The historical evolution of the wealth distribution:
A quantitative-theoretic investigation

Joachim Hubmer, Per Krusell, and Tony Smith

Yale, IIES, and Yale

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Evolution of top wealth inequality (Kopczuk 2015)
Overview: Objective

- calibrate a serious quantitative macro model that explains the full US wealth distribution, including the Pareto tail
- study the transition path: starting in the 1960s, feeding in observed changes in earnings inequality and tax rates
- can the standard macro-inequality framework explain movements in the wealth distribution?
Overview: Findings

- model is partially successful in explaining the evolution of the wealth distribution
  - magnitude of increase in inequality explained for bulk of distribution
  - misses speed of changes at the very top and short-run dynamics
- active channels:
  - decreasing tax progressivity has a dramatic effect on the wealth distribution
  - increase in idiosyncratic labor income risk has in general a dampening effect on wealth inequality via the precautionary savings channel (vanishes at the top)
  - changes in $r - g$ not important, partly working in the opposite direction
- cautious prediction for 21st century: long-term effects of decreasing tax progressivity on wealth inequality
Quantitative model

- Aiyagari ’94 framework:
  - log labor income as sum of persistent and transitory component; adjusted at the top to match the observed Pareto tail in labor income
  - stochastic discount factor follows AR1 process (Krusell Smith ’98 extended)
  - stochastic i.i.d. return on capital
  - progressive taxation: use data on federal effective tax rates for 11 income brackets (Piketty & Saez 2007)
  - parsimonious modeling of social safety-net: 60% of tax revenues rebated as lumpsum transfers

- time-varying tax system and labor income process
Main qualitative mechanism

- stochastic-$\beta$ alone generates a Pareto tail in the wealth distribution
  - add stochastic return to capital and Pareto tail in labor income to improve quantitative properties of the model
  - Pareto tail in labor income alone would be inherited by wealth distribution, but it’s too high
- follows from random growth theory (Kesten 1973, see also Gabaix 2009)
  - mechanism has been employed by Benhabib, Bisin and Zhu (2011), Nirei & Aoki (2015), Piketty & Zucman (2015)
- main alternative calibration (Castañeda, Días-Giménez, Ríos-Rull 2003) cannot deliver this Pareto tail
Stochastic-\(\beta\) yields stochastic, linear savings decisions

Marginal Propensities to Save in the Right Tail

\(\log(k)\)

\(0.92\)\(\quad 0.93\)\(\quad 0.94\)\(\quad 0.95\)\(\quad 0.96\)\(\quad 0.97\)\(\quad 0.98\)\(\quad 0.99\)\(\quad 1\)\(\quad 1.01\)

Marginal Propensities to Save in the Right Tail

\(\beta_{\text{high}, e_{\text{high}}}\)
\(\beta_{\text{high}, e_{\text{low}}}\)
\(\beta_{\text{med}, e_{\text{high}}}\)
\(\beta_{\text{med}, e_{\text{low}}}\)
\(\beta_{\text{low}, e_{\text{high}}}\)
\(\beta_{\text{low}, e_{\text{low}}}\)
Gives rise to a Pareto tail in the wealth distribution
Calibration strategy

- earnings process, tax rates, social safety net calibrated to observables
- randomness in discount factor and return to capital calibrated to replicate the wealth distribution in the initial steady state (1960s)
- focus on tail coefficient alone misleading: even if say the richest 10% can be described exactly by a Pareto distribution, the shape parameter only tells us how wealth is distributed within these 10%, not how much wealth the top 10% control as a fraction of total wealth
Calibration: Stochastic-$\beta$ and $r$

**Stochastic-$\beta$:**
- follows AR(1) process
- $\mu = 0.92$, $\rho = 0.992$, $\sigma = 0.0019$
- i.e. in cross-section, standard deviation = 0.0148
- i.e. over 50 years, mean reversion is $1/3$

**Stochastic Return to Capital:**
- pre-tax return $(1 + r_t\eta_t)$
- $\eta_t \sim^{i.i.d} N(1, 0.725)$
- i.e. in steady state, 90% have return $(1 + r^*\eta_t) \in [0.9874, 1.1437]$
## Matching the wealth distribution

