A Model of the International Monetary System

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Abstract

We propose a simple model of the international monetary system. We study the world supply and demand for reserve assets denominated in different currencies under a variety of scenarios: under a Hegemon vs. a multi-polar world; when reserve assets are abundant vs. scarce; under a gold exchange standard vs. a floating rate system; away from or at the zero lower bound (ZLB). We rationalize the Triffin dilemma which posits the fundamental instability of the system, the common prediction regarding the natural and beneficial emergence of a multi-polar world, the Nurkse warning that a multi-polar world is more unstable than a Hegemon world, and the Keynesian argument that a scarcity of reserve assets under a gold exchange standard or at the ZLB is recessive. We show that competition among few countries in the issuance of reserve assets can have perverse effects on the total supply of reserve assets. Our analysis is both positive and normative.

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1 Introduction

The International Monetary System (IMS) is a topic of enormous importance. Throughout history, it has gone through radical transformations that have shaped global economic outcomes. It has been the constant focus of world powers, has fostered innumerable international policy initiatives, and has captured the imagination of some of the best economic minds. Yet it remains an elusive topic with little or no intellectual organizing framework. A manifestation of this fuzziness is that, even among economists, there is no consensus regarding the defining features of the IMS.

In this paper, we take the IMS to be the collection of three key attributes: (i) the supply and demand for reserve assets; (ii) the exchange rate regime; (iii) international monetary institutions. We provide a theoretical equilibrium framework of the IMS which captures these different aspects. This framework allows us to match the historical evidence, to make sense of the leading historical debates, and to provide new insights.

The key ingredients of our model are the following. The world demand for reserve assets arises from the presence of international investors with risk-averse mean-variance preferences. Risky assets are in elastic supply, but safe (reserve) assets are supplied by one (monopoly Hegemon) or a few (oligopoly multipolar world) risk-neutral reserve countries under Cournot competition. Reserve countries issue reserve assets denominated in their currencies and have limited commitment: Ex-post, they face a trade-off between depreciating their currencies to limit repayment and paying a "default cost". Ex-ante, they issue debt before interest rates are determined.

The model is flexible and modular. It allows to us incorporate a number of important additional features: nominal rigidities, fixed and floating exchange rates, Zero Lower Bound (ZLB), currency of pricing, endogenous reputation, and liquidity preferences. The model is solvable with pencil and paper and delivers closed form solutions.

We start our analysis with the case of a Hegemon issuer. The model features the possibility of self-fulfilling confidence crises à la Calvo (1988). There are three successive regions of the IMS for increasing levels of issuance: a Safety region, an Instability region, and a Collapse region. In the Safety region the Hegemon does not depreciate irrespective of investor expectations. In the Instability region the Hegemon depreciates whenever confronted with unfavorable investor expectations. In the Collapse region the Hegemon depreciates irrespective of investor expectations.

The Hegemon can exploit its monopoly power to capture monopoly rents in the form of a positive endogenous safety premium on reserve assets. The Hegemon optimally trades off issuing more reserve assets at a higher interest rate vs. less at a lower rate. The Hegemon also takes into consideration that levels of issuance beyond the Safety region into the Instability region expose him to confidence crises. The Hegemon therefore faces a stark choice between issuing fewer assets but safe for sure and more but at the risk of a collapse. This provides a rationalization of the famous Triffin Dilemma (Triffin (1961)). In 1959, Triffin exposed the fundamental instability of the Bretton Woods system by predicting its demise, on the grounds that the US, facing a growing demand for reserve assets, would eventually issue so many reserves as to be exposed to a confidence crisis that would lead to a depreciation of the Dollar. Indeed, time proved Triffin right. The Nixon administration, facing a full blown run on the Dollar, first devalued the Dollar vs. gold in 1971 (the “Nixon shock") and ultimately abandoned convertibility and let the Dollar float in 1973.

The deeper logic behind the Triffin Dilemma extends beyond this particular historical episode. Indeed, it can be used to understand how the expansion in reserves provided by Britain under the Gold-Exchange standard system of the 1920s ultimately led to a confidence crisis on the pound, in part lead by France attempting to liquidate their sterling reserves, resulting in Britain going off-gold and depreciating the pound in 1931. Figure 4 illustrates the expansion of monetary reserve assets in the 1920s and the reversal following the collapse in 1931. Our model shows that the Triffin Dilemma is likely to resurface even under the current system of floating exchange rates, because reserve assets embed the implicit promise that reserve currencies will not be devalued in response to world economic disasters.

One avenue to mitigate the Triffin Dilemma and the associated instability of the IMS is to introduce policies that reduce the demand for reserve assets for any level of global savings. Policies to this effect have often been proposed by economists looking to reform the IMS. In their most recent incarnation they have included swap lines amongst central banks, credit lines by the IMF as a Lender of Last Resort (LOLR), and international reserve sharing agreements such as the Chang Mai initiative. Our framework can capture the rationale behind these policies when the demand for safe assets is in part driven by precautionary savings.

Our theoretical foundations allow us to shed a normative light on the Triffin Dilemma. We show that the Hegemon might under- or over- issue from the perspective of social welfare. One might conjecture that, by analogy with standard monopoly problems, the Hegemon under-issues from a social welfare perspective. While this can certainly happen in our model, we also show that it is possible for the Hegemon to over-issue. We trace this surprising result to the fact that the options faced by the Hegemon involve two interrelated dimensions: the traditional quantity dimension analyzed in standard monopoly problems and a novel risk dimension.

