left(\prod_{k=1}^{s} \overline{\gamma}^p \pi_{t+k-1}^{\gamma^p} \right) - \frac{\varepsilon_{t+s}^p}{1-\varepsilon_{t+s}^p} p_{t+s}^w \right] y_{t+s}(j) = 0.$$

By loglinearizing this condition, one can show that p_t^* depends on an expected discounted stream of the retailers nominal marginal cost, given by the nominal wholesale price $p_t p_t^w$.

By inverting the hiring condition derived earlier, one can obtain an expression for the retailers real marginal cost, p_t^w:

$$p_t^w = \frac{1}{f_{nt}} \left[w_t(i) + \kappa_t x_t(i) - \beta E_t \Lambda_{t,t+1} \frac{\kappa_{t+1}}{2} x_{t+1}(i)^2 - \beta E_t \Lambda_{t,t+1} \rho \kappa_{t+1} x_{t+1}(i) \right]. \tag{52}$$

Real marginal costs thus depend on unit labor costs, plus terms that correct for the adjustment costs of hiring workers.

Finally, observe that since we have normalized the relative price of final output at unity, the retailer's markup is given by $p_t / p_t p_t^w = 1/p_t^w$. Since final goods prices are sticky and wholesale prices are flexible, this markup will in general exhibit cyclical behavior, with the direction depending on the nature of the disturbances hitting the economy, as well as other features of the model.

2.7 Government

Monetary policy obeys the following simple Taylor rule:

$$\frac{r_t}{r} = \left(\frac{r_{t-1}}{r} \right)^{\rho^s} \left[\left(\frac{\pi_t}{\pi} \right)^{r_\pi} \left(\frac{y_t}{y_{nt}} \right)^{r_y} \right]^{(1-\rho^s)} \varepsilon_t^r, \tag{53}$$

where

$$\log \varepsilon_t^r = \rho^r \log \varepsilon_{t-1}^r + \varsigma_t^r. \tag{54}$$

Government spending obeys:

$$g_t = \left(1 - \frac{1}{\varepsilon_t^g} \right) y_t, \tag{55}$$

where

$$\log \varepsilon_t^g = (1-\rho^g) \ln \varepsilon^g + \rho^g \log \varepsilon_{t-1}^g + \varsigma_t^g. \tag{56}$$

2.8 Resource Constraint

The resource constraint divides output between consumption, investment and adjustment and utilization costs:

$$y_t = c_t + i_t + g_t + (\kappa_t/2) \int_0^1 x_t(i)^2 n_{t-1} di + a(z_t) k_{t-1}^p. \tag{57}$$

This completes the description of the model.

3 Wage and Hiring Dynamics

The key features of the model that differentiates it from the conventional monetary DSGE model involve wage and hiring dynamics. In this section, accordingly, we derive loglinear relationships for these variables. See the appendix for details.

By converting nominal to real wages and then loglinearizing the expression for the contract wage, we obtain the following loglinear difference equation for real hourly contract wage, $\widehat{w}_t^*$:

$$\widehat{w}_t^* = [(1-\rho\lambda\beta)\widehat{w}_t^o(r) + \rho\lambda\beta E_t\left(\widehat{\pi}_{t+1} - \gamma\widehat{\pi}_t + \widehat{\varepsilon}_{t+1}^a\right)] + \rho\lambda\beta E_t \widehat{w}_{t+1}^*. \tag{58}$$

The contract wage depends on the current and expected future path of the target wage $\widehat{w}_t^o(r)$ and terms that reflect adjustments for indexing.

Let $\widehat{w}_t^o$ be the "spillover free" target wage. Absent the horizon effect, it corresponds to the real wage that would arise if all firms were doing period-by-period Nash bargaining, i.e. the case of perfectly flexible wages. It is possible to show that:

$$\widehat{w}_t^o(r) = \widehat{w}_t^o + \frac{\tau_1}{1-\rho\lambda\beta} E_t\left(\widehat{w}_{t+1} - \widehat{w}_{t+1}^*\right) + \frac{\tau_2}{1-\rho\lambda\beta}\left(\widehat{w}_t - \widehat{w}_t^*\right), \tag{59}$$

with

$$\begin{aligned} \widehat{w}_t^o &= \varphi_{fn}\left(\widehat{p}_t^w + \widehat{f}_{nt}\right) + (\varphi_x + \varphi_s) E_t\widehat{x}_{t+1} + \varphi_s E_t\widehat{s}_{t+1} + \varphi_b\widehat{b}_t \\ &\quad + (\varphi_s + \varphi_x/2) E_t\widehat{\Lambda}_{t,t+1} + \varphi_\chi\left(\widehat{\chi}_t - \rho\beta\widehat{\chi}_{t+1}\right) + \varphi_s(1-\chi)^{-1} E_t\widehat{\chi}_{t+1}, \end{aligned} \tag{60}$$

and where

$$\begin{aligned} \tau_1 &= \varphi_s\Gamma(1-\rho\lambda\beta), \\ \tau_2 &= \left[\varphi_x\lambda - \varphi_\chi(1-\chi)x\Psi\Delta^{-1}\right]\epsilon\Sigma(w/a)(1-\rho\lambda\beta). \end{aligned} \tag{61}$$

As in GT, the parameters τ_1 and τ_2 in equation (59) reflect the influence of "spillovers" on the bargaining process: The second term on the right is direct spillover. If, everything else equal, $E_t\widehat{w}_{t+1}$ exceeds $E_t\widehat{w}_{t+1}^*$, opportunities are unusually good for workers expecting to move into employment next period, and vice-versa if $E_t\widehat{w}_{t+1}$ is below $E_t\widehat{w}_{t+1}^*$. By influencing the worker's outside option

in this way, the expected average market wage at $t+1$ induces a direct spillover effect on the wage bargain. An indirect spillover emerges in the third term because the hiring rate of the renegotiating firm affects the bargaining outcome. It does so by influencing both the firm's saving in adjustment costs and the horizon-adjusted bargaining weight (because it affects the firm's cumulative discount factor.) The difference between hiring rate $\widehat{x}_t(r)$ and average hiring rate $\widehat{x}_t$ depends positively on the difference between the average market wage $\widehat{w}_t$ and the contract wage $\widehat{w}_t^*$. Both spillover effects work to increase the degree of wage rigidity.

The loglinearized real wage index is in turn given by

$$\widehat{w}_t = (1-\lambda)\,\widehat{w}_t^* + \lambda\left(\widehat{w}_{t-1} - \widehat{\pi}_t + \gamma\widehat{\pi}_{t-1} - \widehat{\varepsilon}_t^a\right). \tag{62}$$

Combining these equations along with the relation for $\widehat{w}_t^o(r)$ then yields:

$$\widehat{w}_t = \gamma_b\left(\widehat{w}_{t-1} - \widehat{\pi}_t + \gamma\widehat{\pi}_{t-1} - \widehat{\varepsilon}_t^a\right) + \gamma\widehat{w}_t^o + \gamma_f\left(\widehat{w}_{t+1} + \widehat{\pi}_{t+1} - \gamma\widehat{\pi}_t + \widehat{\varepsilon}_{t+1}^a\right), \tag{63}$$

$$\begin{aligned}
\gamma_b &= (1+\tau_2)\,\phi^{-1}, \\
\gamma &= \varsigma\phi^{-1}, \\
\gamma_f &= (\rho\beta - \tau_1)\,\phi^{-1}, \\
\phi &= 1 + \tau_2 + \varsigma + \rho\beta - \tau_1, \\
\varsigma &= (1-\lambda)(1-\rho\lambda\beta)\,\lambda^{-1}.
\end{aligned}$$

with $\gamma_b + \gamma + \gamma_f = 1$. Note the forcing variable in the difference equation is the "spillover free" target wage $\widehat{w}_t^o$.

Due to staggered contracting, $\widehat{w}_t$ depends on the lagged wage $\widehat{w}_{t-1}$ as well as the expected future wage $E_t\widehat{w}_{t+1}$. Solving out for the reduced form will yield an expression that relates the wage to the lagged wage and a discounted stream of expected future values of $\widehat{w}_t^o$. Note that the spillover effects, measured by τ_1 and τ_2 work to raise the relative importance of the lagged wage (by raising γ_b) and reduce the importance of the expected future wage (by reducing γ_f). In this way, the spillovers work to raise the inertia in the evolution of the wage. In this respect, the spillover effects work in a similar (though not identical) way as to how real relative price rigidities enhance nominal price stickiness in monetary models with time-dependent pricing (see, for example, Woodford, 2003).

Note also that as we converge to $\lambda = 0$ (the case of period by period wage bargaining), both γ_b and γ_f go to zero), implying that $\widehat{w}_t$ simply tracks $\widehat{w}_t^o$ in this instance. Further, as we noted earlier, $\widehat{w}_t^o$, becomes identical to the wage in the flexible case. The model thus nests the conventional period-by-period wage bargaining setup.

Next, loglinearizing the equation for the hiring rate and aggregating yields:

$$\widehat{x}_t = \epsilon p^w\left(f_n/a\right)\left(\widehat{p}_t^w + \widehat{f}_{nt}\right) - \epsilon\left(w/a\right)\left(\widehat{w}_t\right) + \beta E_t\widehat{x}_{t+1} + \upsilon\beta E_t\widehat{\Lambda}_{t,t+1}, \tag{64}$$

with

$$\epsilon = (\kappa x)^{-1},$$

$$\upsilon = \rho + x/2.$$

The hiring rate thus depends on current and expected movements of the marginal product of labor relative to the wage. The stickiness in the wage due to staggered contracting, everything else equal, implies that current and expected movement in the marginal product of labor will have a greater impact on the hiring rate than would have been the case otherwise.

4 Model Estimation

4.1 Estimation Procedure

We consider seven variables in our estimation. To facilitate comparison with the literature, we employ the same seven quarterly series used in CEE and SW. Thus, the variable we use include: (1) per capita real GDP (real GDP/civilian non institutional population); (2) per capita real personal consumption expenditures (deflated with the GDP deflator and divided by civilian non institutional population); (3) per capita fixed private investment (deflated with the GDP deflator and divided by civilian non institutional population); (4) per capita hours (average weekly hours in non farm business sector)*(civilian employment 16 years & over)/(civilian non institutional population)[10]; (5) the real wage (compensation per hour in the nonfarm business sector/civilian non institutional population); (6) inflation (quarter to quarter growth rate of the GDP deflator, in percentage terms); (7) Federal Funds rate (expressed in quarterly terms.). The sample goes from 1960Q1 to 2005Q1.

We first log-linearize the model around a deterministic steady state. The appendix contains the complete log-linear model, as well as the steady state. The coefficients of the log-linear model depend on the primitive parameters of the model, as well as steady state values of variables. We use the steady state conditions of the model to solve out for a number of the parameters. The model also contains seven exogenous shocks, one corresponding to each variable.

There are twenty-two parameters, not including the parameters that characterize the exogenous shocks. Of the twenty-two, there are five new parameters that arise from our modification of the labor market. These include: the steady state flow value of unemployment as fraction of the contribution of the worker to the job, $\overline{b}$, the worker's relative bargaining power, η, the elasticity of new matches with respect to labor market tightness, σ, the job survival rate, ρ, and the steady state job finding rate, s.

Because we are not adding any new variables but are adding new parameters, in this first pass at the data we calibrate three of the five labor market parameters, for which there exists

[10] We note that we use hours per capita to measure labor input rather than employment per capita because the latter is nonstationary over the sample.

independent evidence. In particular,we choose the average monthly separation rate $1-\rho$ based on the observation that jobs last about two years and a half. Therefore, we set $\rho = 1 - 0.035$. We choose the elasticity of matches to unemployment, σ, to be equal to 0.5, the midpoint of the evidence typically cited in the literature.[11] In addition, this choice is within the range of plausible values of 0.5 to 0.7 reported by Petrongolo and Pissarides (2001) in their survey of the literature on the estimation of the matching function. We then set $s = 0.95$ to match recent estimates of the U.S. average monthly job finding rate (Shimer, 2005a).

The two labor market parameters we estimate are $\overline{b}$ and η. Both these parameters are critical determinants of the effective elasticity of labor supply along the extensive margin in the flexible wage case. As emphasized by Hagedorn and Manovskii (2006) and others, the closer $\overline{b}$ is to unity, the better able is the period-by-period Nash bargaining framework to capture unemployment and wage dynamics. For example, when $\overline{b}$ is very close to unity, there is little difference between the value of employment versus unemployment to the worker. Effectively, labor supply along the extensive margin is very elastic in this instance. The response of wages to a shift in the marginal value of a worker to the firm is dampened because in this case a small change in the wage has a large percentage effect on the relative gains to the worker from employment versus unemployment. Indeed, Hagedorn and Manovskii (2006) show that a model calibrated with $\overline{b}$ close to unity can capture the relative volatilities of labor market variables. This calibration however is quite controversial: Shimer (2005) argues in favor of 0.4 based on the interpretation of $\overline{b}$ as unemployment insurance, while Hall (2006) suggest 0.7, based on a broader interpretation that permits utility from leisure. Given the critical role of this parameter, it seems to be a prime candidate to estimate. The worker's bargaining parameter η is similarly important. The smaller is η, the less sensitive are wages to movements in the shadow value of labor, and thus the more sensitive is employment. Indeed, the Hagedorn and Manovskii calculation requires not only a high value of $\overline{b}$ but also a low value of η.

There are four "conventional" parameters that we calibrate: the discount factor, β, the depreciation rate, δ, the "share" parameter on capital in the Cobb-Douglas production function, α, the steady state ratio of government consumption to output $\frac{g}{y}$, and the sensitivity of firm demand elasticity to market share (the Kimball aggregator parameter), ξ. We use conventional values for all these parameters: $\beta = 0.99$, $\delta = 0.025$, $\alpha = 0.33$, $\frac{g}{y} = 0.2$, and $\xi = 10$. Note in contrast to the frictionless labor market model, the term $1-\alpha$ does not necessarily correspond to the labor share, since the latter will in general depend on the outcome of the bargaining process. However, here we simply follow convention by setting $\alpha = 0.33$ to facilitate comparison with the RBC literature.[12]

[11] The values for σ used in the literature are: 0.24 in Hall (2005a), 0.4 in Blanchard and Diamond (1989), Andolfatto (1994) and Merz (1995), 0.45 in Mortensen and Nagypal (2006), 0.5 in Hagedorn and Manovskii (2006), 0.5 in Farmer (2004), 0.72 in Shimer (2005a). See also a brief discussion in Mortensen and Nagypal (2006), p. 10, comparing their value of 0.45 to Shimer's one.

[12] Note that while $1-\alpha$ does not correspond to the labor share, α corresponds to the capital share.

