Managing Expectations: Concepts and Examples*

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Abstract

The idea that monetary policy is principally about "managing expectations" has taken hold in central banks around the world. Discussions of expectations management by central bankers, academics and by financial market participants frequently also include the idea that central bank credibility is imperfect. We use a familiar macroeconomic model to discuss key concepts in the area of expectations management via a series of examples.

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1 Introduction

It is increasingly standard for central bankers, financial market participants, and academic researchers to describe management of expectations as central to monetary policy. The idea of "managing expectations" arises in many specific contexts including the rationale for particular policy actions, the optimal choice of the monetary instrument, the form of central bank behavior during leadership transitions, the desirable central bank response to frequent or unusual shocks, and the nature of monetary policy necessary for disinflation. Analysts who stress management of expectations frequently also suggest that credibility for low and stable inflation is imperfect, making expectations management subtle.

It is therefore desirable to have macroeconomic models in which active management of expectations can be systematically examined. In this paper, we explore the relationship between policy announcements, beliefs, and policy actions that we think is at the heart of the management of expectations, constructing a battery of simple models in which private agents and the monetary authority both have a one period horizon. In extensions of this research,[1] we study the dynamic management of expectations by a central bank with a long horizon, investigating both the transitional dynamics of credibility and its interaction with shocks to the macroeconomy.

In general, our view is that there are four features critical to the analysis of the interplay between expectations management and credibility[2]. We think that private agents do not know the underlying nature of the central bank and, in particular, whether it will take the actions necessary to produce low and stable inflation in the longer run: we define *long-term credibility* of the central bank as the private sector's likelihood that the central bank is of such a low-inflation type. We think that private agents also are uncertain about whether near-term central bank actions will be those consistent with the central bank being of low-inflation type: we define the *short-term credibility* of the central bank as the private sector's likelihood that the central bank will take such actions. We think that short-term credibility is higher than low-term credibility because of *mimicking*, the possibility that the central bank will

[1] King, Lu and Pasten (2007a).

[2] We distinguish between policy that manages expectations and policy that affects the economy via the coordination of expectations. In other work in progress, Lu and Pasten (2007a) explore how fiscal policy can improve economic performance by coordinating expectations. Policy may also affect the incentives that agents have to accumulate information, as in Lu and Pasten (2007b).

take the same near-term actions as a low-inflation central bank even if low long-term inflation will not be the outcome that it produces.[3] We think that the *inflation plans* of a central bank seeking low inflation will influence inflation expectations and the central bank will take an active role in using its plans to manage inflation expectations at a point in time and the evolution of its long-term credibility. More generally, announcements of various types by central banks appear to play a role in the management of expectations.

To display how these core ideas about expectations management can be introduced into macroeconomic models and the consequences of doing so, we use a framework familiar from the work of Kydland and Prescott [1977] and Barro and Gordon [1983] on central bank behavior under commitment and discretion. Thus, we build in an incentive for a committed policymaker to choose low and stable inflation as well as an incentive for a discretionary policymaker to produce inflation surprises. We add two twists on that familiar framework. Our first twist is that we assume that a committed central bank is one which formulates an *observable* inflation plan at the start of each period and then mechanically carries out the inflation rate specified in that plan because it has an internal technology for commitment. That is, as a starting point, we bundle together public information about inflation intentions with commitment for one type of central bank. By contrast, a discretionary central bank simply chooses an optimal inflation action after the private sector has formed its expectations of inflation, in line with the standard approach in the voluminous literature on this topic. Our second twist is that assume that the central bank type is not known to private agents, but it is to the central bank. We suppose that a discretionary central bank will therefore adopt the same publicly inflation plan as a committed central bank, since failure to do so would reveal its type and worsen economic performance. But the discretionary bank's stated inflation plan has no mechanical implication for its inflation decision.

To understand the consequences of the various model features that we incorporate, we begin with simpler models and proceed to more complicated ones. We start our analysis in section 3 by studying macroeconomic outcomes when there is no mimicking. Working with the traditional assumption that central bank type – committed

[3] The idea that mimicking is important for the dynamics of credibility is stressed by Phelan [2006], within a dynamic model in which discretionary governments have long horizons. Our companion research discusses the links between Phelan's work and ours in detail.

or discretionary – is known, we derive the familiar results from the literature and link them to our interest in the management of expectations. Under complete information, a committed central bank chooses its inflation action recognizing that it has a powerful effect on inflation expectations – it can manage them perfectly – and hence chooses low inflation even when there are important real distortions in the economy. Under complete information, a discretionary central bank does not undertake management of expectations, since it chooses its inflation action after private sector beliefs are formed. The discretionary equilibrium features is the inflation bias stressed by Kydland and Prescott [1977] and Barro and Gordon [1983], produced by the discretionary decision-maker's attempt to stimulate the real economy toward efficient levels.

We then extend the analysis to the plausible case in which the private sector does not know whether the central bank is committed or discretionary in its nature. In that simple setting, there is an interesting interplay between credible central bank plans, private agent expectations, and discretionary central bank actions. First, the discretionary central bank will be able to stimulate the real economy through inflation surprises, since expectations are held down by the possibility that a committed central bank is present. Second, a committed central bank's inflation plan manages expectations via two channels: generally, it affects the private sector's beliefs about what both a committed and discretionary central bank will do. Further, with imperfect information, a committed central bank will not choose an inflation plan that has no bias (the planned inflation rate will be positive) so as to avoid causing negative inflation forecast errors and losses in real activity. Committed central bank management of inflation expectations is imperfect because the private sector believes that there is some chance that the inflation action will be chosen by a discretionary central bank. We show how the extent of imperfect short-term credibility affects the rate of inflation, the level of real activity, and the level of social welfare.

While these initial results are interesting, we think that there are two important dimensions to the management of expectations that our basic model does not capture. The first is that the discretionary central bank may mimic the committed one, a possibility that we explore in section 4. We produce partial mimicking by introducing a random fixed cost of deviating by the discretionary authority, so that only low cost central banks deviate. We show how the committed central bank manages expectations when it can affect the likelihood of discretionary outcomes. The

second is that agents may learn about the type of the central bank over time as a result of its actions, which we formalize using Bayesian learning. To explore some implications of this viewpoint, we begin in section 5 by exploring a simple structure in which there is exogenous probability of the discretionary central banking mimicking of the committed central bank's inflation action. We study the evolution of long-term credibility, the private sector's probability that the central bank is of a committed type. A general result is that long-term credibility grows more rapidly when short-term credibility is low as a result of low mimicking, because observed low inflation outcomes are more informative in that case. We then make the mimicking function endogenous along the lines of section 4 and provide a reference analysis of the interplay between long-term credibility, short-term credibility, inflation and real activity.

We finally turn to a set of questions that have direct policy relevance in the U.S. and other countries. For convenience and clarity, as discussed above, our initial model-building assumption is that a committed central bank generates a publicly observable plan as a by-product of its committed nature. This raises the question of how important a commitment technology and the publicly observable plan each are to the management of expectations. In section 6, we modify the model by assuming that the committed central bank is simply a central bank with a deterministic fixed cost of deviating that is the highest that a discretionary central bank might have.[4] Hence, commitment per se is not critical for the management of expectations: inflation reports by a disciplined central can substitute for inflation plans by a committed central bank. In section 6, we also explain that our simple rational expectations model also makes inflation reports unimportant: since all agents understand the structure of the economy, it is not necessary to transfer information via reports. Thus, a systematic pattern of inflation by a disciplined central bank – an inflation rule – can obtain the same desirable real outcomes as an explicitly committed central bank with a publicly observable inflation plan. >From a theoretical perspective, this finding indicates that a role for inflation reports requires more private agent uncertainty than we have so far introduced. From an empirical perspective, the finding accords with recent studies which compare the economic performance of countries with explicit

[4]This simple model is designed to produce a simple version of our results on dynamic expectations management in King, Lu and Pasten [2007a], where there are two types of central banks. One is patient and predictable (high fixed discount factor) and the other can be impatient (it has a stochastic discount factor that may be is as high as that of the patient central bank or may be much lower).

inflation targets to those without.[5] However, whatever the institutional structure, a disciplined central bank seeking low inflation and desirable real performance in the face of imperfect credibility must manage expectations.

2 Commitment, discretion, plans and credibility

The original analysis of Kydland and Prescott [1997] focused on the debate over "rules versus discretion" and on the "time inconsistency of optimal plans" within economies in which private agents form expectations rationally. They stressed that the use of optimal control theory — with its implicit assumption that the policymaker could commit to an entire future path of policy — led to time inconsistency of optimal plans if the policymaker actually operated in a discretionary setting, making decisions on a period-by-period basis. Since that time, it has become standard to contrast macroeconomic equilibrium under the alternative assumptions of commitment and discretion.

Our analysis also focuses on the idea that policymakers can be of a committed or discretionary type, but with some particular features that are important to detail. First, we view policymakers as formulating an inflation plan p at the start of each period within our discrete time model. This plan is assumed to be observable to private agents, but we consider alterations of this assumption in section 6 below. Second, we view committed decisionmakers as always able to execute their plans, producing inflation $\pi = p$ at the end of the period. A discretionary decisionmaker is assumed to state the same plan as a committed decisionmaker would – a provisional restriction that we consider further in several places below – but to be able to depart from the plan if it is subsequently in his self-interest. Third, we view the type of the policymaker, committed or discretionary, as unobserved by the private sector. However, in some versions of our model, the inflation action π reveals useful information about policymaker type.