Despres, Kindleberger and Salant (1966) dismissed the concerns of Triffin regarding the stability of the US international position by providing a “minority view" according to which the US acted as a “world banker" providing financial intermediation services to the RoW. According to this view the external balance sheet of the US is naturally composed of safe-liquid liabilities and risky-illiquid assets. In the recent period of global imbalances (1998 onwards) this view has been brought to prominence and quantified empirically by Gourinchas and Rey (2007*a*,*b*). Despres, Kindleberger and Salant (1966) view this form of intermediation as natural and stable. Our model offers one bridge between the Triffin and the minority views: while our model shares the “world banker" view of the Hegemon, it emphasizes that banking is a fragile activity subject to self-fulfilling runs and panics. Importantly, the runs in our model offer a more considerable challenge than runs on private banks à la Diamond and Dybvig (1983) since there is no natural LOLR with a sufficiently large “war chest" (fiscal capacity) to support a Hegemon of the size of the US.

To analyze the implications of the structure of the IMS for world output we introduce nominal rigidities. The central force captured by our model is that the world natural interest rate increases with the issuance of reserve assets, provided that they are safe. As long as world central banks can adjust interest rates, they can stabilize output and insulate their economies from variations in the supply of reserve assets. However, when they cannot adjust interest rates, such as under a Gold-Exchange standard or under floating exchange rates at the ZLB, world output fluctuates with variations in the supply of reserve assets. Recessions arise when reserve assets are "scarce", i.e. when there is excess demand for reserve assets at full employment and at prevailing world interest rates. Interestingly, we show that in these circumstances, a Hegemon faces a perfectly elastic demand curve, and therefore has strong incentives to increase issuance. In fact, the Hegemon stretches itself and issues all the way up to its debt capacity, which is endogenously limited by its inability to commit. However, not only might this limited amount of reserve assets not be enough to prevent a world recession, but it also exposes the IMS to confidence shocks that, by wiping out the effective stock of safe assets, might trigger an even more severe recession. Nominal rigidities also introduce an extra ex-post incentive for the Hegemon to devalue in order to stimulate its own economy which further curtails its ex-ante credibility. Indeed, such domestic output stabilization considerations played an important role in the decision of the UK in 1931 and US in 1971 to devalue their currencies.

The scarcity of reserve assets in the IMS has often be associated with recessionary pressures. Most famously, Keynes (1923) argued against the return by all countries to a gold standard at pre-WWI parities on the grounds that these would have required a policy of tight money, i.e. high interest rates, leading to an increase in demand for reserve assets that if unaccommodated by an increase in the supply of these assets would have led to a recession. Furthermore, he argued that a peg to gold would have left interest rate policy to be determined by fluctuations (“vagaries") in the demand for reserve assets rather than focusing on domestic macroeconomic stabilization. Similar concerns about the recessionary effects of safe asset shortages have been recently expressed after the developed world hit the ZLB at the onset of the Great Recession.

We have so far focused on an IMS dominated by a Hegemon which has a monopoly over issuance of reserve assets. Of course, this is an idealization and the real world which, while currently dominated by the US issuance of reserve assets, features other competing issuers. Indeed, Figure 6 shows that the Euro and the Yen already play a partial role as reserve currencies and there are speculations that the future of the IMS might involve other reserve currencies, such as the Chinese Renminbi.

We also explore the equilibrium consequences of the presence of multiple issuers of reserve assets for the total quantity of reserves assets and the stability of the IMS. More precisely, we analyze a multipolar world of a few oligopoly issuers of reserve assets competing à la Cournot for the safety premium monopoly rents. Loosely speaking, the thrust of our analysis is that the benefits of a more multipolar world are U-shaped in the number of reserves issuers: a lot of competition is good but a little competition might be worse than monopoly.

With limited commitment and a large number of issuers, the safety premium is small and each issuer finds it optimal to issue in its Safety region. As the number of issuers increases to infinity, the model converges to perfect competition, with no instability and a zero safety premium. This paints a bright picture of a multipolar world as extolled by, among others, Eichengreen (2011).

However, a darker picture emerges in the presence of only a few issuers. We formalize the warning from Nurkse (1944) that a disadvantage of the presence of multiple competing reserve issuers is that it introduces coordination problems across a priori substitutable reserve currencies. Nurkse famously pointed to the instability of the IMS during the interregnum of Dollar and Sterling as reserve currencies in the 1920s. Figure 5 shows that in 1929 60% of world reserves were held in pounds and 37% in dollars. The decade of the 1920s was dominated by fluctuations in the share of reserves denominated in these two currencies (Eichengreen and Flandreau (2009)). It was precisely these frequent switches of rest of the world reserve holdings between the two currencies that lead Nurkse to diagnose an increased difficulty to coordinate on the ultimate reserve asset. The Gold-Exchange standard of the 1920s eventually collapsed with the UK devaluing first in 1931 and the US devaluing in 1933.

We model coordination problems via equilibrium selection involving different investor expectations for each issuer's likelihood of depreciation. We capture the Nurkse (1944) conjecture by assuming that investor expectations are favorable with only a Hegemon, but are only favorable to one or the other of the two issuers with a duopoly. We show that not only the IMS becomes more unstable going from a Hegemon to a duopoly but also that the total supply of reserve assets might fall due to increased coordination problems.