Table 1: Calibrated parameters							
β	δ	α	ξ	σ	g/y	ρ	s
0.99	0.025	0.33	10	0.5	0.2	0.965	0.95

The conventional parameters we estimate include: the elasticity of the utilization rate to the rental rate of capital, η_z[13]; the elasticity of the capital adjustment cost function, η_k; the habit persistence parameter, h; the wage and price rigidity parameters, λ and λ^p; the wage and price indexing parameters, γ and γ_p; and the Taylor rule parameters, r_π and r_y. In addition, we estimate the first order autocorrelations of all the exogenous disturbances, as well as their respective standard deviations.

We estimate the model with Bayesian methods (see An and Schorfheide, 2007, for a comprehensive survey). We combine the likelihood function of the model, $L(\theta, Y)$, with uniform priors for the parameters to be estimated, $p(\theta)$, to obtain the posterior distribution: $L(\theta, Y)p(\theta)$. Draws from the posterior distribution are generated with the Random-Walk Metropolis Hastings (RWMH) algorithm.

4.2 Estimation Results

Table 2 reports the prior and the posterior estimates for each parameter. The table also reports the parameter configuration which maximizes the likelihood (called Max), along with the mean and the values at the 5 and 95 percent tails. Similarly, Table 3 presents the estimates of the prior and posterior distribution of the shock processes.

For the conventional parameters, for the most part we use the same priors as in Primiceri, Schaumburg and Tambalotti (2006), which in turn follow closely those employed by Smets and Wouters (2006). We proceed this way in order to facilitate comparison with the literature. One difference is that we impose a lower prior mean duration of price rigidity (three quarters as opposed to four.) Though, we note that in all instances the priors are reasonably loose. We differ from these authors, however, by proposing uniform priors over the serial correlations and standard deviations of the shock processes. We impose only diffuse priors here, based on the view that existing theory and evidence offers no guidance for the appropriate values of these parameters.

As noted, we estimate two new labor market parameters, η and $\overline{b}$. There is little direct evidence on the worker's bargaining power parameter η. In their survey paper, Mortensen and Nagypal propose a value of 0.5, which appears to reflect conventional thinking in the literature. As we noted earlier, there is considerable debate over the appropriate estimate of the relative flow value of unemployment parameter $\overline{b}$. Given this consideration and given how critical this parameter is, we simply impose a diffuse prior for it: uniform between zero and unity.

[13] We follow SW, define ψ_z such that $\eta_z = \frac{1-\psi_z}{\psi_z}$ and estimate ψ_z. When $\psi_z = 1$, it is very costly to change the capital utilization rate and the utilization rate does not vary. When $\psi_z = 0$, the marginal cost of changing the capital utilization rate is constant and, as a result, the rental rate of capital does not vary.

Table 2: Prior and posterior distribution of structural parameters

	Prior	Posterior			
Parameter	Distribution	Max	Mean	5%	95%
ψ_z	Beta (0.5,0.1)	0.698	0.681	0.559	0.791
η_k	Normal (4,1.5)	3.268	3.288	2.119	4.568
h	Beta (0.5,0.1)	0.700	0.745	0.665	0.817
η	Beta (0.5,0.1)	0.894	0.906	0.858	0.943
$\overline{b}$	Uniform (0,1)	0.754	0.728	0.649	0.797
γ	Uniform (0,1)	0.832	0.807	0.637	0.959
γ^P	Uniform (0,1)	0.0004	0.0162	0.0007	0.0475
λ	Beta (0.75,0.1)	0.739	0.735	0.649	0.819
λ^P	Beta (0.66,0.1)	0.855	0.848	0.799	0.890
ε^P	Normal (1.15,0.05)	1.412	1.410	1.351	1.470
r_π	Uniform (1.1,5)	2.296	2.297	1.907	3.164
r_y	Uniform (0,1)	0.367	0.367	0.288	0.641
ρ_s	Uniform (0,1)	0.805	0.805	0.765	0.876
γ_a	Uniform (1,1.5)	1.0040	1.0040	1.0033	1.0048

Note: For Beta and Normal distributions, the two numbers in parenthesis are respectively the mean and the st. dev. For the Uniform, the two numbers are the lower and the upper bound.

Since the values of the "conventional" parameters are consistent with other studies, we focus on the new parameters we consider here.[14] In particular, we estimate a very reasonable degree of wage rigidity. The estimate of λ is 0.74, which suggests a mean of 4 quarters between wage contracting periods.[15] The evidence from micro-data (Gottshalk, 2006), suggests a modal adjustment time of one year, though is silent (to our knowledge) about medians and means. In addition, the estimates suggest a high degree of indexing of wages to past inflation: The estimate of the indexing parameter is 0.83, which suggests a high degree of effective real wage rigidity.

[14] In Appendix D we present estimates of the Smets and Wouters (2006) model, which has been shown to fit the data nearly as well as a VAR. The precise version of the model we estimate is due to Primiceri, Schaumburg and Tambalotti (2006). Their formulation differs from SW only in some minor details.

[15] It is true that our estimate of the degree of price rigidity ($\lambda^p = 0.85$) is higher than that for wage rigidity. Several points: First, our estimate is similar to what one obtains with the Smets/Wouters model (see Appendix D). Second, our measure of inflation is based on the GDP deflator, which consists of producer prices. While our estimate of λ^p suggests a mean duration of six quarters, it suggests a median of about a year, which is consistent with the estimates of Steinsson and Nakamura (2007). Note that the Poisson process for price adjustment leads to the mean exceeding the median, since the constant hazard process suggests that some prices may not be adjusted for a long time.

The estimate of the key parameter $\overline{b}$, the flow value of unemployment, is 0.75. What it suggests is that in units of consumption goods, the flow value of unemployment is seventy-five percent of the worker's marginal value to the firm. This percentage is close to the value proposed by Hall (2006), who as we suggested earlier motivates $\overline{b}$ as reflecting not only unemployment insurance benefits but also utility gains from leisure. We also note that this estimate is well below the near unity value required for the conventional flexible model to account for the data. Thus, the data seem to prefer a combination of highly sticky wages and (effectively) inelastic labor supply along the extensive margin.

Finally, the estimate of the worker's bargaining power parameter is 0.89. This value lies above the range considered in the literature, typically 0.5 - 0.7. As noted earlier, however, there is virtually no direct evidence on what an appropriate value of this parameter should be. One possibility, is that within our framework it is very difficult to separately identify $\overline{b}$ and η. Both parameters enter the loglinear system via their respective impact on the steady state wage (see the appendix). It may be that to achieve a good identification of these parameters, we may need to introduce additional labor market information.

Table 3: Prior and posterior distribution of shock processes

	Prior	Posterior			
Parameter	Distribution	Max	Mean	5%	95%
ρ_a	Uniform (0,1)	0.074	0.078	0.010	0.170
ρ_r	Uniform (0,1)	0.094	0.143	0.033	0.265
ρ_b	Uniform (0,1)	0.788	0.714	0.581	0.832
ρ_i	Uniform (0,1)	0.628	0.569	0.452	0.672
ρ_p	Uniform (0,1)	0.842	0.826	0.762	0.880
ρ_η	Uniform (0,1)	0.225	0.231	0.105	0.362
ρ_g	Uniform (0,1)	0.999	0.996	0.991	0.999
σ_a	Uniform (0,30)	1.067	1.055	0.968	1.148
σ_r	Uniform (0,30)	0.231	0.231	0.209	0.254
σ_b	Uniform (0,30)	2.070	2.389	1.901	2.942
σ_i	Uniform (0,30)	7.255	7.804	5.330	10.64
σ_p	Uniform (0,30)	10.15	11.24	5.540	20.37
σ_η	Uniform (0,30)	0.603	0.596	0.521	0.672
σ_g	Uniform (0,30)	0.355	0.363	0.331	0.398

Note: The two numbers in parenthesis are the lower and the upper bound.

It is also worth noting that within our framework, the lion's share of the serial correlation in the real wages is accounted for by the wage contracting structure. The exogenous shock to the wage equation (modeled as a shock to bargaining power, η) has a first order serial coefficient of only 0.23.

We next consider the model without wage rigidity. Table 4 presents the parameter estimates and Table 5 presents the estimates of the shock processes in this case.

Table 4: Prior and posterior distribution of structural parameters with no wage rigidity

	Prior	Posterior			
Parameter	Distribution	Max	Mean	5%	95%
ψ_z	Beta (0.5,0.1)	0.863	0.852	0.781	0.913
η_k	Normal (4,1.5)	1.062	1.402	0.881	2.065
h	Beta (0.5,0.1)	0.838	0.844	0.804	0.877
η	Beta (0.5,0.1)	0.543	0.519	0.364	0.680
$\overline{b}$	Uniform (0,1)	0.991	0.992	0.985	0.998
γ	—	—	—	—	—
γ^P	Uniform (0,1)	0.0005	0.0323	0.0018	0.0897
λ	—	—	—	—	—
λ^P	Beta (0.66,0.1)	0.620	0.643	0.555	0.743
ε^P	Normal (1.15,0.05)	1.334	1.339	1.280	1.401
r_π	Uniform (1.1,5)	1.632	1.782	1.500	2.151
r_y	Uniform (0,1)	0.012	0.024	0.003	0.052
ρ_s	Uniform (0,1)	0.687	0.716	0.664	0.764
γ_a	Uniform (1,1.5)	1.0021	1.0019	1.0003	1.0038

Note: For Beta and Normal distributions, the two numbers in parenthesis are respectively the mean and the st. dev. For the Uniform, the two numbers are the lower and the upper bound.

The estimates of the conventional parameters do not change much. There is however now a large change is the estimates of the two key labor market parameters: $\overline{b}$ increases to 0.99 and η falls to 0.54. The former is close to the value we described earlier that Hagedorn and Manovskii used to argue that a flexible wage model could account for labor market volatility.[16] Our estimates confirm

[16] Hagedorn and Manvoski employ a slightly smaller value of $\overline{b}$, 0.95, as opposed to 0.99, and much smaller value of η, 0.05, as opposed to 0.5. Note that our prior on η is sufficiently loose so as not to exclude values well below 0.5.

that absent wage rigidity, it is necessary to have highly elastic labor supply along the extensive margin to account for the facts.[17]

Table 5: Prior and posterior distribution of shock processes with no wage rigidity

	Prior		Posterior		
Parameter	Distribution	Max	Mean	5%	95%
ρ_a	Uniform (0,1)	0.326	0.333	0.195	0.461
ρ_r	Uniform (0,1)	0.197	0.202	0.088	0.309
ρ_b	Uniform (0,1)	0.209	0.232	0.080	0.393
ρ_i	Uniform (0,1)	0.896	0.870	0.825	0.910
ρ_p	Uniform (0,1)	0.941	0.918	0.853	0.966
ρ_η	Uniform (0,1)	0.989	0.989	0.982	0.996
ρ_g	Uniform (0,1)	0.989	0.990	0.983	0.995
σ_a	Uniform (0,30)	1.097	1.120	1.015	1.231
σ_r	Uniform (0,30)	0.234	0.238	0.216	0.261
σ_b	Uniform (0,30)	2.422	2.630	2.034	3.337
σ_i	Uniform (0,30)	2.208	2.784	2.121	3.597
σ_p	Uniform (0,30)	1.397	1.712	1.136	3.007
σ_η	Uniform (0,30)	0.278	0.297	0.199	0.393
σ_g	Uniform (0,30)	0.352	0.357	0.326	0.391

Note: The two numbers in parenthesis are the lower and the upper bound.

One virtue of the Bayesian approach is that it is straightforward to compare the fit of the baseline model versus the model without wage rigidity Table 6 reports the marginal likelihoods for the two models.

Table 6: Log Marginal Likelihood

Baseline Model	Flex Wage Model
-1163	-1186

[17] The large value of $\overline{b}$ suggests a huge response of employment to changes in unemployment insurance benefits, as Hornstein, Krusell and Gianluca Violante (2005) and others have noted.

The baseline model clearly is preferred to the flex wage model. The difference in marginal likelihood is twenty loglikelihood points, which is a significant difference.[18]

Another way to assess how the model captures the data is to portray the autocovariance functions of the model variables against the data. Figure 1 reports this information. The solid line in each panel reports the data. The dashed lines are ninety percent posterior confidence intervals. In this draft, though, we have not taken into account the parameter uncertainty in computing these bands. Overall, the baseline model does well. Note that the empirical autocovariances functions for both hours and wages lie within the model standard error bands. The covariance of output with itself lies slightly below: However once parameter uncertainty it taken into account, we expect the data to lie between model standard error bands.

Figure 2 presents the autocovariance functions for the model without wage rigidity. Overall this model doesn't do as well. The autocovariances for wages and employment often lie outside the bands. The same is true for output.

We next illustrate the properties of the model economy by simulating the response to several key shocks. We analyze the role of wage rigidity, in particular, by examining the model with and without this feature. As Table 7 shows, the estimates suggest that the main driving force is the investment shock which, strictly speaking is interpretable as shock to investment-specific technological change. It accounts for more than half the variation in output growth on impact and more than forty percent at all horizons. This finding is consistent with both SW and PST. Next in importance is the disembodied productivity shock which accounts for roughly seventeen percent of the variation at horizons of a year or greater.

Table 7: Variance decomposition for output ($\Delta \log y$) at different horizons (in percentage)

	Horizons			
Shocks	on impact	1 year	4 years	long run
a shock - technology	2.5	17.4	16.8	16.8
r shock - monetary	6.2	4.8	5.0	5.0
b shock - preferences	12.4	10.0	10.3	10.3
i shock - investment	56.6	42.7	43.1	43.0
g shock - government	8.3	8.6	8.3	8.3
p shock - price markup	2.6	3.1	3.6	3.7
η shock - bargaining power	11.4	13.4	12.9	12.9

[18] Preliminary estimates suggest that our baseline model fits the data nearly as well as the SW model. Because the relatively marginal likelihoods appear sensitive to different assumptions about priors, we delay reporting this information until we have had more time to do robustness checks.

The recent literature on unemployment fluctuations that we alluded to in the introduction almost uniformly treats productivity shocks as the main driving force. Thus for purposes of comparison, we begin with this disturbance. In particular, Figure 3 illustrates the response of the model economy to a productivity shock. The thick line is the model with wage rigidity. The dotted line has wage rigidity turned off.[19] Notice that the response of output and employment is significantly greater with wage rigidity than without. Conversely, due to the staggered contracting the response of wages is much smoother. The smooth response of wages, of course, implies a larger response of profits to the technology shock than otherwise. This leads to a stronger response of output and employment relative to the flexible wage case.