Each of the periods of our discrete time model is broken into three subperiods, as in displayed in panel A of Table 1. In the start of the period, an inflation plan p is formulated. In the middle of the period, the private sector forms inflation expectations e. In the end of the period, the inflation action π is chosen. Panel B of Table 1 also shows the two types of central banks. The committed central bank always chooses

[5] See Ball and Sheridan (2004) and follow-up studies.

an inflation action equal to the planned level: $\pi = p$. The discretionary bank may choose to deviate, selecting $\pi = d$, or it may choose to mimic, selecting $\pi = p$.

Table 1: Basic Model Ingredients

Panel A: subperiod structure

start	middle	end
plan (p)	expectations (e)	action (π)

Panel B: types and actions

		Type: σ	
		committed	discretionary
Action: π	Plan	p	p (mimicking)
	Deviation	–	d

2.1 Expectations and credibility

Our committed central bank is concerned with managing expectations: it therefore must have a model of how expectations will respond to its inflation action p and depend on long-term credibility ρ. We call this the *expectations function*,

$$e(p, \rho). \tag{1}$$

Ultimately, we require that this expectations function is a rational one: it must be consistent with the information, technical, and behavioral structure of the economy.

It is frequently the case that discussions of the management of expectations involve considerations of credibility. In our model, as stressed above, there are actually two notions of credibility which are important; these notions are displayed in Table 2.. First, there is the likelihood that the decisionmaker at date t will actually choose $\pi = p$: we denote this likelihood as ψ and call it *short-term credibility*.

Table 2: Long-term credibility, short-term credibility and mimicking

Concept	definition
Long-term credibility	ρ_t =prob$(\sigma_t = c)$
Short-term credibility	ψ_t =prob$(\pi_t = p_t \vert p_t)$
Mimicking probability	m_t= prob$(\pi_t = p_t) \vert\ (p_t, \sigma_t = d)$

Note to Table 2: All probabilities are minimally conditioned on information up to the start of period t. Long-term credibility is the start-of-the period probability that the decision-maker is of the committed type. Short-term credibility is the probability that the inflation plan will actually be carried out, conditional on the plan's level. The mimicking probability is the likelihood, conditional on the decision-maker actually being of the discretionary type, of inflation being at the planned level.

A central bank also must have a model of how its actions affect its credibility: we call this the *short-term credibility function,*

$$\psi(p, \rho),$$

and also impose a rational expectations requirement on this function below. Second, there is the start-of-the-period likelihood that the central bank is of the committed type that will bring about low long-term inflation, which we denote as ρ and call *long-term credibility.* This probability is a key state variable of models of the variety that we study in our research on the management of expectations. In the framework that we develop below, short-term credibility is important for date t expected inflation and the Phillips curve slope, while ρ would be important for longer-term expected inflation and the term structure of interest rates in extensions of the framework.

Bayes' law implies that long-term credibility evolves according to

$$\rho_{t+1} = \left\{ \begin{array}{cc} \kappa(p_t, \rho_t) = \rho_t / \psi(p_t, \rho_t) & \text{if } \pi_t = p_t \\ 0 & \text{if } \pi_t \neq p_t \end{array} \right\} \tag{2}$$

The notation $\kappa(p, \rho)$ is meant to suggest that long-term credibility is a type of capital for the economy [6]

2.2 Central bank policymaking

There are two types of central banks in our economy, one that is committed and one that is not. Each makes inflation decisions so as to maximize the welfare objective $w(\pi, e)$, which depends positively on inflation and negatively on expected inflation.

2.2.1 Committed policymaking

The committed monetary authority maximizes welfare

$$W(\rho) = \max_{p}\{w(p, e)\} \tag{3}$$

subject to

$$e = e(p, \rho)$$

i.e., subject the inflation expectations function. The solution to this decision problem is a decision rule $p(\rho)$ for inflation that takes into account the management of expectations.

2.2.2 Discretionary policymaking

The discretionary decisionmaker has two parts to his decision problem. First, he decides on an optimal rate of inflation to set if he deviates, d. Second, he decides whether it is desirable to deviate ($\tau = 1$) or not to deviate ($\tau = 0$).

$$V(\rho) = \max_{\tau, d}\{(1 - \tau)w(p, e) + \tau[w(d, e) - \xi]\}.$$

[6] In perhaps more familiar terms, letting $\sigma = c$ be the event in which the decision-maker is of the committed type and $\sigma = n$ be the event in which the type is not committed, then Bayes' rule is that

$$pr(\sigma = c|\pi = p) = \frac{pr(\sigma = c)}{pr(\sigma = c) * pr(\pi = p|\sigma = c) + pr(\sigma = n) * pr(\pi = p|\sigma = n)}$$

with the committed decision-maker always takes the action p, so that $pr(\pi = p|\sigma = c) = 1$.

where ξ is a random utility cost of deviating, governed by a continuous distribution function F with support $[0, \Xi]$.[7] Treating the expectation e as predetermined, the maximization results in decision rules

$$\tau(p, e, \xi)$$
$$d(e)$$

for the optimal discrete and continuos choices.

The decision d simply maximizes momentary utility, $d(e) = \arg\max_{\pi} w(\pi, e)$. There is a critical adjustment cost $\widehat{\xi}$ such that the decisionmaker is just indifferent about whether to deviate or not, given by

$$\widehat{\xi} = [w(d(e), e) - w(p, e)]. \tag{4}$$

The right-hand side of this expression is the one-period gain from behaving in a discretionary manner, sometimes called the *temptation,* so that we define $T(p, e) = w(d, e) - w(p, e)$.

2.3 Rational expectations equilibrium

We now indicate rational expectations restrictions and other consistency conditions in our economy that apply for each date t in our economy. First, short-term and long-term credibility are mechanically connected by

$$\psi(p, \rho) = \rho + (1 - \rho)m(p, \rho), \tag{5}$$

where m is the probability that a discretionary central bank will mimic the committed one by choosing $\pi = p$. That is: specifying a short-term credibility function $\psi(p, \rho)$ is equivalent to specifying a mimicking function $m(p, \rho)$.

Second, imposing one layer of rational expectations, expected inflation is a credibility-weighted average of the two possible outcomes for inflation, the committed government's plan and the deviation by the discretionary government:

[7] The adjustment cost ξ is drawn at the beginning of the period and is private information. These considerations do not play an obvious role in our discussion in the main text, but are important in the context of the "announcement game" studied in appendix D.

$$e(p,\rho) = \psi(p,\rho)\ p + (1-\psi(p,\rho))\ d(e(p,\rho)). \tag{6}$$

Third, imposing another layer of rational expectations, the mimicking function must be consistent with the actual likelihood of deviations, i.e.,

$$m(p,\rho) = 1 - F(T(p, e(p,\rho))$$

where $T(p,e) = w(d,e) - w(p,e)$.

These three conditions serve to determine three "unknown middle sub-period equilibrium functions" $e(p,\rho), \psi(p,\rho)$ and $m(p,\rho)$ as a rational expectations fixed point. Given these functions, the optimization of the committed policymaker determines $p(\rho)$, which in turn implies equilibrium functions $e(\rho)$, $\psi(\rho)$, and $m(\rho)$ as well as the transition rule, $\rho_{t+1} = \rho_t/\psi(\rho_t)$ in line with (2).

Analysis of macroeconomic outcomes under commitment and discretion often exploits the fact that the strategic interaction between the central bank and the private economy can be modelled as a game. From that perspective, the rational expectations equilibrium that we study in this paper is a perfect Bayesian equilibrium, as detailed further in some companion research[8]. In the theory of games, it is fairly standard for there to be multiple perfect Bayesian equilibria and there are well established methods of characterizing the full set of equilibria and various refinements for circumscribing it. When we adopt this approach, we conclude that our rational expectations equilibrium is the unique perfect Bayesian equilibrium, once an alternative pessimistic equilibrium is ruled out using a Bayesian refinement.

3 A basic model of expectations management

To exposit the core ideas about expectations management, we specialize to a simple framework familiar from the work of Kydland and Prescott [1977] and Barro and Gordon [1983a]. In this section, we introduce key aspects of this familiar model and then we study how it works under the assumption that there is no mimicking, so that $\psi = \rho$.

We assume that the central bank – committed or discretionary – maximizes a social welfare function of the form (7)

[8] King, Lu and Pasten [2007b].

$$u(\pi, x) = u^* + \omega_1[x - x^*] - \frac{1}{2}\{\pi^2 + \omega_2[x - x^*]^2\} \tag{7}$$

That is, the central bank likes increases in output towards the efficient level x^* and dislikes departures of inflation (π) and real activity (x) from first-best values of $\pi = 0$ and $x = x^*$. The weights $\omega_i \geq 0$ express the importance of output effects, relative to inflation effects. The parameter u^* captures the level of welfare at the first best, but plays no substantive role in the analysis below. In contrast to some analyses, we are therefore not distinguishing between committed and discretionary central banks in terms of objectives.