Even absent coordination problems, the multipolar model shows that commitment problems limit and in some cases reverse the benefits of competition. On the one hand, competition reduces the quantity of reserve assets issued by each country and alleviates, for a given interest rate, the ex-post temptation of each country to depreciate its currency. On the other hand, competition increases the interest rate on reserves (i.e. decreases the monopoly rents) and increases the ex-post temptation of each country to depreciate its currency. This latter negative effect arises for both static and dynamic reasons. In the static model the higher interest rate, for a given amount of debt, increases the fiscal burden and therefore the benefit of a depreciation. In the dynamic model, we endogenize limited commitment as the threat of losing future cheap financing following a depreciation. In this dynamic set-up the lower monopoly rents due to increased competition lower commitment because they decrease the punishment following a deviation from the commitment outcome. In both the static and dynamic model, the negative effects of competition might outweigh the positive effects when only a few competitors are present, but to illustrate the potential strength of the dynamic negative effects we provide a leading case in which total issuance never increases beyond the maximum amount that a Hegemon would have credibly issued even as the number of competing issuers increases to infinity.

Finally, our model points to two forces that lead to an endogenous emergence of a Hegemon in a multipolar world: fiscal capacity and the currency of pricing in the goods market. To highlight the first force, we consider the case of a duopoly in which the two countries differ in their fiscal capacity. We model fiscal capacity as an additional cost associated with debt repayment due to the distortionary effects of the taxation necessary to raise sufficient funds for repayment. We capture the non-linear effect of fiscal capacity by assuming the extra cost to be zero for lower levels of debt (the Safety region) but positive for higher levels of debt (the Instability Region). We characterize under which configurations even small differences in fiscal capacity lead to an asymmetric equilibrium in which the country with the marginally higher fiscal capacity issues a much larger quantity of reserve assets.

To highlight the second force, the currency of pricing in the goods market, we consider a duopoly and assume that prices are fully rigid in one of the two reserve currencies. In this case the real return of debt denominated in the currency in which the goods are priced is always safe. The crucial consequence is that the country who issues the currency of pricing endogenously acquires de facto full commitment, while the other country still faces limited commitment. As a result, in equilibrium the country who issues the currency of pricing issues more, potentially much more, reserve assets than the other country. Our model offers one rationalization for the association in the empirical regularity that prices of goods are disproportionately quoted and sticky in the dominant reserve currency, in US dollars at present and in British sterling in the 1920s.

Related Literature Early literature on the structure of the IMS focused on discussions of the Gold and Gold-Exchange standards. Most notable in this literature is the intellectually masterful, but ultimately unsuccessful, attempt by Keynes to prevent a return to gold parity following WWI

(Keynes (1923)). Nurkse (1944) offers an insightful retrospective analysis of the instability of the interwar IMS. The focus then shifted to the competing plans of England, as envisioned and represented by Keynes (1943), and the US, as envisioned and represented by White (1943), presented at the Bretton Woods conference in 1944.[1] Finally, a series of contributions arose from Triffin (1961) diagnosis of the Dilemma. Kenen (1960) is an early attempt to assess the logic of the Triffin Dilemma with related contributions by Kenen and Yudin (1965), Hagemann (1969).[2] More recently Farhi, Gourinchas and Rey (2011), Obstfeld (2011) have argued that the logic of the Triffin Dilemma is still relevant to the modern IMS.

The more recent literature has predominantly focused on the asymmetric risk sharing between the US and the rest of the world (Despres, Kindleberger and Salant (1966), Gourinchas and Rey (2007*a*), Caballero, Farhi and Gourinchas (2008), Caballero and Krishnamurthy (2009), Mendoza, Quadrini and Rıos-Rull (2009), Gourinchas, Govillot and Rey (2011), Maggiori (2012)). The literature on sovereign default has developed models in which consumption smoothing or risk sharing generate a desire to borrow internationally (Eaton and Gersovitz (1981), Aguiar and Gopinath (2006), Arellano (2008)) potentially in the presence of self-fulfilling debt crises (Calvo (1988), Cole and Kehoe (2000)). We differ from this literature by incorporating monopoly and oligopoly à la Cournot in an asymmetric risk sharing model. In our model the reserve issuers crucially take into account not only that they can change the riskiness (quantity) of risk of the debt they issue, as in small open economy models of sovereign default, but also the world price that they can obtain for a given quantity of risk (price of risk).

In Section 5 in the instances when we consider sticky prices at the ZLB, our work is related to Caballero and Farhi (2014), Caballero, Farhi and Gourinchas (2016), Eggertsson and Mehrotra (2014), Eggertsson et al. (2016) who also investigate the potential recessionary effect of the scarcity of (reserve) assets. Our contribution is to analyze the optimal provision of these assets from the perspective of a Hegemon who incorporates the effects of its issuance on world output. We show that in the absence of limited commitment the equilibrium amount of safe assets is always sufficient to avoid a recession and characterize the conditions, under limited commitment, in which this is not the case.

In Section 6.3 when we study Nurkse instability as a deterioration of coordination in the presence of multiple competing issuers, our work is complementary to He et al. (2015) who study the selection of reserve assets among two possible candidates in a global games environment that trades off liquidity and relative fundamentals for an exogenous amount of issuance. Our studies are complementary since we do not focus on equilibrium selection, which we take as exogenous,

[1]The paper here cited as Keynes (1943) was presented in Parliament by the Chancellor of the Exchequer but is customarily attributed to J.M. Keynes and we follow this attribution.

[2]See also: Aliber (1964, 1967), Fleming (1966), Cooper (1975, 1987).

but instead focus on endogenous and strategic issuance both under a Hegemon and a multipolar world.