We next turn to the investment shock, as portrayed in Figure 4. In contrast to the case of the productivity shock, in this instance the response of output and employment is very similar across the two models. Note, however, that the response of wages, w, and the price markup, μ_p, is quite different. In the flex wage model, in particular, the responses of these variables appear counterfactually large. In absolute value, the responses of wages and the price markup are three times the standard deviation of output. To our knowledge there do not exist series for wages or the price markup that display this kind of volatility relative to output.[20] Intuitively, the investment shock shifts output demand without directly affecting factor productivities. With nominal price rigidities, markups decline and employment adjusts to meet demand. In the staggered contracting model, because labor costs are sticky, the employment response is associated with only a modest drop in the markup. In the flex wage model, however, the sharp increase in wages implies that markups must drop sharply to accommodate the overall employment response. An added factor is that in the flex wage model the increase in wages generates a strong increase in capital utilization, z, which in turn enhances the employment response in this instance. The response of utilization in this case also appears large: nearly forty percent larger than the output response. In sum, while the flex model appear to account for the responses of output and hours to demand shocks as well as the baseline model, it requires counterfactually large responses of other variables in order to do so.

Finally, we can illustrate how wage rigidity influences the joint dynamics of output and inflation. Imagine that at time zero the lagged real wage is one percent above steady state, with all other predetermined variables at their steady state values. As Figure 5 shows, in the model with sticky wages, inflation at an annual rate jumps roughly 32 basis points. The half life of this increase is roughly five years. The overall impact of the shock is large and persistent, particularly given the size of the shock. The reason for the persistent impact on inflation is the sluggish movement in real wages, which in turn has a persistent impact on marginal cost. At the same time output slowly

[19] To shut off wage rigidity we simply set the probability that wages do not adjust, λ, equal to zero.

[20] It is true that the experiment here is conditional on the investment shock and the evidence to which we are alluding consists of unconditional moments. However, the investment shock accounts for nearly fifty percent of the variation in output growth within the model. If the flex wage model were true then we should observe relatively volatile behavior of real wages and price markups.

declines, reaching a maximum decline of 20 basis points about ten quarters out. The reason for the output decline is the rise in interest rates in response to inflation pressures. In the flexible wage case, there is no discernible impact on inflation or output.

Similarly, we can analyze how the contracting structure leads to persistent effects of lagged inflation. In our estimated model, lagged inflation influences current wages via indexing. (There is no estimated direct effect of lagged indexing on inflation.) Now suppose that we start the model off with lagged inflation 100 basis points above steady state. As Figure 6 shows, there is an immediate impact on the real wage, which increases by 0.8 percent. The increase in real wages in turn triggers movement in inflation and output similar to that occurring in Figure 5.

5 Concluding Remarks

We have developed and estimate a medium scale macroeconomic model that allows for unemployment and staggered nominal wage contracting. In contrast to most existing quantitative models, the employment of existing workers is efficient. Thus, the model is immune to the Barro's (1977) critique that models that rely on wage rigidity to have allocative effects in situations where firms and workers have on-going relationships ignore mutual gains from trade. In our model, in contrast, wage rigidity affects the hiring of new workers. The former is introduced via the staggered Nash bargaining setup of Gertler and Trigari (2006). A robust finding is that the model with rigidity provides a better description of the data than does a flexible wage version. In addition, we are able to quantify the effect of wage rigidity on output and inflation dynamics. We find that it is significant.

More work is necessary, however, to ensure a robust identification of the key labor market parameters. Our preliminary estimates of the degree of wage rigidity and the flow value of unemployment appear to be quite reasonable. The estimate of worker's bargaining power lies above conventional wisdom, though there is little direct evidence on what this parameter should be. One possibility is that it may be difficult to separately identify some of the key labor market parameters that influence employment volatility. Accordingly, it may be necessary to introduce additional labor market information.

References

[1] An, Sungbae and Frank Schorfheide (2007). "Bayesian Analysis of DSGE Models," *Econometric Reviews*, 26(2-4), 113-172.

[2] Bils, Mark (1985). "Real Wages Over the Business Cycle: Evidence from Panel Data," *Journal of Political Economy* 93 (4), 666-689.

[3] Binmore, Ken, Ariel Rubinstein and Asher Wolinsky (1986). "The Nash Bargaining Solution in Economic Modelling," *Rand Journal of Economics* 17, 176-188.

[4] Blanchard, Olivier and Jordi Galí (2006). "A New Keynesian Model with Unemployment," mimeo, MIT.

[5] Calvo, Guillermo (1983). "Staggered Prices in a Utility Maximizing Framework," *Journal of Monetary Economics* 12, 383-398.

[6] Christiano, Lawrence, Martin Eichenbaum and Charles Evans (2005). "Nominal Rigidities and the Dynamics Effects of a Shock to Monetary Policy," *Journal of Political Economy* 113 (1), 1-45.

[7] Christoffel, Kai, Keith Kuester and Tobias Linzert (2006). "Identifying the Role of Labor Markets for Monetary Policy in an Estimated DSGE Model," ECB WP 635.

[8] Cole, Harold and Richard Rogerson (1999). "Can the Mortensen-Pissarides Model Match the Business Cycle Facts?," *International Economic Review* 40, 933-960.

[9] Costain, James and Michael Reiter (2003). "Business Cycles, Unemployment Insurance and the Calibration of Matching Models," CESifo Working Paper No.1008.

[10] Flinn, Christopher (2006). "Minimum Wage Effects on Labor Market Outcomes under Search, Bargaining and Endogenous Contact Rates," *Econometrica* 74, 1013-1062.

[11] Gertler, Mark and Antonella Trigari (2006), "Unemployment Dynamics with Staggered Nash Wage Bargaining," mimeo, NYU

[12] Gottschalk, Peter (2005). "Downward Nominal Wage Flexibility: Real or Measurement Error?," *Review of Economics and Statistics* 87(3), 556-568.

[13] Hagedorn, Marcus and Iourii Manovskii (2006). "The Cyclical Behavior of Equilibrium Unemployment and Vacancies Revisited," mimeo, University of Pennsylvania.

[14] Hall, Robert (2006). "The Labor Market and Macro Volatility: A Nonstationary General Equilibrium Analysis," mimeo, Stanford University.

[15] Hall, Robert (2005a). "Employment Fluctuations with Equilibrium Wage Stickiness," *American Economic Review*, 95 (1), 50-65.

[16] Hall, Robert (2005b). "Employment Efficiency and Sticky Wages: Evidence from Flows in the Labor Market," *Review of Economics and Statistics* 87(3), 397-407.

[17] Hall, Robert (2005c). "Job Loss, Job Finding, and Unemployment in the U.S. Economy over the Past Fifty Years," *NBER Macroeconomics Annual*, M. Gertler and K. Rogoff editors, MIT Press, 101-137.

[18] Hornstein, Andreas, Per Krusell and Gianluca Violante (2005). "Unemployment and Vacancy Fluctuations in the Matching Model: Inspecting the Mechanism," *Economic Quarterly* (Federal Reserve Bank of Richmond) 91(3), 19-51.

[19] Hosios, Arthur (1990). "On the Efficiency of Matching and Related Models of Search and Unemployment," *Review of Economic Studies* 57(2), 279-298.

[20] Kennan, John (2006). "Private Information, Wage Bargaining and Employment Fluctuations," NBER Working Paper No. 11967.

[21] Kimball, Miles (1995). "The Quantitative Analytics of The Basic Neomonetarist Model," *Journal of Money, Credit and Banking* 27(4-2), 1241-1289.

[22] Krause, Michael and Thomas Lubik (2003). "The (Ir)relevance of Real Wage Rigidity in the New Keynesian Model with Search Frictions," *Journal of Monetary Economics*, forthcoming.

[23] Krause, Michael and Thomas Lubik (2005). "The Cyclical Upgrading of Labor and On-the-Job Search," mimeo, Johns Hopkins University.

[24] Menzio, Guido (2005). "High Frequency Wage Rigidity," mimeo, University of Pennsylvania.

[25] Menzio, Guido and Espen Moen (2006), "Incomplete Self-Enforcing Labor Contracts," mimeo, University of Pennsylvania.

[26] Merz, Monika (1995). "Search in the Labor Market and the Real Business Cycle," *Journal of Monetary Economics* 36, 269-300.

[27] Mortensen, Dale and Eva Nagypal (2006). "More on Unemployment and Vacancy Fluctuations," mimeo, Northwestern University.

[28] Mortensen, Dale and Christopher Pissarides (1994). "Job Creation and Job Destruction in the Theory of Unemployment," *Review of Economic Studies* 61, 397-415.

[29] Nakamura, Emi and Jon Steinsson (2007)."Five Facts About Prices: A Reevalutaion of Menu Cost Models", manuscript.

[30] Petrongolo, Barbara and Christopher Pissarides (2001). "Looking into the Black Box: A Survey of the Matching Function," *Journal of Economic Literature* 39(2), 390-431.

[31] Pissarides, Christopher (1985). "Short-Run Equilibrium Dynamics of Unemployment, Vacancies and Real Wages," *American Economic Review* 75 (4), 676-690.

[32] Pissarides, Christopher (2000). Equilibrium Unemployment Theory, The MIT Press.

[33] Primiceri, Giorgio, Ernst Schaumburg and Andrea Tambalotti (2006). "Intertemporal Disturbances", manuscript.

[34] Rotemberg, Julio (2006). "Cyclical Wages in a Search-and-Bargaining Model with Large Firms," mimeo, HBS.

[35] Shimer, Robert (2005a). "The Cyclical Behavior of Equilibrium Unemployment, Vacancies, and Wages: Evidence and Theory," *American Economic Review* 95 (1), 25-49.

[36] Shimer, Robert (2005b). "Reassessing the Ins and Outs of Unemployment," mimeo, University of Chicago.

[37] Shimer, Robert (2004). "The Consequences of Rigid Wages in Search Models," *Journal of the European Economic Association* (Papers and Proceedings) 2, 469-479.

[38] Shimer, Robert and Randall Wright. (2004), "Competitive Search Equilibrium with Asymmetric Information," mimeo.

[39] Smets, Frank and Raf Wouters (2006). "Shocks and Frictions in US Business Cycles: A Bayesian DGSE Approach," *American Economic Review*, forthcoming.

[40] Stole, Lars and Jeffrey Zwiebel (1996). "Intrafirm Bargaining under Non-Binding Contracts," *Review of Economic Studies* 63, 375-410.

[41] Taylor, John B. (1980). "Aggregate Dynamics and Staggered Contracts," *Journal of Political Economy* 88, 1-23.

[42] Trigari, Antonella, (2004). "Equilibrium Unemployment, Job Flows and Inflation Dynamics," ECB WP 304.

[43] Trigari, Antonella, (2006). "The Role of Search Frictions and Bargaining for Inflation Dynamics," IGIER WP 304.

[44] Taylor, John B. (1999). "Staggered Price and Wage Setting in Macroeconomics," *Handbook of Macroeconomics,* Volume 1, J.B. Taylor and M. Woodford editors, Elsevier.

[45] Yashiv, Eran (2005). "Forward-Looking Hiring Behavior and the Dynamics of the Aggregate Labor Market," mimeo, Tel Aviv University.

[46] Walsh, Carl (2005). "Labor Market Search, Sticky Prices and Interest Rate Rules," *Review of Economic Dynamics* 8, 829-849.

[47] Woodford, Michael (2003). Interest and Prices, Princeton University Press.