The private sector is simply described by

$$x = \alpha(\pi - e), \tag{8}$$

which links variations in real activity (x) to surprise inflation $\pi - e$ as in the analyses of Lucas [1973] and Fischer [1977]. This simple specification is used in many discussions of commitment and discretion in monetary policy, such as Persson and Tabellini [1990]. Moreover, it can be derived from an underlying structural macroeconomic model in which firms set prices one period in advance, as in Woodford [2003], which allows a more detailed interpretation of the parameters α, x^* and ω.

Following the literature, we will treat inflation as the decision variable for the central bank. It is therefore convenient to work with a reduced form objective obtained by combining (7) and (8),

$$w(\pi, e) = \omega_1[\alpha(\pi - e) - x^*] - \frac{1}{2}\{\pi^2 + \omega_2[\alpha(\pi - e) - x^*]^2\}. \tag{9}$$

as we did in section 2 above. To maximize this reduced form objective, it is frequently convenient to use the following first order condition,

$$-\{\pi + \alpha(1 - \Delta)[\omega_1 + \omega_2\alpha(\pi - e) - \omega_2 x^*]\} = 0 \tag{10}$$

where Δ is the partial derivative of expected inflation e with respect to actual inflation π, $\Delta = \partial e/\partial \pi$ when a particular expectations function has been imposed.

3.1 Standard analysis of commitment and discretion

We begin with a quick discussion of standard models of policy under commitment and discretion, by way of review and to highlight aspects of expectations management in each case. Our models are very simple, without any real or nominal shocks, so as to focus attention on the interplay of central bank plans and actions with outcomes for real activity.

3.1.1 Commitment

When there is commitment, the central bank can perfectly manage expectations, since it is a large player and private agents care only about forecasting the central bank's action. It is standard to formalize this interaction as specifying that the committed central bank takes its action π prior to the private sector. In terms of the framework above, then, $\Delta = 1$ and the first order condition dictates that $\pi = 0$. In our context, we say that the committed central bank formulates an inflation plan of $p = 0$ at the start of the period, that expectations are thus perfectly managed so that $e = 0$, and that actual inflation is $\pi = 0$ since the central bank executes the plan.

With zero actual and expected inflation, the real activity measure x_t is also zero and the welfare measure under commitment is

$$w(0,0) = u^* - \omega_1 x^* - \frac{1}{2}\{\omega_2 [x^*]^2\}.$$

so that there is the level of utility under commitment is less than u^* due to real distortions present in the economy that the monetary authority cannot eliminate.

3.1.2 Discretion

When there is discretion, the central bank takes its action after the private sector, so that it cannot manage expectations at all ($\Delta = 0$). Using (10), we find that the central bank's decision rule is

$$d = d(e) = \frac{\omega_2 \alpha^2}{1 + \omega_2 \alpha^2} e + \frac{1}{1 + \omega_2 \alpha^2}(\omega_1 \alpha + \omega_2 \alpha x^*) \tag{11}$$

so that it partially accommodates expectations of inflation and partly tries to stimulate the economy because output is inefficiently low, as stressed by Kydland and Prescott [1977] and Barro and Gordon [1983a].

Imposing rational expectations ($e = d$), the equilibrium discretionary inflation rate is given by

$$d = \omega_1 \alpha + \omega_2 \alpha x^* = B \tag{12}$$

where B is a compound parameter representing extent of the celebrated "inflation bias under discretion", which depends on the deeper parameters $\omega_1, \omega_2, \alpha$, and x^*.

Under pure discretion, there is no sense in which the inflation plan is relevant to either the private sector or the monetary authority: it could be anything. Moreover, with inflation accurately forecasted in equilibrium, there are no effects on real economic activity ($x = 0$) and the level of welfare is

$$w(B, B) = u^* - \omega_1 x^* - \frac{1}{2}\{B^2 + \omega_2 [x^*]^2\}.$$

As stressed in the literature, welfare in equilibrium is consequently lower under discretion than under commitment due to the cost of fully anticipated inflation.

3.2 Uncertainty about commitment or discretion

We are interested in environments in which private agents do not know the type of the central bank – whether it will act in a committed or discretionary manner – when they form expectations. If a committed central bank is in place at the start of the period, then it will choose an inflation plan p knowing that it will subsequently carry through on this plan as its inflation action. In line with our discussion in section 2, we analyze this model by working backwards from the actions of a discretionary central bank, should one exist, choosing inflation after expectations have been formed. Since there is no mimicking, long-term and short-term credibility are identical: we use ψ_t rather than ρ_t in this section because that is the better interpretation in fully dynamic extensions of our analysis.

3.2.1 End: The discretionary central bank

At the end of the period, the discretionary central bank takes its action after expectations are formed, so that $\Delta = 0$ as in the analysis of pure discretion above. Using the first order condition, we can determine that the discretionary central bank's decision rule is unchanged from (11) above. That is, the discretionary central bank's action continues to be given by (11), $d(e) = \frac{\omega_2 \alpha^2}{1+\omega_2 \alpha^2} e + \frac{1}{1+\omega_2 \alpha^2} B$.

3.2.2 Middle: Expectations and discretion

In the middle of the period, private agents will form expected inflation e knowing that there is some chance that there will be a committed central bank in place (ψ) and some chance that there will be a discretionary central bank in place $(1-\psi)$, choosing the action d. Hence, expected inflation will be $e = \psi p + (1-\psi)d$ in line with (6). Thus, even though the discretionary central bank's decision rule is the same, the equilibrium with imperfect credibility is different because there is a different specification of beliefs in (6). That is, the equilibrium inflation action of the discretionary central bank arises as a fixed point between beliefs and actions, just as with the situation of pure discretion studied above, but with a different structure for beliefs. In the current context, the degree of short-term credibility influences the nature of this fixed point, since it it affects the relative weight placed on the committed central bank's plan and the discretionary central bank's action in the formation of expected inflation. However, it is possible to simply and explicitly solve for the equilibrium discretionary central bank action since the equations that define the fixed point are both linear, with the result being,

$$d(p;\psi) = \frac{\omega_2\alpha^2\psi}{1+\omega_2\alpha^2\psi}p + \frac{1}{1+\omega_2\alpha^2\psi}B = \theta p + (1-\theta)B \tag{13}$$

with B defined above as the inflation bias if there is a discretionary central bank with certainty (that is, if $\psi = 0$).

There is thus an "middle subperiod equilibrium function" $d(p;\psi)$ describing the discretionary central bank's response to the inflation plan under the requirement that expectations are rational: the intensity of this reaction, which we call θ, is between 0 and 1 so long as $\omega_2 > 0$ since $\theta = \frac{\omega_2\alpha^2\psi}{1+\omega_2\alpha^2\psi}$. Further, θ increases with the degree of short-term credibility ψ. Correspondingly, with $\psi > 0$ and thus $\theta < 1$, monetary discretion gives rise to a smaller degree of inflation bias – at a given p – because expected inflation is held down by the chance that there is a committed central bank in place.

3.2.3 Start: The committed central bank

We next consider the inflation plan formulated by the committed central bank, which optimizes taking as given the equilibrium expectations rule

$$\begin{aligned} e(p;\psi) &= \psi p + (1-\psi)d \\ &= [\psi + (1-\psi)\theta]p + (1-\psi)(1-\theta)B \\ &= \Delta p + (1-\Delta)B \end{aligned}$$

where $\Delta(\psi) = [\psi + (1-\psi)\theta(\psi)]$ and thus $1-\Delta = (1-\psi)(1-\theta)$.

Notice that imperfect short-term credibility ($\psi < 1$) implies that the central bank can no longer perfectly manage expectations ($\psi < 1$ implies $\Delta < 1$): a part of expectations is invariant to its plan because of the possibility that the inflation action will be taken by a discretionary central bank. However, in its expectations management, the committed central bank takes into account the direct effect of its inflation plan on expectations (ψp) and also the indirect effects that operate through the plan's influence on a private sector beliefs about the discretionary central bank's inflation action, as captured by $(1-\psi)\theta p$.

Overall, the committed central bank's optimal inflation plan is latent in the current setting's version of (10), which is

$$0 = -\{p + \alpha(1-\Delta)[\omega_1 + \omega_2\alpha(p-e) - \omega_2 x^*]\}$$

so that it takes the form

$$p(\psi) = [1 - \frac{\Delta(\psi)}{1+\omega_2\alpha^2(1-\Delta(\psi))^2}] \;\; B \tag{14}$$

There are a number of interesting aspects of the committed central bank's inflation plan in a setting of imperfect short-term credibility. First, there continues to be some inflation bias: a committed central bank does not fully offset the inflation bias from expectations about the discretionary central bank's behavior, because it would face output losses from doing so. Second, the coefficient attached to the bias term B coefficient has the property that it is 1 when $\psi = 0$ (no short-term credibility) and that it is 0 when $\psi = 1$ (full short-term credibility). That is, a committed central bank with low short-term credibility will behave very much like a discretionary central

bank in terms of its inflation plan.

We require that the discretionary central bank announce an inflation plan which is the same as that which the committed central bank will actually adopt. We discuss relaxing this assumption further below.

3.2.4 Implications

We now discuss some aspects of the expectations management equilibrium that we have constructed.