In Section 7 our work on Cournot competition in the issuance of assets is related to the literature on competing monies. Under full commitment in the perfect competition limit, as the number of issuers increases to infinity, the model delivers the efficient outcome of full insurance for the Rest of the World (RoW) and no safety premium. This result is consistent with Hayek (1976) view that competition in the supply of monies is beneficial, and runs counter to the opposite view articulated by Friedman (1960). This limit result breaks down under limited commitment even in the absence of coordination problems among investors. This result is related to arguments by Klein (1974), Tullock (1975), Taub (1990) and, most recently, Marimon, Nicolini and Teles (2012) in the context of competition between monies.[3]

2 The Hegemon Model

In this section we introduce a basic model that allows us to capture the core forces of the IMS. We take the defining characteristic of a reserve asset to be its safety and liquidity. We think of the world financial system as being characterized by a scarcity of reserve assets, which can only be issued by a few countries. We trace the scarcity of reserve assets to commitment problems which prevent most countries from issuing significant amounts. In this section we consider the limit case in which there is a single issuer of reserve assets. We call this configuration the Hegemon model to stress the nature of a IMS dominated by a single country. In Section 6 we consider a multipolar model with several issuers of reserve currencies.

2.1 Model Set-up

There are two periods ($t = 0, 1$) and two classes of agents: the Hegemon country and the Rest of the World (RoW), composed of a competitive fringe of international investors.[4] There is a

[3] Marimon, Nicolini and Teles (2012) analyze monopolistic competition among issuers of differentiated monies in the presence of limited commitment and find that each issuer choice of issuance does not depend on the elasticity of substitution between different monies. The equilibrium is inefficient and associated with too low real balances, too high inflation, and too high nominal interest rates. Our Section 7 analyzes competition in a dynamic model and reaches different but related conclusions. We model competition as an increase in the number of issuers of safe assets in a Cournot equilibrium rather than an increase in the elasticity of substitution between monies. In their model, total issuance, individual issuance, the individual short-term benefits of inflating, and the individual long-term costs in terms of lost future rents, are all independent of the degree of competition. In our model, total issuance is also independent of the degree of competition, but individual issuance, the individual short-term benefits of depreciating, and the individual long-term costs in terms of lost future rents, all decrease with the degree of competition and are are exactly inversely proportional to the number of issuers.

[4] We take the RoW to be composed of many countries and within each country many types of reserve buyers (central banks, private banks, investment managers, etc...). Therefore we assume the RoW to be competitive and take world

single good produced by an endowment at $t = 0$ and the endowment is split equally between the Hegemon and the RoW: $w_0 = w_0^*$. Starred variables denote RoW variables. There are two assets, a risky bond in perfectly elastic supply and a nominal bond issued exclusively by the Hegemon and denominated in its currency. The risky asset exogenous returns between time $t = 0$ and $t = 1$ are $\{R^r_H, R^r_L\}$ with $R^r_H > 1$ and $0 < R^r_L < 1$.[5] The low realization of the risky asset at $t = 1$, which we refer to as a disaster, occurs with probability $\lambda \in (0,1)$.[6]

The RoW representative agent has mean-variance preferences over consumption at time $t = 1$ and does not consume at $t = 0$:

$$U^*(C_1^*) \equiv \mathbb{E}[C_1^*] - \gamma\, Var[C_1^*].$$

The Hegemon representative agent is risk neutral over consumption in both periods:

$$U(C_0, C_1) \equiv C_0 + \delta \mathbb{E}[C_1],$$

with the rate of time preference set such that $\delta^{-1} = \mathbb{E}[R^r]$.[7]

Confidence Crises. At time $t = 1$, after uncertainty about world output is resolved, the Hegemon decides the change in its exchange rate, denoted by e, vis a vis the RoW, with the convention that an increase in e represents a Hegemon currency appreciation. The Hegemon bonds' ex-post return in units of the foreign currency is Re, where R is the nominal yield that was determined at $t = 0$ and e is the exchange rate adjustment. For simplicity, we assume that the Hegemon can only choose two values of $e = \{e_H, e_L\}$, with $e_H = 1$ and $e_L < 1$. We normalized the exchange rate at time zero to be $e_0 = 1$. Therefore $e_H = 1$ corresponds to no depreciation, and $1 - e_L$ is the percentage depreciation of the reserve currency. We assume throughout the paper that $e_L = \frac{R^r_L}{R^r_H}$. The assumption simplifies the analysis at little cost to the economics by making the Hegemon debt, when it is risky, a perfect substitute for the risky asset. [8]

In this basic set-up, we assume that deviation from some "commonly agreed upon" path, a

prices of assets as given.

[5]The reader can think of R^r as the return on a risky bond that is not a reserve asset. For simplicity, we introduce a single risky asset in the model, but one could more generally think of many gradations of riskiness.

[6]We assume that the RoW cannot short the Hegemon bond, i.e. it cannot issue the bond. This clarifies the nature of the monopoly of the Hegemon, but is not a binding constraint since the RoW in equilibrium is a purchaser of the bond.

[7]As will become clear this assumption makes the Hegemon indifferent between different levels of investment in the risky asset.