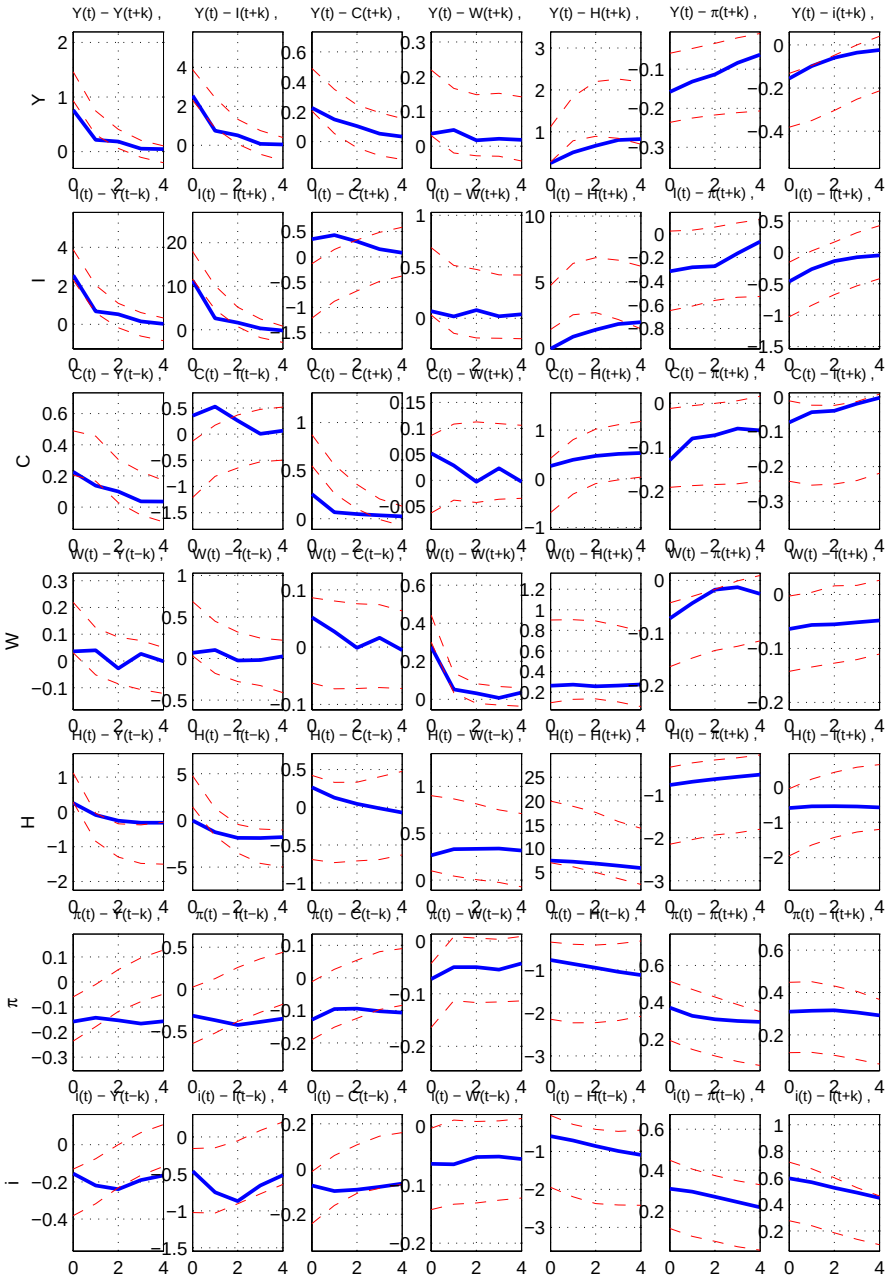

Figure 1: Model against data - autocovariance function

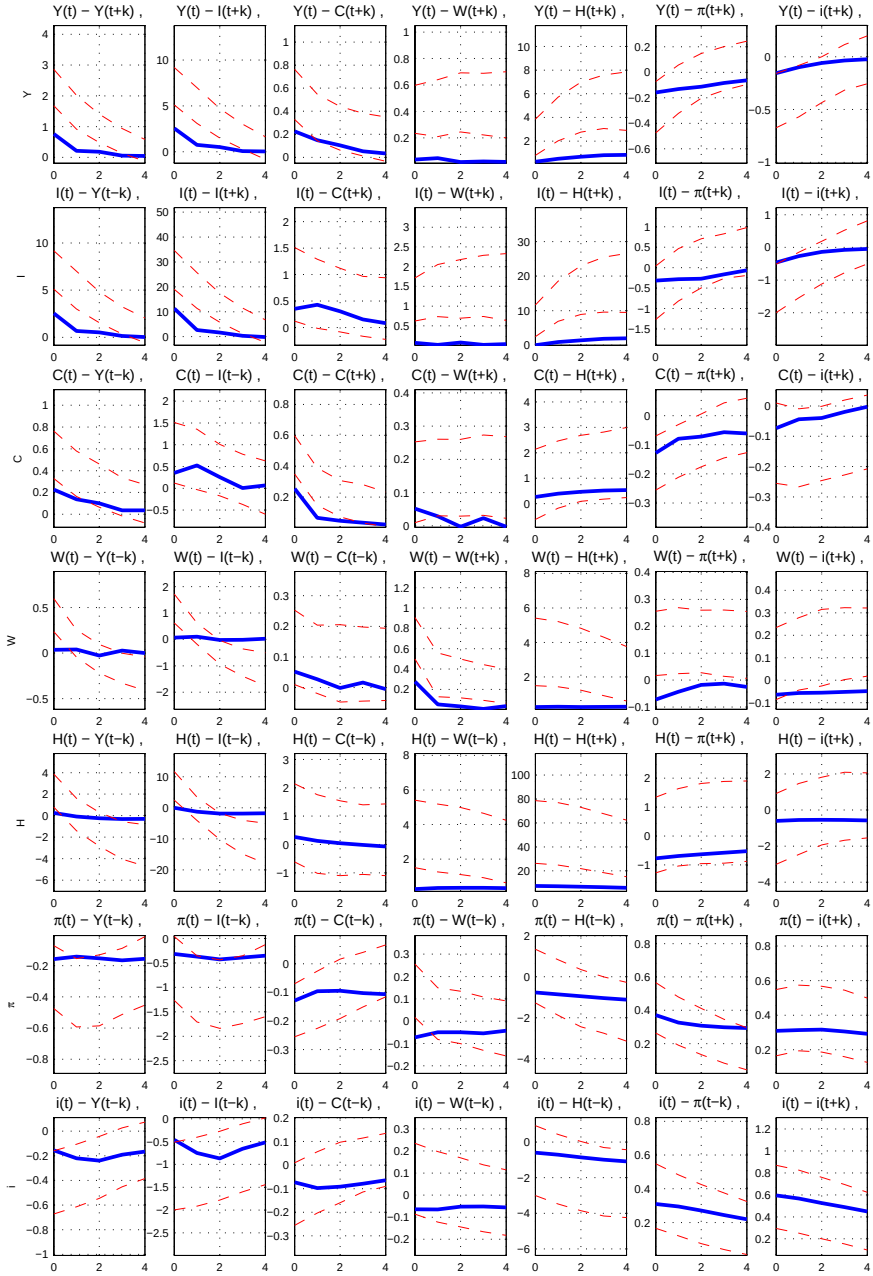

Figure 2: Flex wage model against data - autocovariance function

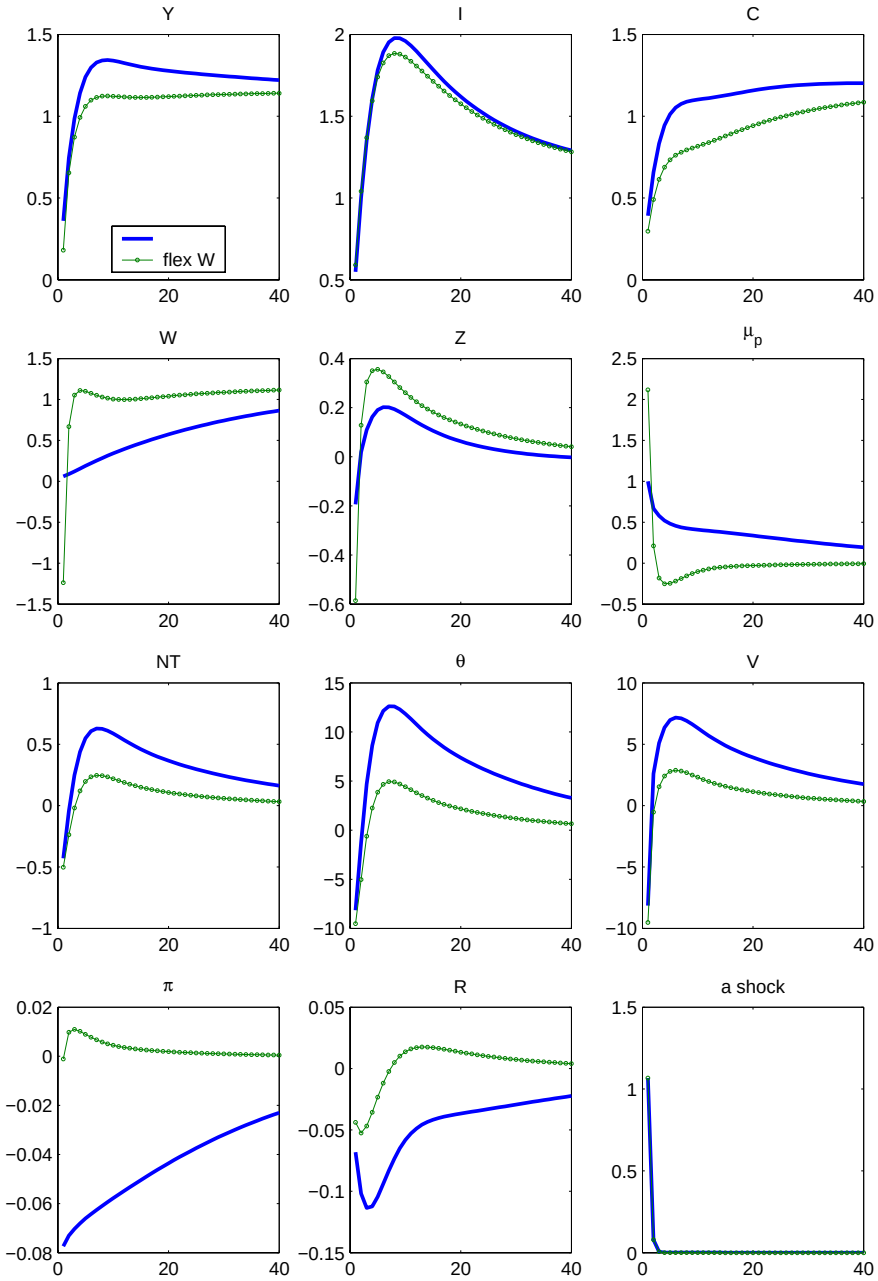

Figure 3: Impulse responses to a one std. dev. shock to technology (ε^a)

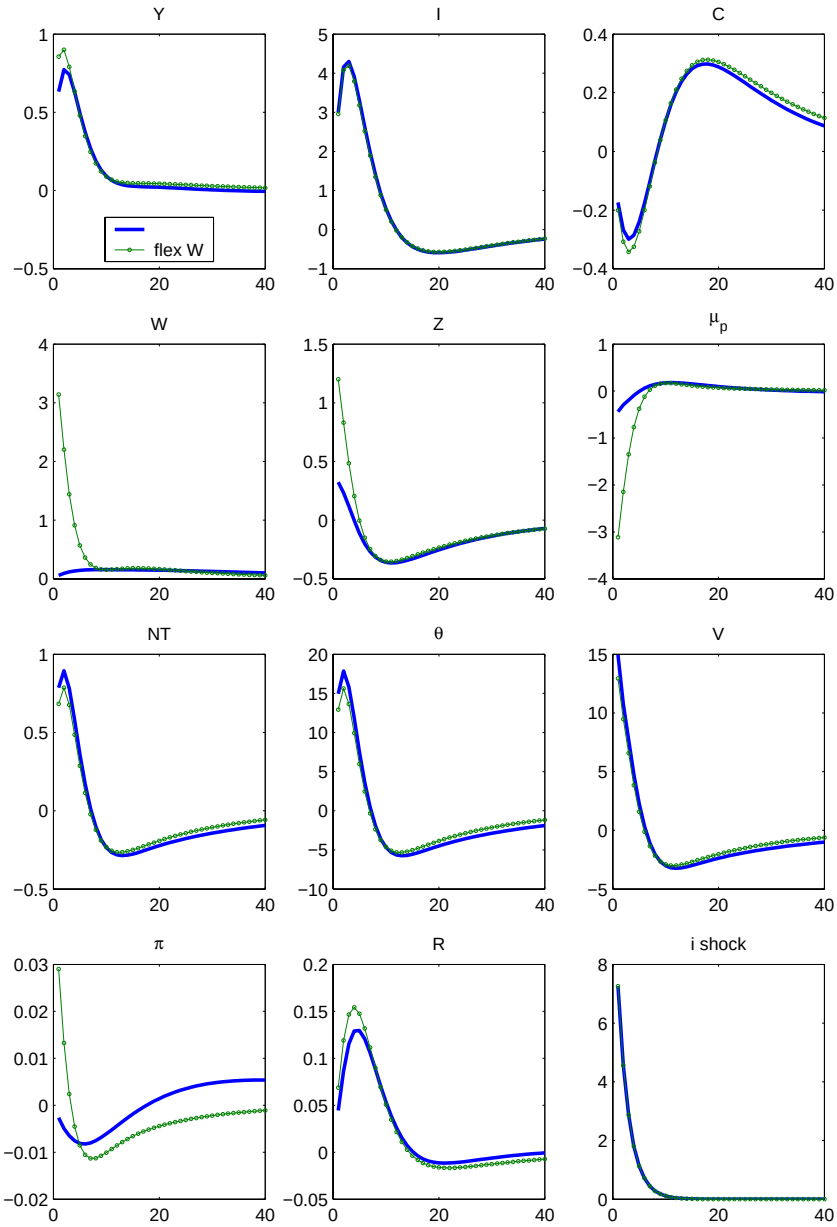

Figure 4: Impulse responses to a one std. dev shock to investment (ε^i)