Equilibrium behavior by the discretionary central bank Given the inflation plan just determined, the discretionary central bank's behavior is

$$d(\psi) = \theta(\psi)p(\psi) + (1 - \theta(\psi))B = p(\psi) + \frac{\Delta(\psi)(1 - \theta(\psi))}{1 + \omega\alpha^2(1 - \Delta(\psi))^2}B \tag{15}$$

Thus, in equilibrium, the discretionary authority chooses a higher rate of inflation than the inflation plan, although the effect is minor when short-term credibility is low (because the coefficient on B is close to zero in that setting).

As we proceed through our analysis of this and more complicated models, there are potential notational confusions which arise because of the three subperiods. In the end subperiod, the discretionary central bank's actions are described by a decision rule $d(e)$. With expectations determined rationally, there is a middle subperiod equilibrium function $d(p; \psi)$. Finally, with an optimal plan determined by the central bank, there will be equilibrium discretionary behavior $d(\psi)$. Throughout our analysis, we choose to maintain notation simplicity, seeking to distinguish these differing functions by their arguments and by the words that are attached in a particular context.

Inflation plan, expectation, and action Our findings about the nature of the inflation plan, the inflation expectation, and the inflation action are summarized in Figure 1.[9]. The top panel (A) shows the reaction coefficient θ that governs how

[9] This figure is drawn under the assumption that $\alpha = 3$, so that a one percent unexpected inflation is associated with a 3% increase in real output. Additionally, the figure is constructed under the assumption that $\omega_1 = .03$, that $\omega_2 = .3$, and that $x^* = .1$ (so that it is desirable to stimulate real activity to 10% above its normal level). Taking these coefficients together, the model produces an inflation bias of B=.18. The percentage counterpart of 18% is used in the figure.

These values are suggested by U.S. annual macroeconomic analysis and data from prior to 1970: if there is an annual Phillips curve linking inflation and unemployment with a slope of negative one,

a discretionary central bank's equilibrium inflation action depends on the inflation plan and the reaction coefficient Δ that governs how expected inflation depends on the inflation plan. As can be verified directly from definitions above, these reaction coefficients are increasing in short-term credibility. The second panel (B).shows how the three measures of the inflation rate depend on short-term credibility ψ: (i) the solid line is the committed central bank's inflation choice p, which declines from the inflation bias level (B=.18) to zero as θ rises; (ii) the discretionary central bank's inflation choice d, which is higher than p but also declining with credibility; and (iii) expected inflation, which is a credibility-weighted average of the plan and the discretionary action ($e = \psi p + (1 - \psi)d$)

Real activity Given the expectations function, $e = \psi p + (1 - \psi)d$, it is now easy to determine the behavior of real activity as $x = \alpha[\pi - e] = \alpha[\pi - \psi p - (1 - \psi)d]$. This simple expression implies that there is a recession when there is a committed central bank in place,

$$x = -\alpha(1 - \psi)(d - p)$$

since we just saw that $d > p$. The above analysis, though, also shows that this recession will be small when short-term credibility is very high (since $\psi = 1$) and when it is low (since $d - p$ is small because the coefficient on B in (15) is close to zero). By contrast, when there is a discretionary central bank in place, there is a boom

$$x = \alpha\psi(d - p)$$

This is a small effect when short-term credibility is small (ψ close to 0) but it is increasing as short-term credibility increases, both because the "slope" $\alpha\psi$ increases and because the size of $(d - p)$ increases. Hence, the short-term credibility benefits the discretionary government when he produces an inflation surprise. The third panel (C) of Figure 1 provides more detail on these real activity outcomes. First, the magnitude of the real contraction brought about by a committed government is largest when there is an intermediate level of short-term credibility, since this is when

as was widely believed at the time, and if there is an Okun's law linkage between unemployment to real output with a coefficient of three, then $\alpha = 3$. The idea that output was perhaps 10% too low in a noninflationary environment is captured by $x^* = .1$. The welfare weights are selected to display a high discretionary inflation rate (18%) and also to generate a period in which discretionary inflation remains high for some time during the transition (see Figure 6 below).

the effective slope ($\alpha(1-\psi)$) and the size of the inflation difference ($d-p$) are both large in magnitude. With high short-term credibility, alternatively, there is a large gap between d and p, for example, but there is a small effective slope.

3.3 Expectations and alternative central banks

We have discussed the interaction between the alternate central banks as a sort of "leader-follower" game between committed and discretionary central banks. Of course, both of these central banks will not be present during the single period under study. Instead, it is private agent uncertainty about which central bank will be present, along with the sequencing of inflation plan and inflation action, that gives rise to this leader-follower structure. In this subsection, we comment on various aspects of the strategic interaction between expectations and the alternative central banks.

3.3.1 Benchmark analysis with imperfect credibility

The committed central bank actively manages beliefs given the expectations function that it faces. However, with imperfect credibility, it cannot exert the same degree of control on expectations as with perfect credibility: private agent think that there is some chance that inflation will be chosen in a discretionary manner. This simple model delivers a number of interesting implications about managing expectations in an environment of imperfect credibility, as follows:

Management of expectations: A committed central bank in a setting of imperfect credibility will manage the beliefs of agents in part by changing what is rational to believe that a discretionary central bank will do;

Imperfect credibility and inflation bias: Even a committed and forward-looking central bank will not eliminate the inflation bias arising from discretionary policy-making if it is in a setting of imperfect credibility;

Failure of inflation plans: Inflation plans will turn out to fail, when central banks turn out to be discretionary in their actions, even though these plans were consistent with optimal management of expectations by a committed central bank;

Real effects of inflation policies: Under imperfect credibility, there will be a real boom when the central bank is discretionary and a real recession when there is a committed central bank.

3.3.2 Alternative equilibria

In discussing the interaction between the committed and discretionary central banks above, we suggested that it is appropriate to consider there being a "leader follower" interaction between the two banks, along the lines of the Stackelberg equilibrium familiar from duopoly theory. Further, our procedure for finding an equilibrium was to maximize the committed central bank's objective, subject to rationality of private sector expectations and the best-response function of the discretionary central bank. The alternative game-theoretic approach would be to determine all of the Bayesian perfect equilibria of our model, a route which we pursue in some companion research.[10] In our context, this approach turns up one other equilibrium that we have so far not discussed: a pessimistic expectations equilibrium in which $e = B$ and $p = d = B$ for all values of ψ. This is also a Nash equilibrium in a simultaneous move game between the two governments and the private sector; it is a weak Bayesian perfect equilibrium as well in our three subperiod game. However, if we use a stronger concept of Bayesian perfect equilibrium which applies Bayes' rule to off-equilibrium beliefs, then this no longer is an equilibrium. In any event, we do not think of this as a candidate equilibrium of interest because does not permit the management of expectations (expectations manage policy, rather than the other way around) .

3.3.3 Discretionary monetary authority signalling

We have so far assumed that the discretionary monetary authority formulates an observable inflation plan that is identical to that which a committed authority would select. This seems like an arbitrary restriction from the standpoint of the model, but it is not. Essentially, with a discretionary central bank able to freely send a signal by selection of a plan, then it faces a cost of deviating from a noninformative signal of p. If it could reduce inflation expectations by announcing a lower plan, then this would be raise its welfare because expected inflation is bad for it. But a signal of $a < p$ would be interpreted by rational, Bayesian agents as indicating that the authority was discretionary and the result would be $e = B$, with expected inflation rising rather than falling. In our companion research, we outline a simple "announcement game" which makes clear that the signal must be noninformative: in

[10] King, Lu and Pasten [2007b]. The existence of a number of subgame perfect equilibria is standard in Stackelberg-type models (see, for example, the discussion in Osborne and Rubenstein [1994, pp. xx]).

the lingo of signalling games, the only outcome is a pooling one. The announcement game that we study is unusual from the standpoint of signalling games in that the committed authority generates a publicly observable inflation plan as a by-product of his action; it is optimal for the discretionary authority to send the same signal.

4 Managing mimicking

So far, we have discussed expectations management when credibility is exogenous. We now consider an extension of our static model that allows for management of an endogenously determined short-term credibility measure. To generate endogenous mimicking, we also introduce the idea that it is costly for the discretionary-type central bank to deviate. As discussed in section 2 above, we suppose that a monetary discretionary authority faces a utility cost of ξ if it deviates, with this fixed cost being drawn from a continuous distribution F with support $[0, \Xi]$. Accordingly, an authority will deviate if the gross gain exceeds the cost, i.e., $w(d, e) - w(p, e) > \xi$ and otherwise it will mimic. Consequently, we can write the mimicking function as

$$m = 1 - F(T(p, e)).$$

with the temptation being $T(p, e) = w(d(p, e), e) - w(p, e)$. The key implication of this condition is that the committed authority partly controls expected inflation by affecting the probability that a discretionary authority will mimic, adopting the committed authority's inflation plan as its inflation action. That is: $e = e(p, \rho) = \psi p + [1 - \psi]d(p, \rho)$ with $\psi = \psi(p, \rho)$.

4.1 An analytical example

For the purpose of our initial discussion of endogenous mimicking and its influence on the inflation plan, it is useful to abstract from some of the other complexities that are present in the full model that we outlined in section 2. For example, if the objective function is linear in real activity ($\omega_2 = 0$) then it is the case that the discretionary government's decision rule is just $d = \omega_1 \alpha = B$. Further, as a consequence, it is not necessary to solve a fixed point problem to determine the equilibrium behavior of the discretionary authority. Finally, there is no impact of the committed authority's plan (p) on the discretionary authority's plan (d). These are clearly very special results

relative to the model explored in section 3 above, but they serve an important role in the current subsection by focusing attention just on the effect of the inflation plan on the likelihood of a discretionary government deviating from the plan.