[8]In an extension, one can consider a different configuration $e_H > 1$ and $e_L < 1$ to allow for the possibility that the reserve asset can be a hedge, a negative "beta" asset, rather than a risk-less asset. In this paper we consider the risk-less configuration since it provides most of the economics while making the model as simple as possible.

state-contingent plan, of the exchange rate generates a utility loss (at $t = 1$) for the Hegemon. We assume that the Hegemon can only decide to depreciate after a disaster, and if it chooses to do so it pays a utility cost proportional to the depreciation: $\tau(1 - e_L)$, with $\tau > 0$. While in the present one period set-up this cost is exogenous, we show in Section 7 that it can be further rationalized as the loss of future monopoly rents, in the form of cheaper financing, that the Hegemon suffers after a deprecation of its currency.[9]

The timing of decision follows the self-fulfilling crisis model of Calvo (1988). The timeline is summarized in Figure 1. Starting from the last decision and proceeding backward we have that at $t = 1$, after observing the realization of the disaster, the Hegemon sets its exchange rate taking as given the interest rate on debt, R, and the amount of outstanding debt to be repaid to the RoW, $b \geq 0$. At $t = 0^+$ a sunspot is realized and the interest rate R on the quantity of debt being sold by the Hegemon, b, is determined and the RoW forms its portfolio. The sunspot can takes value safe (s) with probability α and risky (r) with the complement probability. At time $t = 0^-$ the Hegemon decides how much debt b to issue and its investment in the risky asset.

The crucial element in this Calvo timing is that the amount of real debt being sold, b, is decided before the interest rate to be paid on this debt, R, is determined and cannot be adjusted depending on the interest rate. This timing generates the possibility of multiple equilibria depending on the RoW investors expectations regarding the future Hegemon exchange rate e if the disaster occurs. Indeed, in equilibrium, there will be three regions for b, a Safety region, an Instability region, and a Collapse region. In the Safety region, $e = 1$ independently of the realization of the sunspot so that the Hegemon debt is safe. In the Collapse region, $e = e_L$ independently of the sunspot so that the Hegemon debt is risky. In the Instability region, $e = 1$ and hence the Hegemon debt is safe if the sunspot realization is s, and $e = e_L$ and hence the Hegemon debt is risky if the sunspot realization is r.

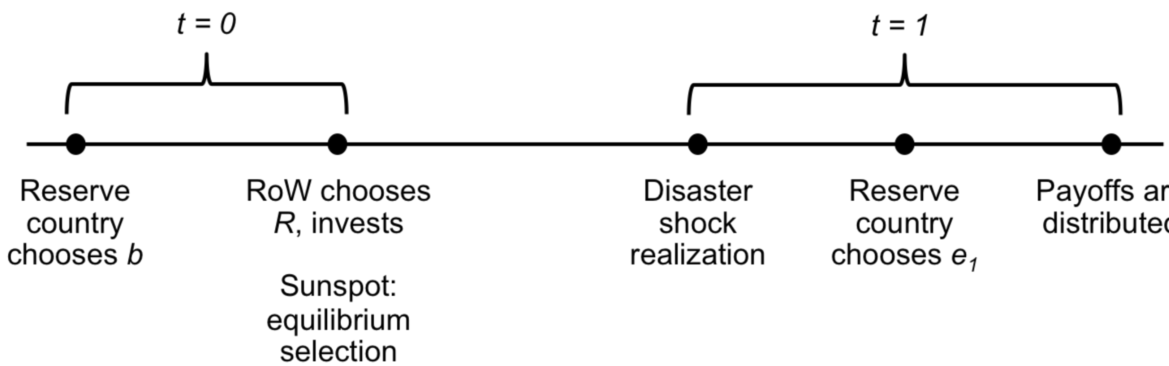
Given the importance of this timing, it is useful to define short hand notation for expectation operators.

Definition 1 *We define $\mathbb{E}^+[x_1]$ to denote the expectation taken at time $t = 0^+$ of random variable x_1, the realization of which will occur at $t = 1$. We further define $\mathbb{E}^s[x_1]$ to be the expectation taken at $t = 0^+$ conditional on the safe realization of the sunspot, and $\mathbb{E}^r[x_1]$ to be the expectation taken at $t = 0^+$ conditional on the risky realization of the sunspot. We define $\mathbb{E}^-[x_1]$ to be the expectation taken at $t = 0^-$ before the sunspot realization.*

At each stage agents make their decisions to maximize their expected utilities subject to their

[9] We focus on the incentives of the Hegemon to depreciate in bad rather than good times. This is a stylized way to capture the notion that the temptation to depreciate is higher after a bad shock. This would happen if the Hegemon were also risk averse but to a lesser extent of RoW.

Figure 1: Timeline



Note: The timeline of decisions for the one-period Hegemon model.

respective budget constraints. The RoW budget constraints are:

$$w^* = s^* + b,$$

$$s^* R^r + bRe = C_1^*,$$

where s^* is the real value invested in the world risky asset.

The Hegemon budget constraints similarly are:

$$w - C_0 = s - b, \tag{1}$$
$$sR^r - bRe = C_1. \tag{2}$$

We abuse the notation and already include the zero net-supply constraint on Hegemon debt. We emphasize that the reader should think of b as the subset of total debt that is bought by RoW residents. Both countries are also subject to the restriction $b \geq 0$ and $s \geq 0,\ s^* \geq 0$.[10]

RoW Demand Function for Debt The RoW optimization problem is given by :

$$\max_b \quad \mathbb{E}^+[C_1^*] - \gamma\, Var^+(C_1^*),$$

$$s.t. \quad w^* = s^* + b \quad s^* \geq 0 \quad b \geq 0,$$

$$s.t. \quad s^* R^r + bRe = C_1^*.$$

[10]We restrict C_0 to be positive but allow C_1 to be negative.