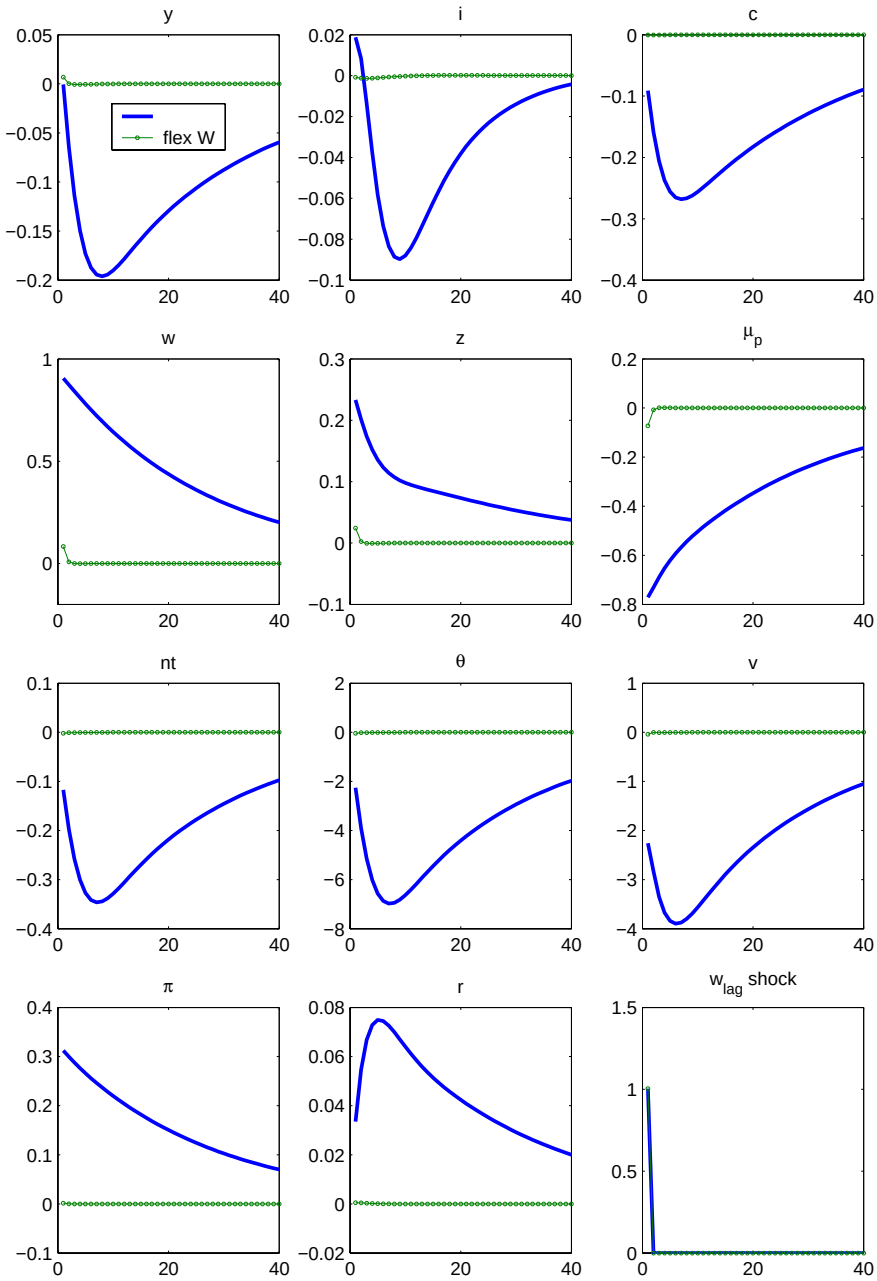

Figure 5: Impulse responses to a unit shock to w_{t-1}

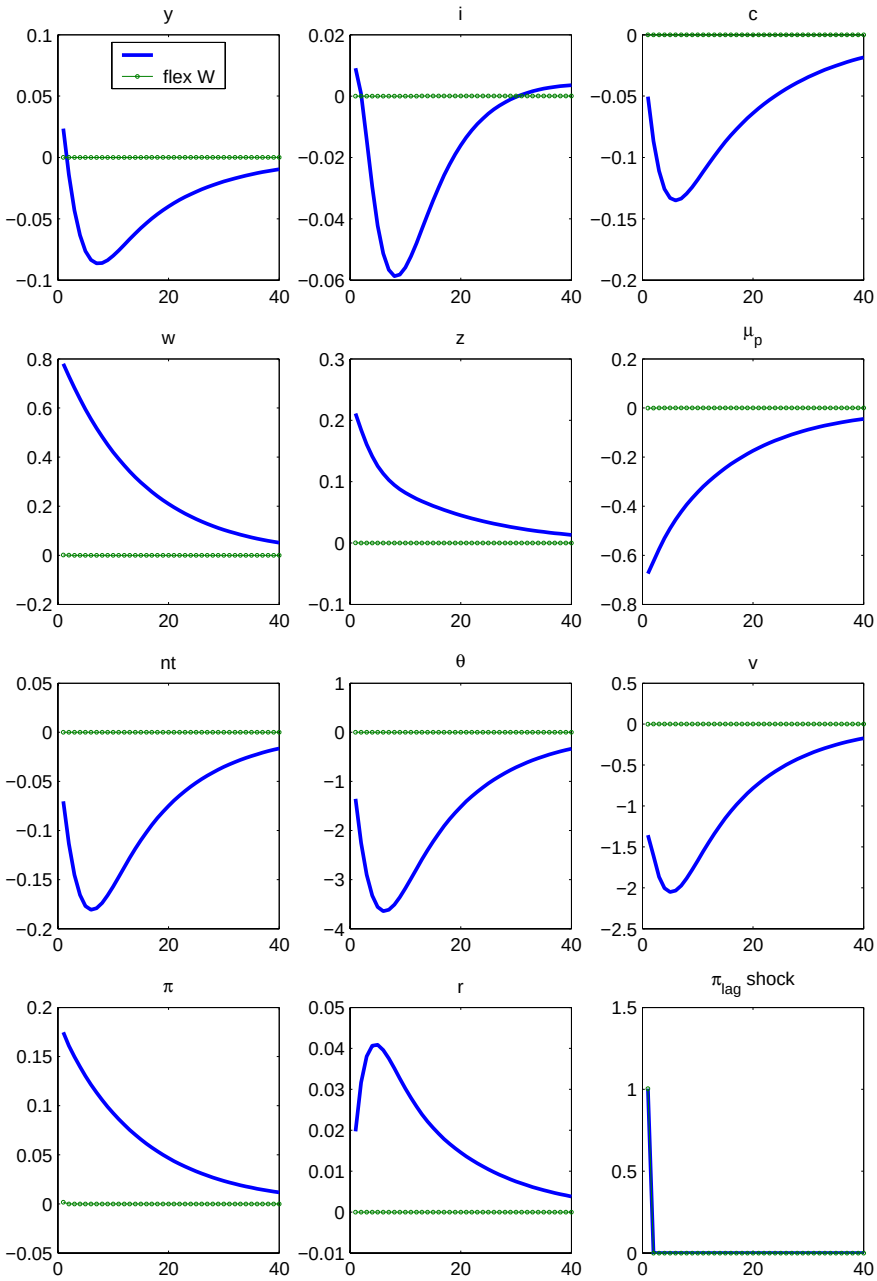

Figure 6: Impulse responses to a unit shock to π_{t-1}

APPENDIX A

A1. Sum of expected future wages for a worker at a firm renegotiating at t, $W_t^w(r)$

- Let $W_t^w(r)$ denote the discounted sum of expected future wages to be received by a worker over the life of the relationship at a firm renegotiating at t:

$$\begin{aligned} W_t^w(r) &= E_t \sum_{s=0}^{\infty} (\rho\beta)^s \Lambda_{t,t+s} w_{t+s}(r) \\ &= w_t(r) + (\rho\beta) E_t \Lambda_{t,t+1} w_{t+1}(r) + (\rho\beta)^2 E_t \Lambda_{t,t+2} w_{t+2}(r) + ... \end{aligned}$$

- At a firm renegotiating at time t, the current and future expected wages are given by:

$$w_t(r) = \frac{w_t^{*n}}{p_t}$$

$$E_t w_{t+1}(r) = E_t \frac{1}{p_{t+1}} \left[\lambda w_t^{*n} \overline{\gamma} \pi_t^{\gamma} + (1-\lambda) w_{t+1}^{*n}\right]$$

$$\begin{aligned} E_t w_{t+2}(r) &= E_t \frac{1}{p_{t+2}} \left\{\lambda \left[\lambda w_t^{*n} \overline{\gamma} \pi_t^{\gamma} + (1-\lambda) w_{t+1}^{*n}\right] \overline{\gamma} \pi_{t+1}^{\gamma} + (1-\lambda) w_{t+2}^{*n}\right\} \\ &= E_t \frac{1}{p_{t+2}} \left[\lambda^2 w_t^{*n} \overline{\gamma} \pi_{t+1}^{\gamma} \overline{\gamma} \pi_t^{\gamma} + \lambda(1-\lambda) w_{t+1}^{*n} \overline{\gamma} \pi_{t+1}^{\gamma} + (1-\lambda) w_{t+2}^{*n}\right] \end{aligned}$$

and so on....

- Using these expressions, we can write:

$$\begin{aligned} W_t^w(r) &= \frac{w_t^{*n}}{p_t} \\ &+ (\rho\beta) E_t \Lambda_{t,t+1} \frac{1}{p_{t+1}} \left[\lambda w_t^{*n} \overline{\gamma} \pi_t^{\gamma} + (1-\lambda) w_{t+1}^{*n}\right] \\ &+ (\rho\beta)^2 E_t \Lambda_{t,t+2} \frac{1}{p_{t+2}} \left[\lambda^2 w_t^{*n} \overline{\gamma} \pi_t^{\gamma} \overline{\gamma} \pi_{t+1}^{\gamma} + \lambda(1-\lambda) w_{t+1}^{*n} \overline{\gamma} \pi_{t+1}^{\gamma} + (1-\lambda) w_{t+2}^{*n}\right] \\ &+ ... \end{aligned}$$

$$\begin{aligned} W_t^w(r) &= w_t^* \\ &+ (\rho\beta) E_t \Lambda_{t,t+1} \left[\lambda w_t^* \frac{p_t}{p_{t+1}} \overline{\gamma} \pi_t^{\gamma} + (1-\lambda) w_{t+1}^*\right] \\ &+ (\rho\beta)^2 E_t \Lambda_{t,t+2} \left[\lambda^2 w_t^* \frac{p_t}{p_{t+2}} \overline{\gamma} \pi_t^{\gamma} \overline{\gamma} \pi_{t+1}^{\gamma} + \lambda(1-\lambda) w_{t+1}^* \frac{p_{t+1}}{p_{t+2}} \overline{\gamma} \pi_{t+1}^{\gamma} + (1-\lambda) w_{t+2}^*\right] \\ &+ ... \end{aligned}$$

- Collecting terms:

$$\begin{aligned} W_t^w(r) &= \left[1+(\rho\lambda\beta)\Lambda_{t,t+1}\frac{p_t}{p_{t+1}}\overline{\gamma}\pi_t^{\gamma}+(\rho\lambda\beta)^2\Lambda_{t,t+2}\frac{p_t}{p_{t+2}}\overline{\gamma}\pi_t^{\gamma}\overline{\gamma}\pi_{t+1}^{\gamma}+...\right]w_t^* \\ &+(1-\lambda)(\rho\beta)E_t\Lambda_{t,t+1}\left[1+(\rho\lambda\beta)\Lambda_{t+1,t+2}\frac{p_{t+1}}{p_{t+2}}\overline{\gamma}\pi_{t+1}^{\gamma}+...\right]w_{t+1}^* \\ &+(1-\lambda)(\rho\beta)^2E_t\Lambda_{t,t+2}\left[1+(\rho\lambda\beta)\Lambda_{t+2,t+3}\frac{p_{t+2}}{p_{t+3}}\overline{\gamma}\pi_{t+2}^{\gamma}+....\right]w_{t+2}^* \end{aligned}$$

- Letting

$$\Delta_t = E_t\sum_{s=0}^{\infty}(\rho\lambda\beta)^s\Lambda_{t,t+s}\frac{p_t}{p_{t+s}}\left(\prod_{k=1}^{s}\overline{\gamma}\pi_{t+k-1}^{\gamma}\right)$$

  we have

$$\begin{aligned} W_t^w(r) &= \Delta_t w_t^* \\ &+(1-\lambda)(\rho\beta)E_t\Lambda_{t,t+1}\Delta_{t+1}w_{t+1}^* \\ &+(1-\lambda)(\rho\beta)^2E_t\Lambda_{t,t+2}\Delta_{t+2}w_{t+2}^* \\ &+... \end{aligned}$$

- Finally, rearranging:

$$W_t^w(r) = \Delta_t w_t^* + (1-\lambda)E_t\sum_{s=1}^{\infty}(\rho\beta)^s\Lambda_{t,t+s}\Delta_{t+s}w_{t+s}^*$$

A2. Sum of expected future wages for a firm renegotiating at t, $W_t^f(r)$

- Let $W_t^f(r)$ denote the discounted sum of expected future wage payments by a firm renegotiating at t over both the existing contract and subsequent contracts:

$$\begin{aligned} W_t^f(r) &= E_t\sum_{s=0}^{\infty}\frac{n_{t+s}}{n_t}(r)\beta^s\Lambda_{t,t+s}w_{t+s}(r) \\ &= w_t(r)+E_t\frac{n_{t+1}}{n_t}(r)\beta\Lambda_{t,t+1}w_{t+1}(r)+E_t\frac{n_{t+2}}{n_t}(r)\beta^2\Lambda_{t,t+2}w_{t+2}(r)+... \end{aligned}$$

- Using the expressions for the future expected wages, we can write:

$$\begin{aligned} W_t^f(r) &= \frac{w_t^{*n}}{p_t} \\ &+E_t\frac{n_{t+1}}{n_t}(r)\beta\Lambda_{t,t+1}\frac{1}{p_{t+1}}\left[\lambda w_t^{*n}\overline{\gamma}\pi_t^{\gamma}+(1-\lambda)w_{t+1}^{*n}\right] \\ &+E_t\frac{n_{t+2}}{n_t}(r)\beta^2\widetilde{\Lambda}_{t,t+2}\frac{1}{p_{t+2}}\left[\lambda^2\frac{w_t^{*n}}{a_t}\frac{\overline{\gamma}\pi_t^{\gamma}\overline{\gamma}\pi_{t+1}^{\gamma}}{\varepsilon_{t+1}^a\varepsilon_{t+2}^a}+\lambda(1-\lambda)\frac{w_{t+1}^{*n}}{a_{t+1}}\frac{\overline{\gamma}\pi_{t+1}^{\gamma}}{\varepsilon_{t+2}^a}+(1-\lambda)\frac{w_{t+2}^{*n}}{a_{t+2}}\right] \\ &+... \end{aligned}$$

$$
\begin{aligned}
W_t^f(r) &= w_t^* \\
&+E_t \frac{n_{t+1}}{n_t}(r)\,\beta\Lambda_{t,t+1}\left[\lambda w_t^* \frac{p_t}{p_{t+1}}\overline{\gamma}\pi_t^{\gamma} + (1-\lambda)\,w_{t+1}^*\right] \\
&+E_t \frac{n_{t+2}}{n_t}(r)\,\beta^2\Lambda_{t,t+2}\left[\lambda^2 w_t^* \frac{p_t}{p_{t+2}}\overline{\gamma}\pi_t^{\gamma}\overline{\gamma}\pi_{t+1}^{\gamma} + \lambda(1-\lambda)\,w_{t+1}^* \frac{p_{t+1}}{p_{t+2}}\overline{\gamma}\pi_{t+1}^{\gamma} + (1-\lambda)\,w_{t+2}^*\right] \\
&+...
\end{aligned}
$$

- Collecting terms:

$$
\begin{aligned}
W_t^f(r) &= \left[1 + (\rho\lambda\beta)\,\Lambda_{t,t+1}\frac{p_t}{p_{t+1}}\overline{\gamma}\pi_t^{\gamma} + (\rho\lambda\beta)^2\,\Lambda_{t,t+2}\frac{p_t}{p_{t+2}}\overline{\gamma}\pi_t^{\gamma}\overline{\gamma}\pi_{t+1}^{\gamma} + ...\right] w_t^* \\
&+ (1-\lambda)\,E_t \frac{n_{t+1}}{n_t}(r)\,\beta\Lambda_{t,t+1}\left[1 + (\rho\lambda\beta)\,\Lambda_{t+1,t+2}\frac{p_{t+1}}{p_{t+2}}\overline{\gamma}\pi_{t+1}^{\gamma} + ...\right] w_{t+1}^* \\
&+ (1-\lambda)\,E_t \frac{n_{t+2}}{n_t}(r)\,\beta^2\Lambda_{t,t+2}\left[1 + (\rho\lambda\beta)\,\Lambda_{t+2,t+3}\frac{p_{t+2}}{p_{t+3}}\overline{\gamma}\pi_{t+2}^{\gamma} + ...\right] w_{t+2}^*
\end{aligned}
$$

- Letting

$$
\Sigma_t(r) = E_t \sum_{s=0}^{\infty} \frac{n_{t+s}}{n_t}(r)\,(\lambda\beta)^s\,\Lambda_{t,t+s}\frac{p_t}{p_{t+s}}\left(\prod_{k=1}^{s}\overline{\gamma}\pi_{t+k-1}^{\gamma}\right)
$$

  we have

$$
\begin{aligned}
W_t^f(r) &= \Sigma_t(r)\,w_t^* \\
&+ (1-\lambda)\,E_t \frac{n_{t+1}}{n_t}(r)\,\beta\Lambda_{t,t+1}\Sigma_{t+1}(r)\,w_{t+1}^* \\
&+ (1-\lambda)\,E_t \frac{n_{t+2}}{n_t}(r)\,\beta^2\Lambda_{t,t+2}\Sigma_{t+2}(r)\,w_{t+2}^* \\
&+...
\end{aligned}
$$

- Finally, rearranging

$$
W_t^f(r) = \Sigma_t(r)\,w_t^* + (1-\lambda)\,E_t \sum_{s=1}^{\infty} \frac{n_{t+s}}{n_t}(r)\,\beta^s\Lambda_{t,t+s}\Sigma_{t+s}w_{t+s}^*
$$

A3. Worker surplus at a firm renegotiating at t, $H_t(r)$

- The worker surplus at a firm renegotiating at t is

$$
\begin{aligned}
H_t(r) &= w_t(r) - b_t + \rho\beta E_t\Lambda_{t,t+1}H_{t+1}(r) - s_{t+1}\beta E_t\Lambda_{t,t+1}H_{x,t+1} \\
&= W_t^w(r) - E_t \sum_{s=0}^{\infty} (\rho\beta)^s\,\Lambda_{t,t+s}\left[b_{t+s} + s_{t+s+1}\beta\Lambda_{t+s,t+s+1}H_{x,t+s+1}\right]
\end{aligned}
$$