### US Wealth distribution in 1967:

<table>
<thead>
<tr>
<th></th>
<th>Top 10% Share</th>
<th>Top 1%</th>
<th>Top 0.1%</th>
<th>Top 0.01%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data*</td>
<td>70.8%</td>
<td>27.8%</td>
<td>9.4%</td>
<td>3.1%</td>
</tr>
<tr>
<td>Model</td>
<td>70.6%</td>
<td>28.1%</td>
<td>9.5%</td>
<td>2.9%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>fraction w negative wealth</th>
<th>Bottom 50% share</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data*</td>
<td>8.0%</td>
<td>4.0 %</td>
</tr>
<tr>
<td>Model</td>
<td>7.0%</td>
<td>3.1 %</td>
</tr>
</tbody>
</table>


- model matches wealth distribution well on its entire domain
Observed change 1: decrease in tax progressivity

- federal effective tax rates (Piketty & Saez 2007): income, payroll, corporate and estate taxes
Observed change 2: increase in labor income risk

Observed change 3: increase in top labor income shares

- adjust standard AR1 in idiosyncratic productivity by imposing a Pareto tail for the top 10% earners: calibrated tail coefficient decreases from 2.8 to 1.9 (Piketty & Saez, 2003 [updated series -2011])
Main result: evolution of top wealth shares

- **Top 10% Wealth Share**
  - 1970: 70
  - 1980: 75
  - 1990: 80
  - 2000: 85
  - 2010: 90

- **Top 1% Wealth Share**
  - 1970: 30
  - 1980: 35
  - 1990: 40
  - 2000: 45
  - 2010: 50

- **Top 0.1% Wealth Share**
  - 1970: 2
  - 1980: 4
  - 1990: 6
  - 2000: 8
  - 2010: 10

- **Top 0.01% Wealth Share**
  - 1970: 1
  - 1980: 2
  - 1990: 4
  - 2000: 6
  - 2010: 8
Other statistics

- captures dynamics of capital stock (but capital ≠ wealth) and share of wealth held by asset-poor
Summary of transitional dynamics

- model captures the salient features of the evolution of the US wealth distribution
- perfect foresight assumption does not seem to be critical
- robust to CES production function with elasticity $> 1$
- shortcomings:
  - miss on short-run dynamics (heterogeneous portfolios and valuation effects?)
  - explosion of wealth concentration at the very top (0.1% and above) as measured by Saez & Zucman (2014) not explained well
Main channels

What fraction of the increase in the top 1% wealth share do the three channels account for?

<table>
<thead>
<tr>
<th>earnings risk</th>
<th>top earnings</th>
<th>taxes</th>
<th>combined</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.19</td>
<td>0.05</td>
<td>0.82</td>
<td>0.65</td>
</tr>
</tbody>
</table>

- larger earnings risk induces higher precautionary savings (vanishes for the rich), depressing the interest rate and thus increasing the Pareto tail coefficient (i.e. decreasing top wealth inequality)

- in general equilibrium, the average tax level does not matter much for wealth inequality, but changing progressivity has a large effect
Capital in the 21st Century?

- long-run effects of decrease in tax progressivity
Other channels: what about $r - g$?

- Increase in $r - g$ decreases wealth inequality in the medium run (a few decades).
- Pareto tail coefficient decreases (i.e., top wealth inequality increases), but very slowly.
- More important in short-run: low-asset agents’ savings decisions more elastic w.r.t. the interest rate.
- Random growth models generally feature slow transitions, it takes long to fill a thick long tail (see Gabaix, Lasry, Lions, Moll [2015]).
Conclusion: where next?

- speed of changes at the very top hard to match
- asset price movements and portfolio choice?
  - why are portfolios heterogeneous?
  - why are asset prices moving that much? (outside the scope of our model - What would Shiller say?)

What would Shiller say?
Price-earnings ratio (Shiller)
Perfect foresight vs myopic transition

- **Top 10% Wealth Share**
- **Top 1% Wealth Share**
- **Top 0.1% Wealth Share**
- **Top 0.01% Wealth Share**
CES with Elasticity of Substitution $> 1$

- $\sigma = 1.25$ (Karabarbounis and Neiman, 2014)
$r - g$?

- model increase in $r - g$ as temporary 50% increase in interest rate
- partial equilibrium, holding wage and transfers constant

![Graph showing shock to interest rate over time](image)
The image shows graphs of various economic indicators over time, specifically focusing on the bottom 50% wealth share, top 1% wealth share, Gini Wealth, and Gini Income. The graphs illustrate the changes in wealth distribution and income inequality as measured by the Gini coefficient, with pre-tax and post-tax comparisons. The data is presented over 100 years, highlighting trends and changes in wealth distribution and income inequality.