If the Hegemon debt is expected to be safe, then the optimality condition for the portfolio of the RoW is:

$$R^s(b) = \bar{R}^r - 2\gamma(w^* - b)\sigma^2, \tag{3}$$

where $\bar{R}^r = \mathbb{E}[R^r]$. This demand function for Reserve currency debt is linear and increasing in the amount of debt being bought b.[11] Interest rates on debt increase the more debt is being bought, and decrease in the risk aversion of the RoW (γ), the background riskiness of the economy (σ^2), and the savings/endowments of the RoW.[12]

If instead the Hegemon debt is expected to be risky, then it is a perfect substitute for the risky asset. No arbitrage then requires that $R = R^r_H$ so that $\mathbb{E}^r[Re] = \mathbb{E}[R^r]$, and the demand for the Hegemon debt is infinitely elastic.[13]

Liquidity and Networks Effects We have derived the linear demand curve for reserve assets in equation (3) on the grounds of risk and risk aversion (mean variance). The reader is encouraged not to take γ as a deep parameter of household risk aversion but as a proxy for features (institutional constraints, regulatory requirements, etc...) of the world economy that lead the rest of world to demand reserve assets. In this spirit we now show that our model can also capture elements of liquidity and network effects while maintaining the simplicity of the linear demand curve.

We extend the model by adding a “reserve asset in the utility function" component which captures the extra utility benefits arising from the holdings of reserve assets. Importantly we follow Stein (2012) in assuming that these liquidity benefits of holding bonds only arise if they are safe, and hence reserve assets.[14] We further allow for network effects by assuming that the liquidity benefits not only depend on individual but also on aggregate holdings.

[11] While technically the demand curve for debt expresses the demand quantity $b(R^s)$ as a function of R^s, in the interest of convenience we will abuse the convention and often refer to the inverse demand function $R^s(b)$ as the demand curve and to $R'^s(b) = 1/b'(R^s)$ as the slope of the demand curve. Whenever the distinction is meaningful in understanding the paper, we shall be explicit about the concept of demand function that we employ.

[12] The demand for safe assets as a macroeconomic force has been analyzed in different contexts also by: Dang, Gorton and Holmstrom (2015), Gorton and Ordonez (2014), Moreira and Savov (2014), Gorton and Penacchi (1990), Gorton and Ordonez (2013), Hart and Zingales (2014), Greenwood, Hanson and Stein (2015), Gennaioli, Shleifer and Vishny (2012).

[13] The Online Appendix, Proposition A.1, provides more details on the exclusion of the possibility of a backward bending demand for risky debt. We impose the parameter restriction $\bar{R}^r - 2\gamma w^* \sigma^2 > 0$ to guarantee that the demand function never violates free disposal. The restriction ensures that yields on risk-free debt are always greater than -100%: i.e. prices of debt must be strictly positive. If this condition was violated there would be cases of arbitrage: debt could have negative prices despite having strictly positive payoffs.

[14] Similarly, a linear demand function could have also been originated by limits to arbitrage theories (Shleifer and Vishny (1997), Gabaix and Maggiori (2015)).

Formally, the RoW utility function now takes the form:

$$\mathbb{E}^+[C_1^*] - \gamma Var^+(C_1^*) + (B^T\omega + B^T\Omega B)\mathbf{1}_{\{\mathbb{E}^+[e]=1\}},$$

where $B = (b,\ \tilde{b})^T$ is a vector in which b is individual holdings and $\tilde{b}$ is aggregate holdings, ω and Ω are a 2×1 vector and a 2×2 matrix, respectively, and $\mathbf{1}_{\{\mathbb{E}^+[e]=1\}}$ is an indicator function that takes value 1 if the debt is safe, i.e. $E^+[e] = E^s[e] = 1$, and zero otherwise. We assume that $\omega_1 \geq 0$ and $\Omega_{11} \leq 0$, capturing positive but decreasing marginal liquidity benefits from individual bond holdings. We also assume that $\Omega_{12} = \Omega_{21} \geq 0$, capturing the increase in the marginal liquidity benefits from individual bond holdings with aggregate bond holdings, and that $\Omega_{11} + \Omega_{12} \leq 0$ so that this effect is not too strong.

If the debt is expected to be safe, then the optimality condition for individual portfolios is

$$R^s(b) = \bar{R}^r - 2\gamma\sigma^2(w^* - b) - \omega_1 - 2\Omega_{11}b - (\Omega_{12} + \Omega_{21})\tilde{b}.$$

Imposing the equilibrium condition $b = \tilde{b}$ we obtain the demand curve for reserve assets:

$$R^s(b) = \bar{R}^r - 2\gamma\sigma^2 w^* - \omega_1 + 2(\gamma\sigma^2 - \Omega_{11} - \Omega_{12})b,$$

which can be rewritten as

$$R^s(b) = \bar{R}^r - 2\hat{\gamma}\sigma^2(\hat{w}^* - b), \tag{4}$$

where $\hat{\gamma} \equiv \gamma - \frac{\Omega_{11}+\Omega_{12}}{\sigma^2}$ and $\hat{w}^* \equiv w^*\frac{\gamma}{\hat{\gamma}} + \frac{\omega_1}{2\hat{\gamma}\sigma^2}$. Hence under this formulation, liquidity benefits and network effects from bond holdings modify the level and the slope of the demand curve $R^s(b)$. They are isomorphic to a renormalized version of the baseline model with different values of w^* and γ. Larger marginal liquidity benefits ($\uparrow \omega_1$) decrease the level of $R^s(b)$ while stronger decreasing returns in liquidity benefits ($\downarrow \Omega_{11}$) increase the level and the slope of $R^s(b)$. Similarly, larger network effects ($\uparrow \Omega_{12}$) decrease the level and the slope of $R^s(b)$.[15] If the debt is expected to be risky, then the demand curve is the same as the one in the basic mean-variance case ($R = R^r_H$).