Given the discretionary inflation rate, it then follows that the gross welfare under discretion is

$$w(B,e) = u^* - \frac{1}{2}B^2 + \omega_1\alpha(B-e)$$

and that under the inflation plan is

$$w(p,e) = u^* - \frac{1}{2}p^2 + \omega_1\alpha(p-e)$$

so that the gain from deviating is

$$\begin{aligned} w(B,e) - w(p,e) &= -\frac{1}{2}(B^2 - p^2) + \omega_1\alpha(B-p) \\ &= \frac{1}{2}(B-p)^2 \end{aligned}$$

which does not depend on expected inflation. Hence, mimicking is a function just of the inflation plan,

$$m = 1 - F[\frac{1}{2}(B-p)^2]$$

Further, there is a good deal that we can say about this mimicking function. First, if planned inflation is zero, then mimicking will be positive so long as $\Xi > \frac{1}{2}B^2$ Second, mimicking will be zero when $p = B$. Third, if the distribution of costs takes the form $F(x) = (x/\Xi)^{1/2}$ then mimicking is just a linear function of the deviation between discretionary and planned inflation,

$$m = 1 - \frac{1}{\sqrt{2\Xi}}(B-p).$$

which is increasing for p between 0 and B. Given a mimicking function, short-term credibility is then

$$\psi(p;\rho) = \rho + (1-\rho)m(p)$$

By increasing the planned inflation rate, the monetary authority increases short-term credibility because it increases mimicking, i.e., it increases the likelihood that the inflation plan will be carried out even if a discretionary government is in place.

4.1.1 Expected and unexpected inflation

In general, expected inflation is a credibility-weighted sum of the inflation actions p and $d = B$. With the specific objective used in the previous subsection, the discretionary authority chooses $d = \omega_1\alpha$ so that expected inflation takes the form

$$e(p) = \psi(p)p + (1 - \psi(p))B.$$

>From the standpoint of the committed monetary authority, then, there is an expectations function that depends on its inflation plan, as it did above, but it now does so for a new reason, which is that credibility is endogenous. Further, with the distribution assumption made above, it follows that expected inflation is a quadratic function of p. Therefore, unexpected inflation is given by

$$\begin{aligned} p - e(p) &= (1 - \psi(p))(p - B) = (1 - \rho)(1 - m(p))(p - B) \\ &= -(1 - \rho)\frac{1}{\sqrt{2\Xi}}(p - B)^2. \end{aligned}$$

Unexpected inflation will always be negative if the committed central bank selects a lower planned rate of inflation than its discretionary counterpart would choose. The ex ante likelihood that the central bank is of a discretionary type $(1 - \rho)$ dictates the scale of unexpected inflation.

4.1.2 Optimal inflation

The committed central bank will maximize

$$u(p, x) = u^* - \frac{1}{2}(p)^2 + \omega_1(x - x^*)$$

subject to

$$x = \alpha(p - e(p))$$

with the expectations function given above. An efficient inflation plan requires that $0 = -p + \alpha(1 - \Delta)\omega_1 = -p + \alpha[(1 - \rho)\frac{1}{\sqrt{2\Xi}}(d - p)]\omega_1$ so that optimal inflation is

$$p = \frac{\alpha\omega_1(1 - \rho)}{\sqrt{\Xi/2} + \alpha\omega_1(1 - \rho)}B$$

since $d = B$. Hence, a central bank that is operating in a setting of imperfect information about its type ($\rho < 1$) will choose an inflation plan that does not fully eliminate the inflation bias, even when it can manage expectations. The committed central bank recognizes that a policy of zero inflation would be more credible than its optimal plan, but also that higher output losses would occur in that setting. An increase in ρ implies a lower optimal plan with $p = 0$ arising in the full credibility case $\rho = 1$. However, in the pure discretionary case, i.e. $\rho = 0$, the optimal p does not necessarily equal B, contrasting with the analysis in section 3 where no mimicking is allowed. This is because a discretionary type will not find deviation optimal with a sufficiently high fixed cost. Therefore, even though the government is of a discretionary type, there is still some probability that an inflation plan will be fulfilled.

4.2 Managing mimicking in the full model

We now evaluate mimicking in the full model which adds costly mimicking to the structure of section 3, so that there is an effect of p on the action of the discretionary authority d if it deviates. We also assume that there is a mimicking cost distribution of the form displayed in Figure 2.[11]

As discussed in section 2, the mimicking probability and short-term credibility are solutions to a fixed point problem. As shown in upper panel of Figure 3, if the planned inflation rate p is higher, then it is more likely that a discretionary monetary authority will mimic, with the increase in the likelihood of mimicking due to a reduction in temptation. Given m and ρ, higher p reduces the size of the deviation from the plan implied by the optimal response of the discretionary government, which in turn reduces the reward from deviating. The lower panel of Figure 3 illustrates

[11] The fixed cost is distributed according to a beta distribution, with support [0,1]. There are two parameters of the beta distribution, which can be adjusted to give it a wide variety of shapes. If these parameters are η_1 and η_2 then the beta density is

$$f(z;\eta_1,\eta_2) = \frac{\Gamma(\eta_1+\eta_2)}{\Gamma(\eta_1)\Gamma(\eta_2)} z^{\eta_1-1}(1-z)^{\eta_2-1}$$

where Γ is the gamma function. An additional convenient property of the beta distribution is that there are simple analytical formulae for the moments: the mean is $\eta_1/(\eta_1+\eta_2)$.

In our analysis, the distribution above governs the standardized random variable $z = \xi/\Xi$. When we choose $\eta_1 = 1$, $\eta_2 = 5$ and $\Xi = .05$ so that the mean is is $\Xi/6$ or just under .01. The $\Xi = .05$ value exceeds $\frac{1}{2}B^2 = \frac{1}{2}(.18)^2 = .0162$, so that there is indeed mimicking when there is zero inflation.

the effect of varying p from 2% to 2.5% on m given two different level of long term credibility $\rho = 0.2$ and $\rho = 0.5$.

There is a particularly strong effect of the inflation plan on mimicking when credibility is low. The upper of Figure 3 shows an illustrative case, where ρ is respectively 0.2, 0.5 and 0.95. When the inflation plan is around 2.5%, a variation of only 0.5% in inflation creates a difference of 8% in mimicking probability as a result of the steep slope associated with that low ρ. By contrast, if ρ is at 0.5, the 0.5% variation in the inflation plan only creates a 1% difference in m. The relatively big impact of the inflation plan on the mimicking probability when ρ is low makes the choice of the plan p particularly important and subtle for an authority with low credibility, perhaps a new authority, since it is not very trusted by the private sector.

In turn, the effect of the plan on short-term credibility also depends on p in a similar manner as mimicking (panel A in Figure 4), since $\psi = \rho + (1-\rho)\, m\,(p, \rho)$. Therefore, a higher planned inflation rate is more credible because it is more likely to be mimicked. Moreover, when long-term credibility ρ is low, the impact of p on ψ is particularly significant. Having this optimal response of ψ to p in hand, we now turn to its consequence for the optimum inflation plan.

We have previously stressed that the optimizing committed monetary authority evaluates the consequences of its plan on inflation expectations. Relative to our analysis in section 3 and in Figure 1, there is new channel of influence displayed in this section: short-term credibility. Since inflation expectations take the form (**??**), the effect of a marginally higher inflation plan is

$$\frac{\partial e(p,\rho)}{\partial p} = \psi + [1-\psi]\frac{\partial d(p,\rho)}{\partial p} + [p-d]\frac{\partial \psi(p,\rho)}{\partial p}$$

The first term in this expression is simply the direct effect of the plan on expectations: it produce a higher rate of expected inflation. The second one looks familiar from the discussion in section 3 above: it is the indirect effect via the discretionary government's response to the plan. However, due to the endogenous ψ, the discretionary government' response is more complicated. Recall that the response function (13) specifies that d is the credibility-weighted average of p and B. In section 3, the weight is exogenous so that an increase in inflation plan also increases the inflation action of the discretionary government. But, now, the weights are affected by the inflation plan through its effects on ψ: an increase in p raises ψ, and in turn lowers

the weight on B in d. If this effect is strong enough, as in Figure 4, d drops rather than rises for some values of ρ and p.

The third term is new to the current setup: it also concerns the effect of a marginally higher inflation plan on the likelihood that the plan will be carried out, i.e., on its credibility. We have seen above that a higher rate of inflation raises the mimicking rate thus increases short-term credibility. Since the discretionary inflation rate d is always higher than the inflation plan p it follows that this third channel involves a heterodox effect: a higher planned inflation rate lowers expected inflation because it raises the likelihood that the plan will be carried out. A committed monetary authority with low long-term credibility that had an inflation plan of about 8 percent in Figure 4 would be bedeviled by rising rapidly output losses if it sought further reductions in inflation, as dramatically illustrated by panel D of Figure 4.