- Substituting the expression for $W_t^w(r)$, we get

$$\begin{aligned} H_t(r) &= \Delta_t w_t^* - E_t \sum_{s=0}^{\infty} (\rho\beta)^s \Lambda_{t,t+s} \left[b_{t+s} + s_{t+s+1} \beta \Lambda_{t+s,t+s+1} H_{x,t+s+1} \right. \\ &\quad \left. - (1-\lambda)(\rho\beta) \Lambda_{t+s,t+s+1} \Delta_{t+s+1} w_{t+s+1}^* \right] \end{aligned}$$

A4. Firm marginal surplus for a firm renegotiating at t, $J_t(r)$

- The value of a marginal worker for a firm renegotiating at t is

$$\begin{aligned} J_t(r) &= p_t^w f_{nt} - w_t(r) + \beta E_t \Lambda_{t,t+1} \frac{\kappa_{t+1}}{2} x_{t+1}(r)^2 + \rho\beta E_t \Lambda_{t,t+1} J_{t+1}(r) \\ &= E_t \sum_{s=0}^{\infty} (\rho\beta)^s \Lambda_{t,t+s} \left[p_{t+s}^w f_{nt+s} + \beta \Lambda_{t+s,t+s+1} \frac{\kappa_{t+s+1}}{2} x_{t+s+1}(r)^2 \right] - W_t^w(r) \end{aligned}$$

- Substituting the expression for $W_t^w(r)$, we get

$$\begin{aligned} J_t(r) &= E_t \sum_{s=0}^{\infty} (\rho\beta)^s \Lambda_{t,t+s} \left[p_{t+s}^w f_{nt+s} + \beta \Lambda_{t+s,t+s+1} \frac{\kappa_{t+s+1}}{2} x_{t+s+1}(r)^2 \right. \\ &\quad \left. - (1-\lambda)(\rho\beta) \Lambda_{t+s,t+s+1} \Delta_{t+s+1} w_{t+s+1}^* \right] - \Delta_t w_t^* \end{aligned}$$

- Using the vacancy posting condition, the value of a marginal worker can similarly be expressed as discounted profits per worker:

$$\begin{aligned} J_t(r) &= p_t^w f_{nt} - w_t(r) - \beta E_t \Lambda_{t,t+1} \frac{\kappa_{t+1}}{2} x_{t+1}(r)^2 + E_t (\rho + x_{t+1}(r)) \beta \Lambda_{t,t+1} J_{t+1}(r) \\ &= p_t^w f_{nt} - w_t(r) - \beta E_t \Lambda_{t,t+1} \frac{\kappa_{t+1}}{2} x_{t+1}(r)^2 + E_t \frac{n_{t+1}}{n_t}(r) \beta \Lambda_{t,t+1} J_{t+1}(r) \\ &= E_t \sum_{s=0}^{\infty} \frac{n_{t+s}}{n_t}(r) \beta^s \Lambda_{t,t+s} \left[p_{t+s}^w f_{nt+s} - \beta \Lambda_{t+s,t+s+1} \frac{\kappa_{t+s+1}}{2} x_{t+s+1}(r)^2 \right] - W_t^f(r) \end{aligned}$$

A5. The contract wage

- The Nash first-order condition is

$$\chi_t(r) J_t(r) = (1 - \chi_t(r)) H_t(r)$$

with

$$\chi_t(r) = \frac{\eta_t}{\eta_t + (1-\eta_t) \Sigma_t(r) / \Delta_t}$$

- This can be rewritten as

$$(\chi_t(r) - \chi)[J_t(r) + H_t(r)] + \chi J_t(r) = (1-\chi) H_t(r)$$

or

$$\chi J_t(r) = (1-\chi) H_t(r) - \Omega_t(r)$$

with

$$\Omega_t(r) = (\chi_t(r) - \chi)[J_t(r) + H_t(r)]$$

- Substituting $J_t(r)$ and $H_t(r)$ and rearranging, we obtain:

$$\begin{aligned}\Delta_t w_t^* - \Omega_t(r) &= E_t \sum_{s=0}^{\infty} (\rho\beta)^s \Lambda_{t,t+s} \Big\{ \Big[\chi \left(p_{t+s}^w f_{nt+s} + \beta \Lambda_{t+s,t+s+1} \frac{\kappa_{t+s+1}}{2} x_{t+s+1}(r)^2 \right) + \\ &\quad (1-\chi)(b_{t+s} + s_{t+s+1}\beta\Lambda_{t+s,t+s+1} H_{x,t+s+1})] \\ &\quad - (1-\lambda)\rho\beta\Lambda_{t+s,t+s+1}\Delta_{t+s+1} w_{t+s+1}^* \Big\}\end{aligned}$$

- The above equation can be written in a recursive form in the following way:

$$\begin{aligned}\Delta_t w_t^* - \Omega_t(r) &= \chi \left(p_t^w f_{nt} + \beta E_t \Lambda_{t,t+1} \frac{\kappa_{t+1}}{2} x_{t+1}(r)^2 \right) + (1-\chi)(b_t + s_{t+1}\beta E_t \Lambda_{t,t+1} H_{x,t+1}) \\ &\quad - (1-\lambda)\rho\beta E_t \Lambda_{t,t+1}\Delta_{t+1} w_{t+1}^* + \rho\beta E_t \Lambda_{t,t+1} \left(\Delta_{t+1} w_{t+1}^* - \Omega_{t+1}(r) \right)\end{aligned}$$

- Simplifying, we obtain

$$\Delta_t w_t^* = w_t^o(r) + \rho\lambda\beta E_t \Lambda_{t,t+1} \Delta_{t+1} w_{t+1}^*$$

with $w_t^o(r)$ denoting the target wage:

$$\begin{aligned}w_t^o(r) &= \chi \left(p_t^w f_{nt} + \beta E_t \Lambda_{t,t+1} \frac{\kappa_{t+1}}{2} x_{t+1}(r)^2 \right) \\ &\quad + (1-\chi)(b_t + s_{t+1}\beta E_t \Lambda_{t,t+1} H_{x,t+1}) + \Phi_t\end{aligned}$$

and

$$\begin{aligned}\Phi_t &= \Omega_t(r) - \rho\beta E_t \Lambda_{t,t+1} \Omega_{t+1}(r) \\ &= (\chi_t(r) - \chi)[J_t(r) + H_t(r)] - \rho\beta E_t \Lambda_{t,t+1} \left(\chi_{t+1}(r) - \chi \right) [J_{t+1}(r) + H_{t+1}(r)]\end{aligned}$$

A6. The loglinearized target wage

- Let V_t be the unconditional average value of employment at t and $V_{x,t}$ the average value of employment at t conditional on being a new hire:

$$V_t = \int_0^1 V_t(i) \frac{n_t(i)}{n_t} di$$

$$V_{x,t} = \int_0^1 V_t(i) \frac{x_{t-1}(i) n_{t-1}(i)}{x_{t-1} n_{t-1}} di$$

Note that V_t and $V_{x,t}$ are identical up to a first order approximation:

$$\widehat{V}_t = \widehat{V}_{x,t}$$

This implies that $H_t = V_t - U_t$ and $H_{x,t} = V_{x,t} - U_t$ are also identical up to a first order:

$$\widehat{H}_t = \widehat{H}_{x,t}$$

- Loglinearizing the target wage then yields

$$\begin{aligned} \widehat{w}_t^o(r) &= \varphi_{fn}\left(\widehat{p}_t^w + \widehat{f}_{nt}\right) + \varphi_x E_t \widehat{x}_{t+1}(r) + (\varphi_x/2) E_t \widehat{\Lambda}_{t,t+1} + \\ & \quad \varphi_b \widehat{b}_t + \varphi_s E_t \left(\widehat{s}_{t+1} + \widehat{H}_{t+1} + \widehat{\Lambda}_{t,t+1}\right) + \varphi_\chi \left[\widehat{\chi}_t(r) - \rho\beta E_t \widehat{\chi}_{t+1}(r)\right] \end{aligned}$$

where

$$\overline{w} = (w/a)$$

$$\varphi_{fn} = \chi p^w (f_n/a) \overline{w}^{-1} \quad \varphi_x = \chi\beta\kappa x^2 \overline{w}^{-1}$$

$$\varphi_b = (1-\chi) b (k/a) \overline{w}^{-1} \quad \varphi_s = \chi\beta s\kappa x \overline{w}^{-1}$$

$$\varphi_\chi = \chi (1-\chi)^{-1} \epsilon^{-1} \overline{w}^{-1}$$

$$\epsilon = (\kappa x)^{-1}$$

- Finally, the weight in the target wage is

$$\chi_t(r) = \frac{\eta_t}{\eta_t + (1-\eta_t)\Sigma_t(r)/\Delta_t}$$

with

$$\Delta_t = 1 + \rho\lambda\beta E_t \Lambda_{t,t+1} \Delta_{t+1} \frac{p_t}{p_{t+1}} \overline{\gamma} \pi_t^\gamma$$

$$\Sigma_t(r) = 1 + E_t(\rho + x_{t+1}(r))(\lambda\beta)\Lambda_{t,t+1}\Sigma_{t+1}(r) \frac{p_t}{p_{t+1}} \overline{\gamma} \pi_t^\gamma$$

Loglinearizing yields

$$\widehat{\chi}_t(r) = -(1-\chi)\left(\widehat{\Sigma}_t(r) - \widehat{\Delta}_t\right)$$

with

$$\widehat{\Delta}_t = \rho\lambda\beta E_t\left(\widehat{\Lambda}_{t,t+1} - \widehat{\pi}_{t+1} + \gamma\widehat{\pi}_t + \widehat{\Delta}_{t+1} - \widehat{\varepsilon}^a_{t+1}\right)$$

$$\widehat{\Sigma}_t(r) = x\lambda\beta E_t\widehat{x}_{t+1}(r) + \lambda\beta E_t\left(\widehat{\Lambda}_{t,t+1} - \widehat{\pi}_{t+1} + \gamma\widehat{\pi}_t + \widehat{\Sigma}_{t+1}(r) - \widehat{\varepsilon}^a_{t+1}\right)$$

A7. Hiring rate at a firm renegotiating at t, $x_t(r)$

- Let x_t be the unconditional average value of the hiring rate:

$$x_t = \int_0^1 x_t(i)\frac{n_t(i)}{n_t}di$$

- Using the job creation condition, x_t can be written as

$$\kappa_t x_t = p_t^w f_{nt} - w_t + \beta E_t\Lambda_{t,t+1}\frac{\kappa_{t+1}}{2}{x_{t+1}}^2 + \rho\beta E_t\Lambda_{t,t+1}\kappa_{t+1}x_{t+1} + \varsigma_t^x$$

with

$$\varsigma_t^x = \beta E_t\Lambda_{t,t+1}\left[\int_0^1\left(\frac{\kappa_{t+1}}{2}x_{t+1}(i)^2 + \rho\kappa_{t+1}x_{t+1}(i)\right)\frac{n_t(i)}{n_t}di - \left(\frac{\kappa_{t+1}}{2}{x_{t+1}}^2 + \rho\kappa_{t+1}x_{t+1}\right)\right]$$

- Loglinearizing yields:

$$\widehat{x}_t = \epsilon p^w(f_n/a)\left(\widehat{p}_t^w + \widehat{f}_{nt}\right) - \epsilon\overline{w}\widehat{w}_t + \beta E_t\widehat{x}_{t+1} + \upsilon\beta E_t\widehat{\Lambda}_{t,t+1}$$

with

$$\upsilon = \rho + x/2$$

and where

$$\widehat{\varsigma}_t^x = 0$$

- Consider now a firm renegotiating at time t. We can write:

$$\widehat{x}_t - \widehat{x}_t(r) = -\epsilon\overline{w}\left(\widehat{w}_t - \widehat{w}_t(r)\right) + \beta E_t\left(\widehat{x}_{t+1} - \widehat{x}_{t+1}(r)\right)$$

which can be iterated forward to give:

$$\begin{aligned}\widehat{x}_t - \widehat{x}_t(r) &= -\epsilon\overline{w}\left(\widehat{w}_t - \widehat{w}_t(r)\right)\\ &\quad -\beta\epsilon\overline{w}E_t\left(\widehat{w}_{t+1} - \widehat{w}_{t+1}(r)\right)\\ &\quad -\beta^2\epsilon\overline{w}E_t\left(\widehat{w}_{t+2} - \widehat{w}_{t+2}(r)\right)\\ &\quad -...\end{aligned}$$

- Using the loglinear version of the real wage index:

$$\widehat{w}_t = (1-\lambda)\,\widehat{w}_t^* + \lambda\left(\widehat{w}_{t-1} - \widehat{\pi}_t + \gamma\widehat{\pi}_{t-1} - \widehat{\varepsilon}_t^a\right)$$

together with the loglinear expressions for the expected future wages at a firm renegotiating at time t (see section A1), we obtain

$$\widehat{w}_t - \widehat{w}_t\,(r) = \widehat{w}_t - \widehat{w}_t^*$$

$$E_t\left(\widehat{w}_{t+1} - \widehat{w}_{t+1}\,(r)\right) = \lambda\left(\widehat{w}_t - \widehat{w}_t^*\right)$$

$$E_t\left(\widehat{w}_{t+2} - \widehat{w}_{t+2}\,(r)\right) = \lambda^2\left(\widehat{w}_t - \widehat{w}_t^*\right)$$

and so on....