2.2 The Full Commitment Equilibrium

To build intuition and a reference point for the outcomes of the basic model, we first solve it under full commitment on the part of the Hegemon. That is, we assume that the Hegemon can commit to future choices of the exchange rate when deciding how much debt to issue at time $t = 0^-$ or equivalently that $\tau \uparrow \infty$ so that there is an infinite penalty from depreciating. In this case

[15] For risk based empirical assessment of dollar currency premia see Hassan (2013), Verdelhan (2016). For a liquidity/safety assessment of the demand for US treasuries see Krishnamurthy and Vissing-Jorgensen (2012).

the Hegemon always sets $e = 1$ and the debt is safe.

The maximization problem for the Hegemon can be written in the following intuitive form:

$$\max_{b \geq 0} \quad V^{FC}(b) \equiv b(\bar{R}^r - R^s(b)), \tag{5}$$

which states that utility maximization is the same as maximizing the expected wealth transfer that the Hegemon receives from the RoW.[16] The wealth transfer is the product of two terms: the amount of debt issued (b) and the safety premium on that debt $\bar{R}^r - R^s(b)$. Note that the Hegemon is indifferent between investing in the risky asset to consume at time $t = 1$ or consuming the proceeds of the debt sale b at time $t = 0$. The term $b\bar{R}^r$ in equation (5) captures these benefits.[17]

The Hegemon trades off a larger debt issuance against a lower safety premium, leading to the optimality condition:

$$\bar{R}^r - R^s(b) - b\, R'^s(b) = 0 \quad or \quad b = 0. \tag{6}$$

This condition shows that the country, since it is a monopolist, takes into account the effect of its debt issuance on the interest rate. This optimality condition is a type of Lerner formula, the monopolist issues debt at a mark-up over marginal cost that depends on the elasticity of the demand function:

$$\frac{\bar{R}^r - R^s(b)}{R^s(b)} = \frac{bR'^s(b)}{R^s(b)}.$$

From the demand function for safe debt in equation (3), the slope of the demand curve is:

$$R'^s(b) = 2\gamma\sigma^2.$$

Substituting this in Equation (6), we have:

$$b = \frac{1}{2\gamma} \frac{\bar{R}^r - R^s(b)}{\sigma^2} > 0 \qquad \text{and otherwise} \quad b = 0.$$

Interestingly, the optimal supply of debt decision from the Hegemon is given by the portfolio demand for the risky asset by a mean variance (CARA-normal) agent. It depends positively on the Sharpe ratio of the risky asset, and negatively on the coefficient of risk aversion. Intuitively, the Hegemon optimally chooses to supply more debt, the more a mean-variance agent would have

[16] See Online Appendix, Lemma A.1, for details. One could extend this objective function to capture distortionary costs of taxation. Indeed, one could introduce a social cost of public funds $\phi > 1$ such that it costs $bR^s(b)\phi$ for the government to repay $bR^s(b)$. In this case the objective function of the Hegemon would become $b(\bar{R}^r - \phi R^s(b))$. The analysis can be carried out almost identically with this extension.

[17] For example, our model is consistent but does not require the Hegemon to issue debt and at the same time hold a large portfolio of risky assets against it. The model is equally consistent with a Hegemon that borrows to finance immediate government spending.

liked to invest in the risky asset given equilibrium prices. It is as if the Hegemon, that is risk-neutral, had incorporated the risk aversion of the RoW in its demand for risky investment financed by risk-less debt. Notice that this "transfer" of preferences only occurs because of monopoly power. To see why consider the perfect competition equilibrium under full commitment:

Lemma 1 ***Perfect Competition Equilibrium.*** *Under perfect competition, when the Hegemon takes the interest rate as given, and under full commitment, the equilibrium is characterized by:*

$$R^s(b) = \bar{R}^r,$$

$$b = w^*.$$

The Hegemon provides full insurance to the RoW and there is no safety premium.

Proof. Optimal portfolio choice given risk neutrality of the Hegemon implies that expected returns on all assets have to be equalized, hence $\mathbb{E}[R^r] - R^s(b) = 0$. Imposing zero excess returns in the demand function of the RoW for Hegemon currency debt (Equation (3)) pins down equilibrium debt supply $b = w^*$. ■

Equilibrium Under Full commitment. Equating demand (Equation (3)) and supply (Equation (6)) for reserve assets, we solve for the equilibrium interest rate:

$$R^s(b^{FC}) = \bar{R}^r - \gamma\sigma^2 w^*.$$

There is a safety premium on reserve assets $\gamma\sigma^2 w^*$, which is increasing in RoW risk aversion (γ), the riskiness of the risky asset (σ), and the wealth of the RoW (w^*).