Relative to the monetary authority described in section 3, the committed decision-maker will choose a higher planned rate of inflation, because he knows that low rates of inflation will decrease the current credibility of the inflation plan and move expected inflation against him, so that there will be higher output losses when he sticks to his plan. Hence, the endogenous reaction of ψ to p sets a barrier for the committed government in selecting a low inflation plan. We note that the optimal p (indicated by '$*$') always lies on the right of the nonmonotonicity regime of e. Thus, for a short-horizon decision-maker, the effect of management of expectations is to promote greater gradualism.

Since we can determine the optimal p and the associated levels of m, e, d, and ψ at each level of ρ, it is also possible to depict the way in which these variables depend on the state, long-term credibility, as in Figure 5. Note that increasing long-term credibility reduces each of the three inflation variables p, e and d, while reducing the output loss that a committed central bank faces (this is x^c in the bottom panel). Notice also that the extent of mimicking is initially high at low credibility, then decreasing at higher levels of ρ.

5 Evolution of long-term credibility

We have provided basic examples of two aspects of expectations management. In section 3, we studied how a committed monetary authority with a short-horizon would optimally choose inflation when there was exogenous imperfect credibility. In section

4, we show this same authority would manage credibility with endogenous mimicking. In each case, the examples were structured so that the expectations function and credibility function could be worked out in closed form. However, the simple model does not describe the dynamic evolution of credibility that seems essential to understanding many issues in macroeconomics. It is to this topics that we now turn, working under the assumption that central banks and private agents are only concerned with single period objectives but that the economy lives forever. In this introduction to the interplay between mimicking, credibility and learning, we thus take two steps. First, we study the mechanical dynamics of the evolution of credibility, by specifying a mimicking function $m(\rho)$. Second, we study how long-term credibility evolves when the committed central bank maximizes momentary welfare, without regard to the implications of its actions for its long-term credibility.

5.1 Mechanical dynamics and general principles

Specification of a mimicking function allows us to write *short-term credibility* – the probability that the action will be that of a committed central bank – as

$$\psi(\rho_t) = \rho_t + (1 - \rho_t) m(\rho_t).$$

where we initially treat $m(\rho)$ as an arbitrary mimicking function, not specifically restricted by the analysis above. This formula makes clear that short-term credibility will be higher than long-term credibility because of mimicking behavior. As stressed in the discussion in section 3 above, it is short-term credibility that plays a role in the determination of output and inflation in the economy at a given point in time.

Further, as discussed above, since the observed inflation rate is the only information in the economy about the nature of central bank type, then Bayesian learning implies that

$$\rho_{t+1} = \begin{cases} \frac{\rho_t}{\psi_t} = \frac{\rho_t}{\rho_t + (1-\rho_t) m(\rho_t)} & \text{if } \pi_t = p_t \\ 0 & \text{if } \pi_t \neq p_t \end{cases}$$

That is, in terms of the structure discussed in section 2, it is Bayes' rule that is governs the linkage between inflation plans – which influence the rate of mimicking – and the evolution of the state variable of our model, which is long-term credibility

In terms of general principles, there is thus an interesting interplay between short-term credibility ψ and long-term credibility ρ. If short-term credibility is high (close

to one) then long-term credibility evolves very slowly (if $\psi = 1$, then Bayesian learning specifies that $\rho_{t+1} = \rho_t$). This is because there is little information in the observation that inflation is at the planned rate p. On the other hand, if short-term credibility is low, then there is a good deal of information in the fact that inflation is at the planned rate. Further, short-term credibility can be high either because long-term credibility is high or because the likelihood of mimicking (m) is high. In terms of the latter case, $\psi = 1$ if $m = 1$ irrespective of the level of ρ. So, if there is substantial mimicking, then long-term credibility will evolve very slowly. More specifically, the high rate of mimicking at low levels of long-term credibility displayed in panel A of Figure 5 means that short-term credibility is relatively high, so that long-term credibility grows slowly when it is low.

5.2 A reference dynamic analysis

The equilibrium functions just displayed in Figure 5 and described above make it possible to trace out the dynamic path followed if there is a committed monetary authority in place. In panel A of Figure 6, there is a relatively high level of short-term credibility throughout, which means that long-term credibility accumulates slowly; mimicking also declines over time as long-term credibility rises. In panel B of Figure 6, the time paths for the inflation plan (p), the inflation deviation (d) and expected inflation (e) are displayed. The inflation plan declines slowly from an initial level of about 6%, with expected inflation mirroring its decline. The committed government faces a lengthy period of below average output, initially of about 2.5% and maintaining that low level for a number of periods (years).

The extremely gradual transition raises an interesting question: would a long-horizon monetary authority, who maximized discounted expected utility, take more aggressive inflationary actions so as to more rapidly build credibility? Our companion analysis deals with this question, as well as considering how shocks to the economy affect the timepath of credibility.[12]

[12] King, Lu and Pasten [2007a].

6 Reports and rules as inflation commitments

To this point, we have depicted a committed monetary authority which chooses an inflation plan with two forms of knowledge. First, he knows that he can execute this plan because he has an internal structure for doing so: a commitment technology. Second, he knows that private agents will respond to his announced plan knowing that there are two types of monetary authorities, one type that will execute the inflation plan for sure and another that may execute or may deviate from the plan depending on a privately observed shock. We have drawn a strong distinction between the committed authority, who decides on the inflation outcome prior to the formation of inflation expectations, and the discretionary authority, who decides on inflation after the formation of inflation expectations.

We now drop the assumption of a commitment technology, supposing that there are simply two types of monetary authorities, disciplined and discretionary. The disciplined monetary authority has a deviation fixed cost of $\Xi > \frac{1}{2}B^2$, which is assumed to be known by all participants in the economy. The discretionary monetary authority continues to have a fixed cost that is stochastic, with support $0 \leq \xi \leq \Xi$. Hence, one decision-maker is almost surely less likely to deviate than the other.

6.1 Inflation reports

To think about the role that an inflation report might play, we reinterpret our prior equilibrium under the following alternative assumptions.

First, at the start of each period, the monetary authority issues an inflation report which depends on the state of the economy: this report is given by the equilibrium function $p(\rho)$ displayed in panel B of Figure 5: it is the equilibrium inflation plan derived above. This announcement is accordingly optimal for the disciplined decision-maker if his plan is assumed credible by private agents. A discretionary decision-maker would never issue any alternative report, for to do so would reveal its type and produce an inflation bias equilibrium at $\pi = B$.

Second, in the middle of the period, private agents form inflation expectations according to $e(\rho)$, also displayed in panel B of Figure 5, because they believe that a disciplined decision-maker's plan will be followed with a probability $\psi(\rho)$ and that there will be deviation to $d(\rho)$ with probability $(1 - \psi(\rho))$.

Third, at the end of the period, all monetary decisionmakers choose whether to

follow the plan or deviate. The equilibria that we displayed above made it optimal for a relatively high fixed cost discretionary monetary authorities to follow the inflation plan, which we called mimicking above. That is, in Figures 3-5, there is always some likelihood that a discretionary monetary authority would follow the plan for all values of long-term credibility, with such patient authorities having ξ above the critical value $\hat{\xi}(\rho) < \Xi$. The highest cost discretionary monetary authority has a fixed cost of Ξ, identical to that of the disciplined authority in the current context. Hence, a disciplined decisionmaker will not depart from the announced inflation plan.

There thus is a rational expectations equilibrium in which the committed government is simply a disciplined government – one with a high cost of changing its plan, but not higher than that for some discretionary authorities – which uses an inflation report to signal its planned actions and then executes these as if it had access to a commitment technology.[13]

6.2 Inflation rules

We have just seen that it is possible to drop the assumption that it is technologically feasible to commit and to replace it with the assumption that a disciplined central bank issues an inflation report and then finds it desirable *ex post* to produce that planned rate of inflation.

The inflation report, while natural, also is not a crucial element of the model as it is presently structured: we are studying a rational expectations model with imperfect credibility, but the participants in the model all understand exactly that a disciplined central bank would issue the inflation report $p(\rho)$; and that a discretionary central bank would issue the same report, so that the report is *not informative*. Accordingly, a disciplined central bank that uses the inflation rule $p(\rho)$, implemented with an appropriate interest rate or monetary aggregate instrument policy, would also secure the same performance. In the jargon of current policy discussions, there could be a system of implicit inflation targets.

Hence, although we began with a model in which there are explicit observable inflation plans as part of a committed monetary authority's operating policy, our

[13] While this distinction between commitment and discipline may seem somewhat artificial in the current context, it is a simple example of a finding in King, Lu and Pasten [2007a], where the disciplined monetary authority is a patient one (a fixed, high discount factor) and the discretionary monetary authority has a fluctuating discount factor.

setup does not deliver a case for either explicit inflation reports or, indeed, require any policy announcements as a result of the assumption that the structure of the economy is common knowledge for all participants.

6.3 Managing expectations once again

A disciplined central bank working with either of these systems – explicit inflation reports or implicit inflation targets – must design its policy, however, taking into account that its actions affect expectations and that its credibility is imperfect. While the assumption of a commitment technology can be dropped, this does not mean that a disciplined central bank seeking to deliver low inflation can ignore imperfect credibility and the consequent necessity of managing expectations. Instead, it must adopt the particular report or action $p(\rho)$ identical to that which an authority would if it were feasible to manage expectations through a publicly observable, fully committed inflation plan.