- Substituting and rearranging yields:

$$\widehat{x}_t\,(r) = \widehat{x}_t + \epsilon\Sigma\overline{w}\left(\widehat{w}_t - \widehat{w}_t^*\right)$$

with

$$\Sigma = \frac{1}{1-\beta\lambda}$$

A8. Expected average worker surplus at firm renegotiating at t, $E_t H_{t+1}$

A8a. Average worker surplus H_t and firm marginal surplus J_t

- The unconditional average value of worker surplus H_t can be written as:

$$H_t = w_t - b_t + E_t\left(\rho - s_{t+1}\right)\beta\Lambda_{t,t+1}H_{t+1} + \varsigma_t^w$$

with

$$\varsigma_t^w = \beta E_t\Lambda_{t,t+1}\left[\rho\left(\int_0^1 V_{t+1}\,(i)\,\frac{n_t(i)}{n_t}di - V_{t+1}\right) - s_{t+1}\left(\int_0^1 V_{t+1}\,(i)\,\frac{x_{t+1}(i)n_t(i)}{x_{t+1}n_t} - V_{t+1}\right)\right]$$

- The unconditional average value of firm marginal surplus J_t can be written as:

$$J_t = p_t^w f_{nt} - w_t + \beta E_t\Lambda_{t,t+1}\frac{\kappa_{t+1}}{2}x_{t+1}^2 + \rho\beta E_t\Lambda_{t,t+1}J_{t+1} + \varsigma_t^f$$

with

$$\varsigma_t^f = \beta E_t\Lambda_{t,t+1}\left(\int_0^1 \frac{\kappa_{t+1}}{2}x_{t+1}^2\,(i)\,\frac{n_t(i)}{n_t}di - \frac{\kappa_{t+1}}{2}x_{t+1}^2\right) + \rho\beta E_t\Lambda_{t,t+1}\left(\int_0^1 J_{t+1}\,(i)\,\frac{n_t(i)}{n_t}di - J_{t+1}\right)$$

- Loglinearizing H_t and J_t and rearranging

$$\widehat{H}_t = (1-\chi)\chi^{-1}\epsilon\left(\overline{w}\widehat{w}_t - b(k/a)\widehat{b}_t\right) + \rho\beta E_t\left(\widehat{\Lambda}_{t,t+1} + \widehat{H}_{t+1}\right) - s\beta E_t\left(\widehat{s}_{t+1} + \widehat{\Lambda}_{t,t+1} + \widehat{H}_{t+1}\right)$$

$$\widehat{J}_t = \epsilon p^w (f_n/a)\left(\widehat{p}_t^w + \widehat{f}_{nt}\right) - \epsilon\overline{w}\widehat{w}_t + x\beta E_t\widehat{x}_{t+1} + \rho\beta E_t\widehat{J}_{t+1} + \beta\upsilon E_t\widehat{\Lambda}_{t,t+1}$$

where

$$\widehat{\varsigma}_t^w = \widehat{\varsigma}_t^f = 0$$

- Then we can write the following expressions[21]:

$$\widehat{H}_t - \widehat{H}_t(i) = (1-\chi)\chi^{-1}\epsilon\overline{w}\left(\widehat{w}_t - \widehat{w}_t(i)\right) + \rho\beta E_t\left(\widehat{H}_{t+1} - \widehat{H}_{t+1}(i)\right)$$

$$\widehat{J}_t - \widehat{J}_t(i) = x\beta E_t\left(\widehat{x}_{t+1} - \widehat{x}_{t+1}(i)\right) - \epsilon\overline{w}\left(\widehat{w}_t - \widehat{w}_t(i)\right) + \rho\beta E_t\left(\widehat{J}_{t+1} - \widehat{J}_{t+1}(i)\right)$$

A8b. Expected worker surplus at a firm renegotiating at $t+1$, $E_t H_{t+1}(r')$

- Consider a firm renegotiating at time $t+1$. We can write:

$$\begin{aligned} E_t\left(\widehat{H}_{t+1} - \widehat{H}_{t+1}(r')\right) &= (1-\chi)\chi^{-1}\epsilon\overline{w}E_t\left(\widehat{w}_{t+1} - \widehat{w}_{t+1}(r')\right) + \rho\beta E_t\left(\widehat{H}_{t+2} - \widehat{H}_{t+2}(r')\right) \\ &= (1-\chi)\chi^{-1}\epsilon\overline{w}E_t\left[\left(\widehat{w}_{t+1} - \widehat{w}_{t+1}(r')\right) + \rho\beta\left(\widehat{w}_{t+2} - \widehat{w}_{t+2}(r')\right) + ...\right] \end{aligned}$$

- Note that, for a firm renegotiating at time $t+1$, we have:

$$E_t\left(\widehat{w}_{t+1} - \widehat{w}_{t+1}(r')\right) = E_t\left(\widehat{w}_{t+1} - \widehat{w}_{t+1}^*\right)$$

$$E_t\left(\widehat{w}_{t+2} - \widehat{w}_{t+2}(r')\right) = \lambda E_t\left(\widehat{w}_{t+1} - \widehat{w}_{t+1}^*\right)$$

$$E_t\left(\widehat{w}_{t+3} - \widehat{w}_{t+3}(r')\right) = \lambda^2 E_t\left(\widehat{w}_{t+1} - \widehat{w}_{t+1}^*\right)$$

and so on....

- Substituting these expressions and rearranging:

$$E_t\widehat{H}_{t+1}(r') = E_t\widehat{H}_{t+1} - (1-\chi)\chi^{-1}\epsilon\Delta\overline{w}E_t\left(\widehat{w}_{t+1} - \widehat{w}_{t+1}^*\right)$$

with

$$\Delta = \frac{1}{1-\rho\lambda\beta}$$

[21] Note we are using $E_t\widehat{H}_{x,t+1} = E_t\widehat{H}_{t+1}$.

A8c. Expected firm marginal surplus for a firm renegotiating at $t+1$, $E_t J_{t+1}(r')$

- We can write:

$$\begin{aligned} E_t\left(\widehat{J}_{t+1}-\widehat{J}_{t+1}(r')\right) &= x\beta E_t\left(\widehat{x}_{t+2}-\widehat{x}_{t+2}(r')\right)-\epsilon\overline{w}E_t\left(\widehat{w}_{t+1}-\widehat{w}_{t+1}(r')\right) \\ &\quad +\rho\beta E_t\left(\widehat{J}_{t+2}-\widehat{J}_{t+2}(r')\right) \\ &= x\beta E_t\left[\left(\widehat{x}_{t+2}-\widehat{x}_{t+2}(r')\right)+\rho\beta\left(\widehat{x}_{t+3}-\widehat{x}_{t+3}(r')\right)+...\right] \\ &\quad -\epsilon\overline{w}E_t\left[\left(\widehat{w}_{t+1}-\widehat{w}_{t+1}(r')\right)+\rho\beta\left(\widehat{w}_{t+2}-\widehat{w}_{t+2}(r')\right)+...\right] \end{aligned}$$

- Moreover, we have:

$$E_t\left(\widehat{x}_{t+1}(r')-\widehat{x}_{t+1}\right)=\epsilon\Sigma\overline{w}E_t\left(\widehat{w}_{t+1}-\widehat{w}^*_{t+1}\right)$$
$$E_t\left(\widehat{x}_{t+2}(r')-\widehat{x}_{t+2}\right)=\lambda\epsilon\Sigma\overline{w}E_t\left(\widehat{w}_{t+1}-\widehat{w}^*_{t+1}\right)$$
$$E_t\left(\widehat{x}_{t+3}(r')-\widehat{x}_{t+3}\right)=\lambda^2\epsilon\Sigma\overline{w}E_t\left(\widehat{w}_{t+1}-\widehat{w}^*_{t+1}\right)$$

and so on....

- Substituting the expressions for the expected future wages and hiring rates and rearranging:

$$E_t\widehat{J}_{t+1}(r')=E_t\widehat{J}_{t+1}+\epsilon\Sigma\overline{w}E_t\left(\widehat{w}_{t+1}-\widehat{w}^*_{t+1}\right)$$

with

$$\Sigma=\frac{1}{1-\lambda\beta}$$

A8d. Weight in the Nash bargaining first order condition

- Recall that

$$\widehat{\chi}_t(r)=-(1-\chi)\left(\widehat{\Sigma}_t(r)-\widehat{\Delta}_t\right)$$

with

$$\widehat{\Delta}_t=\rho\lambda\beta E_t\left(\widehat{\Lambda}_{t,t+1}-\widehat{\pi}_{t+1}+\gamma\widehat{\pi}_t+\widehat{\Delta}_{t+1}-\widehat{\varepsilon}^a_{t+1}\right)$$

$$\widehat{\Sigma}_t(r)=x\lambda\beta E_t\widehat{x}_{t+1}(r)+\lambda\beta E_t\left(\widehat{\Lambda}_{t,t+1}-\widehat{\pi}_{t+1}+\gamma\widehat{\pi}_t+\widehat{\Sigma}_{t+1}(r)-\widehat{\varepsilon}^a_{t+1}\right)$$

- Averaging across all firms the firm cumulative discount factor yields:

$$\widehat{\Sigma}_t=(x\lambda\beta)E_t\widehat{x}_{t+1}+(\lambda\beta)E_t\left(\widehat{\Lambda}_{t,t+1}-\widehat{\pi}_{t+1}+\gamma\widehat{\pi}_t+\widehat{\Sigma}_{t+1}-\widehat{\varepsilon}^a_{t+1}\right)$$

- Taking differences and iterating forward yields

$$\begin{aligned}\widehat{\Sigma}_t(r) - \widehat{\Sigma}_t &= (x\lambda\beta) E_t(\widehat{x}_{t+1}(r) - \widehat{x}_{t+1}) \\ &\quad + (\lambda\beta)(x\lambda\beta) E_t(\widehat{x}_{t+2}(r) - \widehat{x}_{t+2}) \\ &\quad + (\lambda\beta)^2 (x\lambda\beta) E_t(\widehat{x}_{t+3}(r) - \widehat{x}_{t+3}) \\ &\quad + ...\end{aligned}$$

- Substituting the expressions for the future hiring rates and collecting terms:

$$\widehat{\Sigma}_t(r) - \widehat{\Sigma}_t = x\Psi\epsilon\Sigma\overline{w}(\widehat{w}_t - \widehat{w}_t^*)$$

with

$$\Psi = \frac{\lambda^2\beta}{1-\lambda^2\beta}$$

- Finally, we have

$$\widehat{\chi}_t(r) = \widehat{\chi}_t - (1-\chi) x\Psi\epsilon\Sigma\overline{w}(\widehat{w}_t - \widehat{w}_t^*)$$

- Similarly, we have

$$E_t\widehat{\chi}_{t+1}(r') = E_t\widehat{\chi}_{t+1} - (1-\chi) x\Psi\epsilon\Sigma\overline{w} E_t\left(\widehat{w}_{t+1} - \widehat{w}_{t+1}^*\right)$$

A8e. Using the expected Nash condition at time $t+1$

- The expected Nash condition for firms renegotiating at time $t+1$ is

$$E_t\chi_{t+1}(r') J_{t+1}(r') = E_t\left(1 - \chi_{t+1}(r')\right) H_{t+1}(r')$$

- Loglinearizing

$$E_t\widehat{J}_{t+1}(r') + (1-\chi)^{-1} E_t\widehat{\chi}_{t+1}(r') = E_t\widehat{H}_{t+1}(r')$$

- Substituting the expressions found in sections A8b, A8c and A8d and rearranging yields

$$E_t\widehat{H}_{t+1} = E_t\widehat{J}_{t+1} + \Gamma E_t\left(\widehat{w}_{t+1} - \widehat{w}_{t+1}^*\right) + (1-\chi)^{-1} E_t\widehat{\chi}_{t+1}$$

with

$$\Gamma = (1 - \eta(1-\rho)\Psi)\eta^{-1}\epsilon\Sigma\overline{w}$$

- Using the loglinear expression for the hiring rate averaged across all firms:

$$\widehat{x}_t = \widehat{J}_t$$

we finally obtain

$$E_t\widehat{H}_{t+1} = E_t\widehat{x}_{t+1} + \Gamma E_t\left(\widehat{w}_{t+1} - \widehat{w}^*_{t+1}\right) + (1-\chi)^{-1} E_t\widehat{\chi}_{t+1}$$

A9. Spillover effects

- Consider the loglinear target wage

$$\begin{aligned}\widehat{w}^o_t(r) &= \varphi_{fn}\left(\widehat{p}^w_t + \widehat{f}_{nt}\right) + \varphi_x E_t\widehat{x}_{t+1}(r) + (\varphi_x/2) E_t\widehat{\Lambda}_{t,t+1} + \\ &\quad \varphi_b\widehat{b}_t + \varphi_s E_t\left(\widehat{s}_{t+1} + \widehat{H}_{t+1} + \widehat{\Lambda}_{t,t+1}\right) + \varphi_\chi\left[\widehat{\chi}_t(r) - \rho\beta E_t\widehat{\chi}_{t+1}(r)\right]\end{aligned}$$

- Substituting the following expressions in the target wage:

$$E_t\widehat{H}_{t+1} = E_t\widehat{x}_{t+1} + \Gamma E_t\left(\widehat{w}_{t+1} - \widehat{w}^*_{t+1}\right) + (1-\chi)^{-1} E_t\widehat{\chi}_{t+1}$$

$$E_t\widehat{x}_{t+1}(r) = E_t\widehat{x}_{t+1} + \lambda\epsilon\Sigma\overline{w}\left(\widehat{w}_t - \widehat{w}^*_t\right)$$

$$\widehat{\chi}_t(r) = \widehat{\chi}_t - (1-\chi)(1-\rho)\Psi\epsilon\Sigma\overline{w}\left(\widehat{w}_t - \widehat{w}^*_t\right)$$

$$\widehat{\chi}_{t+1}(r) = \widehat{\chi}_{t+1} - (1-\chi)(1-\rho)\Psi\lambda\epsilon\Sigma\overline{w}\left(\widehat{w}_t - \widehat{w}^*_t\right)$$

we obtain

$$\widehat{w}^o_t(r) = \widehat{w}^o_t + \frac{\tau_1}{1-\rho\lambda\beta}E_t\left(\widehat{w}_{t+1} - \widehat{w}^*_{t+1}\right) + \frac{\tau_2}{1-\rho\lambda\beta}\left(\widehat{w}_t - \widehat{w}^*_t\right)$$

with

$$\begin{aligned}\widehat{w}^o_t &= \varphi_{fn}\left(\widehat{p}^w_t + \widehat{f}_{nt}\right) + (\varphi_x + \varphi_s) E_t\widehat{x}_{t+1} + \varphi_s E_t\widehat{s}_{t+1} + \varphi_b\widehat{b}_t \\ &\quad + (\varphi_s + \varphi_x/2) E_t\widehat{\Lambda}_{t,t+1} + \varphi_\chi\left(\widehat{\chi}_t - \rho\beta\widehat{\chi}_{t+1}\right) + \varphi_s(1-\chi)^{-1} E_t\widehat{\chi}_{t+1}\end{aligned}$$

and

$$\begin{aligned}\tau_1 &= \varphi_s\Gamma(1-\rho\lambda\beta) \\ \tau_2 &= \left[\varphi_x\lambda - \varphi_\chi(1-\chi)x\Psi\Delta^{-1}\right]\epsilon\Sigma\overline{w}(1-\rho\lambda\beta)\end{aligned}$$

APPENDIX B

Steady state calculation

- First obtain

$$n = \frac{s}{1-\rho+s}$$
$$u = 1-n$$
$$x = \frac{su}{n}$$

and

$$\chi = \frac{\eta}{\eta + (1-\eta)\,\Sigma/\Delta}$$

- Then get

$$r^k = \gamma_a/\beta - 1 + \delta$$
$$\frac{k/a}{y/a} = \frac{\alpha p^w}{r^k}$$
$$\frac{i/a}{y/a} = \left(1 - \frac{1-\delta}{\gamma_a}\right)\gamma_a \frac{k/a}{y/a}$$
$$\frac{k/a}{n} = \left(\frac{k/a}{y/a}\right)^{\frac{1}{1-\alpha}}$$
$$\frac{y/a}{n} = \left(\frac{k/a}{n}\right)^{\alpha}$$
$$f_n/a = (1-\alpha)\,\frac{y/a}{n}$$