7 Summary and conclusions

We provide a basic analysis of expectations management in an environment of imperfect credibility, extending a standard macroeconomic framework for this purpose. In our model, a monetary authority pursuing low inflation but concerned about real activity manages expectations taking into account imperfect credibility and, in particular, gauging the effect of his policy action on private sector beliefs about the likelihood and intensity of discretionary policy actions that would be taken by an alternative type of policymaker.

Our analysis highlights the nature of optimal expectations management by a central bank in a very simple model and displays its implications for the evolution of credibility over time. It leaves open a number of important topics, which never-theless seem feasible to explore in extensions of our approach.

First, our analysis concerns a model in which all public and private decision-makers have one period horizons, even though the economy operates for many periods. Modern central banks consider inflation policy from the standpoint of intertemporal objectives that incorporate the consequences of current actions for future economic performance. It is therefore important to extend the analysis to situations in

which central banks have present discounted value objectives: this is the topic of our companion research, which explores expectations management influences transitional credibility dynamics and the response of the economy to shocks.

Second, our analysis introduces imperfect information about central bank type only in a very limited manner. It delivers the conclusion that expectations management can be undertaken via explicit binding plans, announcements such as inflation reports, or simply via a well understood regular operating method such as an implicit inflation target. It seems important to enrich the extent of private agent uncertainty about central bank type in future work, with an eye to understanding when each of these institutional arrangements is desirable.[14]

Third, the macroeconomic model that we employ is deliberately "old school" in its inflation dynamics and does not spell out detailed microeconomic foundations. It seems important to explore the nature of inflation management in more realistic macroeconomic models with better micro foundations.

Fourth, issues of expectations management frequently are suggested to be important in the choice of the monetary instrument in general and the setting of the chosen instrument at a point in time. By assuming that the central bank directly controls inflation, we have abstained from consideration of these issues, but it seems important to do so systematically.

Fifth, imperfect credibility and associated topics of expectations management have been suggested to be important for understanding specific historical periods in the monetary histories of the U.S., the U.K., and other countries. It is of interest to begin a more detailed exploration of these connections.

[14] Stein (1989) shows that unobserved decision-maker type may lead to imprecise policy announcements, using the "cheap talk" approach of game theory.

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Notation Table

Symbol	Definition
π	inflation
p	inflation plan
d	inflation deviation
e	expected inflation
x	real activity
x^*	efficient real activity
u	welfare ($u(\pi, x)$)
w	reduced form welfare ($w(\pi, e)$)
ω_1, ω_2	welfare weight on output terms
α	slope of Lucas-Fischer supply function
e	expected inflation
Δ	partial derivative of e wrt π
B	inflation bias ($B = \omega_1\alpha + \omega_2\alpha x^*$)
ψ	short-term credibility
θ	reaction coefficient ($\frac{\omega_2\alpha^2\psi}{1+\omega_2\alpha^2\psi}$)
ρ	long-term credibility
m	mimicking probability
W	value function for committed central bank
V	value function for discretionary central bank
T	temptation
ξ	random fixed cost of deviating
Ξ	largest fixed cost
F	distribution of fixed costs

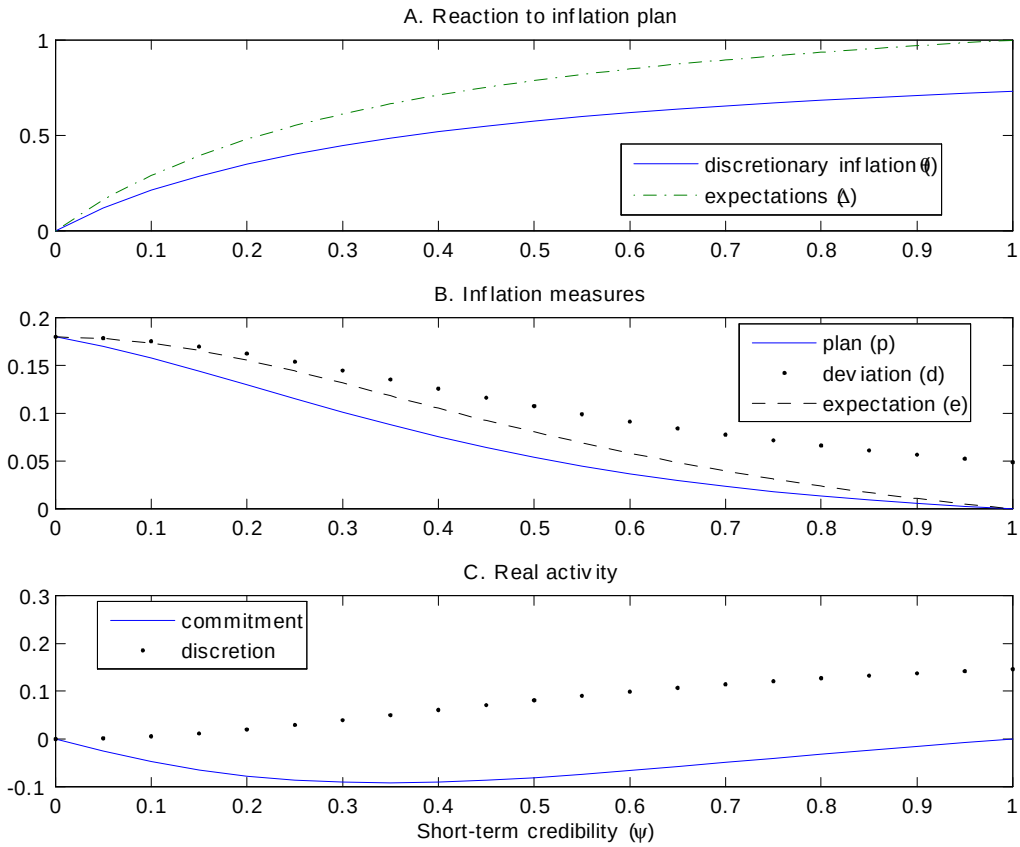

Figure 1: Inflation and real activity with imperfect credibility

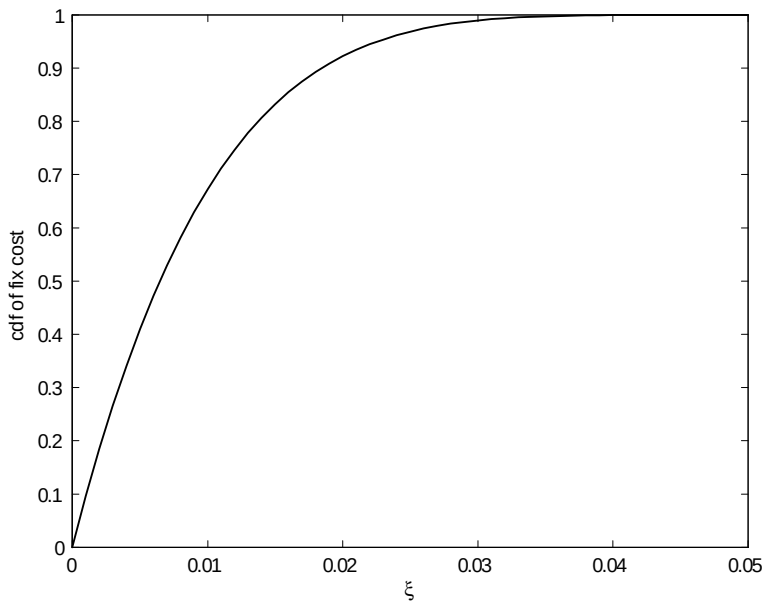

Figure 2: Distribution of fixed costs

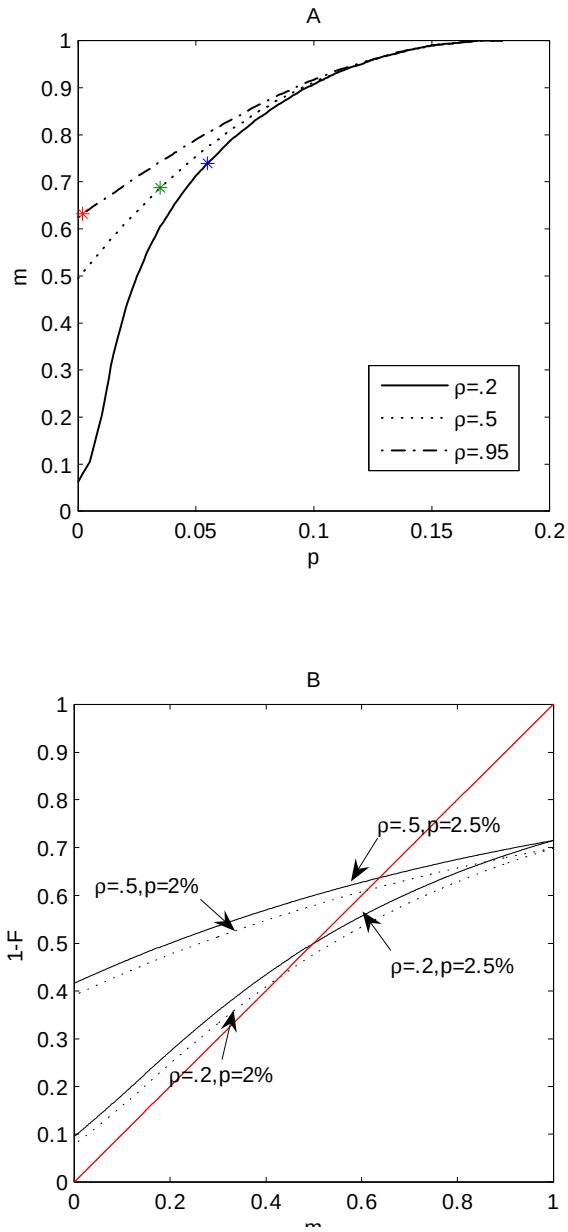

Figure 3: Short-horizon monetary authority. Panel A shows the effect of plan (p) on mimicking (m) at three different levels of credibility. Panel B shows the effect of changes in the inflation plan on the fixed point for mimicking at two different levels of credibility.

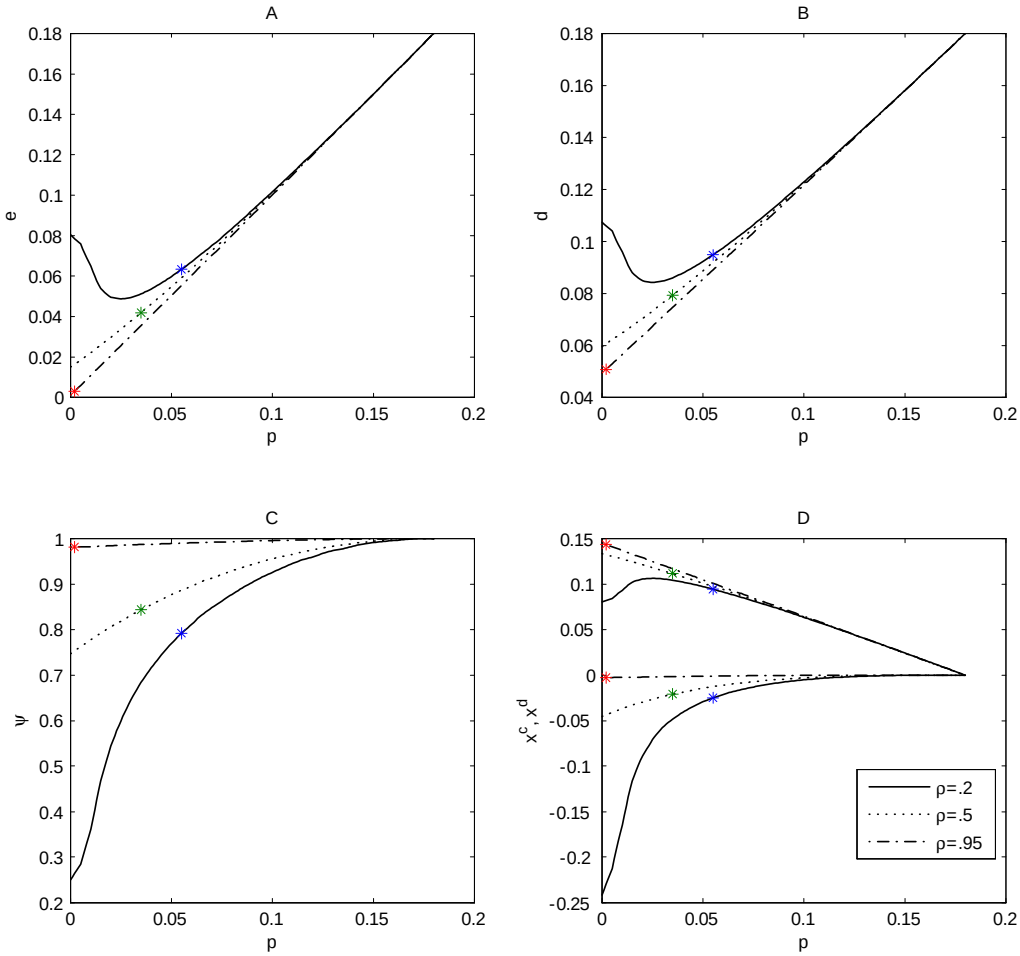

Figure 4: Short-horizon monetary authority: Panel A effect of plan on expected inflation (e), Panel B on discretionary inflation (d), Panel C on short-term credibility (ψ) and Panel D real activity (x).

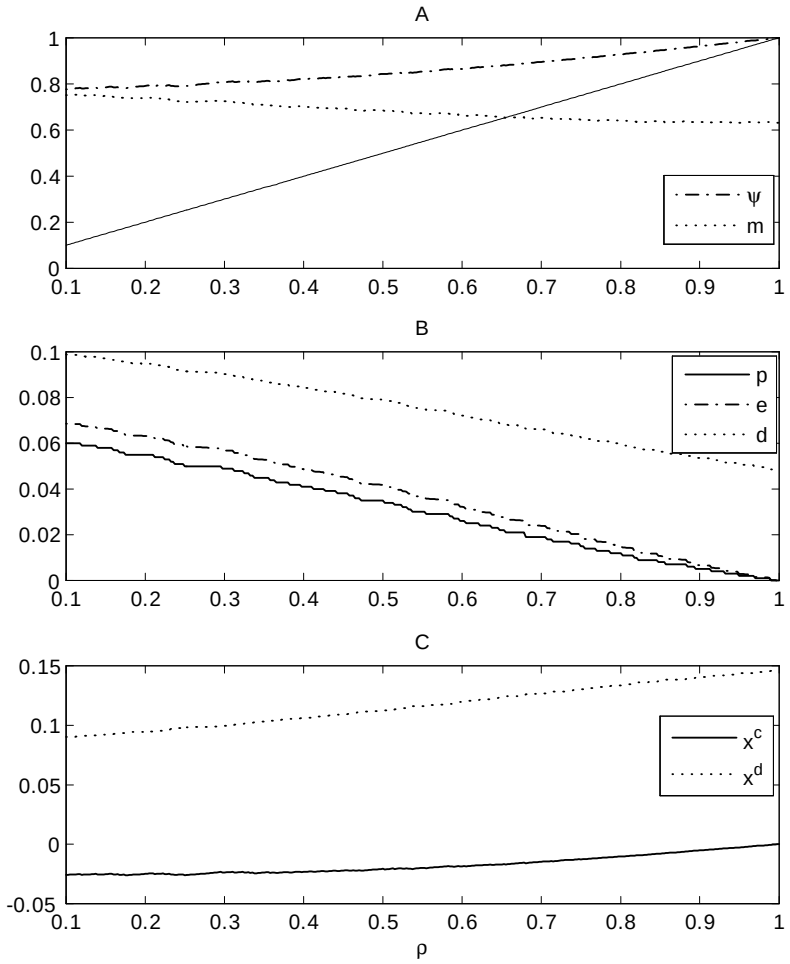

Figure 5: Short-horizon monetary authority: equilibrium functions.

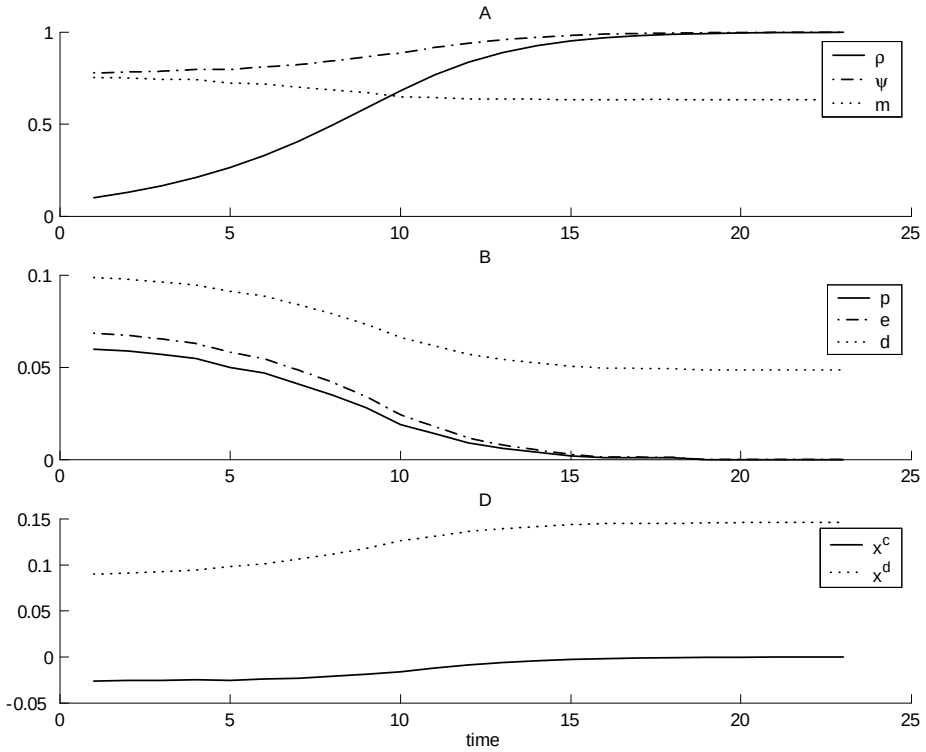

Figure 6: Short-horizon monetary authority: time series response from low initial long-term credibility. Panel A shows long-term credibility (ρ), short-term credibility (ψ) and mimicking (m); Panel B shows the inflation plan (p), expected inflation (e) and discretionary inflation (d); Panel C shows real activity under commitment and discretion.