- We also have

$$z = q^k = 1$$

- Then κ and w solve the following system

$$\begin{cases} \kappa x = p^w\,(f_n/a) - (w/a) + (\beta/\gamma_a)\,(\kappa/2)\,x^2 + (\beta/\gamma_a)\,\rho\kappa x \\ (w/a) = \chi\left[p^w\,(f_n/a) + (\beta/\gamma_a)\,(\kappa/2)\,x^2 + (\beta/\gamma_a)\,s\kappa x\right] + (1-\chi)\left[\overline{b}\,(w/a)\right] \end{cases}$$

where

$$\overline{b} = \frac{b\,(k/a)}{(w/a)}$$

- The flow value of unemployment is given by

$$b^{tot} = b\,(k/a) = \overline{b}\,(w/a)$$

- The steady state labor share is calculated from

$$ls = \frac{w/a}{(y/a)/n}$$

- Finally

$$\frac{c/a}{y/a} = 1 - \frac{g/a}{y/a} - \frac{i/a}{y/a} - \frac{\kappa}{2}\frac{x^2 n}{y/a} = 1 - \frac{g/a}{y/a} - \frac{i/a}{y/a} - \frac{\kappa}{2}\frac{x^2}{(y/a)/n}$$

APPENDIX C

The complete loglinear GST model

- Technology

$$\widehat{y}_t = \alpha \widehat{k}_t + (1-\alpha)\,\widehat{n}_t \tag{E1}$$

- Resource constraint

$$\widehat{y}_t = cy\widehat{c}_t + iy\widehat{i}_t + gy\widehat{g}_t + zy\widehat{z}_t + xy\,(2\widehat{x}_t + \widehat{n}_{t-1}) \tag{E2}$$

where $cy = \frac{c/a}{y/a}$, $iy = \frac{i/a}{y/a}$, $gy = \frac{g/a}{y/a}$, $zy = r^k \frac{k/a}{y/a}$ and $xy = \frac{\kappa}{2}\frac{x^2 n}{y/a}$.

- Matching

$$\widehat{m}_t = \sigma \widehat{u}_t + (1-\sigma)\,\widehat{v}_t \tag{E3}$$

- Employment dynamics

$$\widehat{n}_t = \rho \widehat{n}_{t-1} + (1-\rho)\,\widehat{m}_t \tag{E4}$$

- Transition probabilities

$$\widehat{q}_t = \widehat{m}_t - \widehat{v}_t \tag{E5}$$

$$\widehat{s}_t = \widehat{m}_t - \widehat{u}_t \tag{E6}$$

- Unemployment

$$\widehat{u}_t = -\frac{n}{u}\widehat{n}_{t-1} \tag{E7}$$

- Effective capital

$$\widehat{k}_t + \widehat{\varepsilon}^a_t = \widehat{z}_t + \widehat{k}^p_{t-1} \tag{E8}$$

- Physical capital dynamics

$$\widehat{k}^p_t = \xi\left(\widehat{k}^p_{t-1} - \widehat{\varepsilon}^a_t\right) + (1-\xi)\left(\widehat{i}_t + \widehat{\varepsilon}^i_t\right) \tag{E9}$$

where $\xi = \frac{1-\delta}{\gamma_a}$

- Aggregate vacancies

$$\widehat{x}_t = \widehat{q}_t + \widehat{v}_t - \widehat{n}_{t-1} \tag{E10}$$

- Consumption-saving

$$\widehat{\lambda}_t = E_t\widehat{\lambda}_{t+1} + (\widehat{r}_t - E_t\widehat{\pi}_{t+1}) - E_t\widehat{\varepsilon}^a_{t+1} \tag{E11}$$

- Marginal utility

$$(1-h/\gamma_a)(1-\beta h/\gamma_a)\widehat{\lambda}_t = (h/\gamma_a)(\widehat{c}_{t-1}-\widehat{\varepsilon}_t^a) - \left(1+\beta(h/\gamma_a)^2\right)\widehat{c}_t + \beta(h/\gamma_a)E_t\left(\widehat{c}_{t+1}+\widehat{\varepsilon}_{t+1}^a\right) + (1-h/\gamma_a)\left(\widehat{\varepsilon}_t^b - \beta h/\gamma_a E_t\widehat{\varepsilon}_{t+1}^b\right) \tag{E12}$$

where h measures the degree of habit persistence in consumption.

- Capital utilization

$$\widehat{z}_t = \eta_z \widehat{r}_t^k \tag{E13}$$

where $\eta_z = a'(z)/a''(z) = \frac{1-\psi_z}{\psi_z}$

- Investment

$$\widehat{i}_t = \frac{1}{1+\beta}\left(\widehat{i}_{t-1}-\widehat{\varepsilon}_t^a\right) + \frac{1/\left(\eta_k\gamma_a^2\right)}{1+\beta}\left(\widehat{q}_t^k+\widehat{\varepsilon}_t^i\right) + \frac{\beta}{1+\beta}E_t\left(\widehat{i}_{t+1}+\widehat{\varepsilon}_{t+1}^a\right) \tag{E14}$$

where $\eta_k = s''$

- Capital renting

$$\widehat{p}_t^w + \widehat{y}_t - \widehat{k}_t = \widehat{r}_t^k \tag{E15}$$

- Tobin's q

$$\widehat{q}_t^k = (\beta/\gamma_a)(1-\delta)E_t\widehat{q}_{t+1}^k + [1-(\beta/\gamma_a)(1-\delta)]E_t\widehat{r}_{t+1}^k - (\widehat{r}_t - E_t\widehat{\pi}_{t+1}) \tag{E16}$$

- Aggregate hiring rate

$$\widehat{x}_t = \epsilon p^w(f_n/a)\left(\widehat{p}_t^w+\widehat{f}_{nt}\right) - \epsilon(w/a)\widehat{w}_t + \beta E_t\widehat{x}_{t+1} + \upsilon\beta E_t\widehat{\Lambda}_{t,t+1} \tag{E17}$$

where

$$\epsilon = (\kappa x)^{-1}$$

$$\upsilon = \rho + x/2$$

- Marginal product of labor

$$\widehat{f}_{nt} = \widehat{y}_t - \widehat{n}_t \tag{E18}$$

- Weight in Nash bargaining

$$\widehat{\chi}_t = -(1-\chi)\left(\widehat{\Sigma}_t - \widehat{\Delta}_t\right) \tag{E19}$$

with

$$\widehat{\Delta}_t = \rho\lambda\beta E_t\left(\widehat{\Lambda}_{t,t+1} - \widehat{\pi}_{t+1} + \gamma\widehat{\pi}_t + \widehat{\Delta}_{t+1} - \widehat{\varepsilon}_{t+1}^a\right) \tag{E20}$$

$$\widehat{\Sigma}_t = x\lambda\beta E_t\widehat{x}_{t+1} + \lambda\beta E_t\left(\widehat{\Lambda}_{t,t+1} - \widehat{\pi}_{t+1} + \gamma\widehat{\pi}_t + \widehat{\Sigma}_{t+1} - \widehat{\varepsilon}_{t+1}^a\right) \tag{E21}$$

- Spillover-free target wage

$$\begin{aligned}\widehat{w}_t^o &= \varphi_{fn}\left(\widehat{p}_t^w + \widehat{f}_{nt}\right) + (\varphi_x + \varphi_s) E_t\widehat{x}_{t+1} + \varphi_s E_t\widehat{s}_{t+1} + \varphi_b\widehat{b}_t \\ &\quad + (\varphi_s + \varphi_x/2) E_t\widehat{\Lambda}_{t,t+1} + \varphi_\chi\left(\widehat{\chi}_t - \rho\beta\widehat{\chi}_{t+1}\right) + \varphi_s (1-\chi)^{-1} E_t\widehat{\chi}_{t+1}\end{aligned} \tag{E22}$$

where

$$\overline{w} = (w/a)$$
$$\varphi_{fn} = \chi p^w (f_n/a)\,\overline{w}^{-1} \quad \varphi_x = \chi\beta(1-\rho)\,\epsilon^{-1}\overline{w}^{-1}$$
$$\varphi_b = (1-\chi)\,b\,(k/a)\,\overline{w}^{-1} \qquad \varphi_s = \chi\beta s\epsilon^{-1}\overline{w}^{-1}$$
$$\varphi_\chi = \chi(1-\chi)^{-1}\epsilon^{-1}\overline{w}^{-1}$$
$$b^{tot} = b\,(k/a)$$

- Aggregate wage

$$\widehat{w}_t = \gamma_b\left(\widehat{w}_{t-1} - \widehat{\pi}_t + \gamma\widehat{\pi}_{t-1} - \widehat{\varepsilon}_t^a\right) + \gamma_o\widehat{w}_t^o + \gamma_f\left(\widehat{w}_{t+1} + \widehat{\pi}_{t+1} - \gamma\widehat{\pi}_t + \widehat{\varepsilon}_{t+1}^a\right) + \widehat{\varepsilon}_t^\eta \tag{E23}$$

where

$$\gamma_b = (1+\tau_2)\,\phi^{-1} \quad \gamma_o = \varsigma\phi^{-1} \quad \gamma_f = (\rho\beta - \tau_1)\,\phi^{-1}$$
$$\phi = 1 + \tau_2 + \varsigma + \rho\beta - \tau_1 \quad \varsigma = (1-\lambda)(1-\rho\lambda\beta)\,\lambda^{-1}$$

$$\tau_1 = \varphi_s\Gamma(1-\rho\lambda\beta)$$
$$\tau_2 = \left[\varphi_x\lambda - \varphi_\chi(1-\chi)(1-\rho)\,\Psi\Delta^{-1}\right]\epsilon\Sigma\overline{w}\,(1-\rho\lambda\beta)$$
$$\Gamma = (1 - x\eta\Psi)\,\eta^{-1}\epsilon\Sigma\overline{w} \qquad \Psi = \beta\lambda^2/\left(1-\beta\lambda^2\right) \qquad \Sigma = (1-\beta\lambda)^{-1}$$

- Phillips curve

$$\widehat{\pi}_t = \iota_b\widehat{\pi}_{t-1} + \iota_o\left(\widehat{p}_t^w + \widehat{\varepsilon}_t^p\right) + \iota_f E\widehat{\pi}_{t+1} \tag{E24}$$

where

$$\iota_b = \gamma^p(\phi^p)^{-1} \quad \iota_o = \varsigma^p(\phi^p)^{-1} \quad \iota_f = \beta(\phi^p)^{-1}$$
$$\phi^p = 1 + \beta\gamma^p \qquad \varsigma^p = (1-\lambda^p)(1-\lambda^p\beta)(\lambda^p)^{-1}$$

- Tayor rule

$$\widehat{r}_t = \rho_s\widehat{r}_{t-1} + (1-\rho_s)\left[r_\pi\widehat{\pi}_t + r_y\left(\widehat{y}_t - \widehat{y}_{nt}\right)\right] + \widehat{\varepsilon}_t^r \tag{E25}$$

- Government spending

$$\widehat{g}_t = \widehat{y}_t + \frac{1-gy}{gy}\widehat{\varepsilon}_t^g \tag{E26}$$

- Labor share

$$\widehat{ls}_t = \widehat{w}_t + \widehat{n}_t - \widehat{y}_t \tag{E27}$$

- Market tightness

$$\widehat{\theta}_t = \widehat{v}_t - \widehat{u}_t \tag{E28}$$

- Benefits

$$\widehat{b}_t = \widehat{k}_t^p \tag{E29}$$

APPENDIX D

Estimates of the Smets/Wouters Model

Note that the parameters ω and ε^w are, respectively, the Frisch elasticity of labor supply and the gross steady state wage markup. The parameters ρ_w and σ_w refer to a wage markup shock.

Table A1: Prior and posterior distribution of structural parameters: Smets/Wouters Model

	Prior	Posterior			
Parameter	Distribution	Max	Mean	5%	95%
ψ_z	Beta (0.5,0.1)	0.638	0.644	0.514	0.756
η_k	Normal (4,1.5)	3.844	4.157	2.728	5.654
h	Beta (0.5,0.1)	0.759	0.786	0.705	0.852
ω	Beta (0.5,0.1)	3.548	3.835	2.655	5.226
γ	Uniform (0,1)	0.829	0.804	0.630	0.961
γ^P	Uniform (0,1)	0.0000	0.0218	0.0012	0.0634
λ	Beta (0.75,0.1)	0.897	0.878	0.798	0.934
λ^P	Beta (0.66,0.1)	0.852	0.857	0.813	0.894
ε^w	Normal (1.15,0.05)	1.134	1.124	1.036	1.214
ε^P	Normal (1.15,0.05)	1.398	1.392	1.336	1.456
r_π	Uniform (1.1,5)	3.409	3.534	2.441	4.697
r_y	Uniform (0,1)	0.622	0.662	0.391	0.939
ρ_s	Uniform (0,1)	0.891	0.893	0.847	0.927
γ_a	Uniform (1,1.5)	1.0039	1.0039	1.0029	1.0047

Note: For Beta and Normal distributions, the two numbers in parenthesis are respectively the mean and the st. dev. For the Uniform, the two numbers are the lower and the upper bound.

Table A2: Prior and posterior distribution of shock processes
Smets/Wouters Model

	Prior		Posterior		
Parameter	Distribution	Max	Mean	5%	95%
ρ_a	Uniform (0,1)	0.012	0.059	0.005	0.149
ρ_r	Uniform (0,1)	0.135	0.154	0.037	0.278
ρ_b	Uniform (0,1)	0.679	0.637	0.493	0.773
ρ_i	Uniform (0,1)	0.520	0.510	0.383	0.639
ρ_p	Uniform (0,1)	0.843	0.824	0.761	0.889
ρ_w	Uniform (0,1)	0.219	0.247	0.109	0.386
ρ_g	Uniform (0,1)	0.998	0.996	0.990	0.999
σ_a	Uniform (0,30)	1.024	1.037	0.944	1.140
σ_r	Uniform (0,30)	0.232	0.233	0.211	0.257
σ_b	Uniform (0,30)	2.414	2.764	2.147	3.571
σ_i	Uniform (0,30)	9.060	9.866	6.471	13.72
σ_p	Uniform (0,30)	9.157	11.85	5.723	20.84
σ_w	Uniform (0,30)	0.218	0.214	0.175	0.255
σ_g	Uniform (0,30)	0.360	0.365	0.332	0.401

Note: The two numbers in parenthesis are the lower and the upper bound.