We can then solve for equilibrium debt issuance b by plugging the interest rate solution in the Reserve currency debt supply function (Equation 6), thus obtaining:

$$b^{FC} = \frac{1}{2} w^*.$$

Equilibrium debt issuance under full commitment only depends on foreign wealth because the parameters γ and σ increase the level and decrease the elasticity of the demand curve with offsetting effects on equilibrium issuance.[18]

[18] To close the equilibrium we note that the above financial market equilibrium is consistent with goods market clearing and investment in the risky asset by the Hegemon $s \in [0, w + \frac{1}{2}w^*]$. Recall that Hegemon investment in the risky asset is indeterminate and, therefore, we simply choose a range consistent with goods market clearing at $t = 0$. hence $w_0 + w_0^* \geq w + \frac{1}{2}w^* - s \geq 0$. The first inequality is satisfied for all $s \geq -\frac{1}{2}w^*$, so it is automatically satisfied by imposing no shorting of the asset $s \geq 0$. The second inequality is satisfied for all $s \leq w + \frac{1}{2}w^*$.

From the Hegemon budget constraints (Equations 1-2) we have that:

$$C_0 + \delta \mathbb{E}[C_1] = w + \delta b(\mathbb{E}[R^r] - R^s(b)),$$

On average the Hegemon consumes more than the average proceeds of its wealth entirely invested in the risky asset. This extra positive (on average) transfer from the RoW is the monopoly rent given by

$$b(\mathbb{E}[R^r] - R^s(b)) = \frac{1}{2}\gamma\sigma^2 w^{*2}. \tag{7}$$

For reasons that will become clear below, we term these monopoly rents the "exorbitant privilege". We collect all results under commitment in the proposition below.[19]

Proposition 1 ***Full Commitment Equilibrium****. Under full commitment the Hegemon chooses to issue risk-free debt and commits to not depreciate the reserve currency in case of a disaster. Equilibrium interest rate, issuance, and exorbitant privilege (monopoly rent) are given by:*

$$R^s(b^{FC}) = \bar{R}^r - \gamma\sigma^2 w^*,$$

$$b^{FC} \equiv \frac{1}{2} w^*,$$

$$b^{FC}(\mathbb{E}[R^r] - R^s(b^{FC})) = \frac{1}{2}\gamma\sigma^2 w^{*2}.$$

French Finance Minister and future President Valery Giscard d'Estaing famously accused the US in 1960s of having an exorbitant privilege due to its reserve status and the ensuing ability to finance itself at cheaper rates than the RoW. In our model, this expected transfer of wealth to the the Hegemon is fair compensation for risk, a feature shared by Gourinchas and Rey (2007*a*), Caballero, Farhi and Gourinchas (2008), Mendoza, Quadrini and Rıos-Rull (2009), Gourinchas, Govillot and Rey (2011), Maggiori (2012), but crucially the Hegemon influences the terms of the compensation via its supply of reserves. In our model there is a sense in which the privilege (equation (7)) is truly exorbitant since it is a pure monopoly rent.

The exorbitant privilege in equation (7) is increasing in the risk aversion (γ) and the the pool of savings (w^*) of the RoW and in the background risk (σ^2). Recalling from equation (4) that liquidity and network effects are isomorphic to changes in γ and w^*, we conclude that higher liquidity benefits ($\uparrow \omega_1$) and stronger network effects ($\uparrow \Omega_{12}$) increase both the level of issuance and the size of the exorbitant privilege.

[19] Online Appendix, Proposition A.2, provides mild conditions under which equilibrium prices are arbitrage free.

Private Issuance of Reserve Assets. Since the size of the exorbitant privilege relies on the amount of reserve assets being issued (in our model b) we discuss here different interpretations of what this stock of assets corresponds to in reality. A narrow interpretation would include only the Hegemon money and short-term government debt, while a broad interpretation would include any asset (including those issued by the private sector to foreigners) denominated in the reserve currency. Under the latter broader interpretation, which we favor, the data counterpart to b is the gross external liabilities of the Hegemon country denominated in the reserve currency.[20]

We extend the model to allow for private issuance of reserve assets. We assume that there is a mass μ of private issuers within the Hegemon country who can each issue one unit of debt denominated in reserve currency. Each issuer can issue at a cost η and for simplicity we assume the cost to be distributed uniform over $[0,\xi]$ across issuers. We denote the total issuance by b^T, and since the marginal private issuer is defined by a cutoff $\bar{\eta} = \bar{R}^r - R^s(b^T)$, we conclude that:

$$b^T = b + \frac{\mu}{\xi}(\bar{R}^r - R^s(b^T)),$$

for $\bar{R}^r - R^s(b^T) \in [0,\xi]$. Solving this equation, we derive a simple relationship between total issuance b^T and public issuance b:

$$b^T = \frac{b + \frac{\mu}{\xi}2\gamma\sigma^2 w^*}{1 + \frac{\mu}{\xi}2\gamma\sigma^2}.$$

We can then rewrite the demand curve for reserve assets as a function of b:

$$\hat{R}^s(b) = \bar{R}^r - 2\hat{\gamma}\sigma^2(w^* - b),$$

where $\hat{\gamma} \equiv \frac{\gamma}{1+\frac{\mu}{\xi}2\gamma\sigma^2}$. Hence, private issuance decreases the slope of the demand curve $R^s(b)$ for reserve assets making it more elastic.

If the Hegemon does not take into consideration the welfare of private issuers, then the Hegemon problem is isomorphic to the one solved in this section replacing γ with $\hat{\gamma}$. If instead the Hegemon takes into the consideration the welfare of private issuers gross of the entry costs, then the Hegemon problem is isomorphic to the one solved in this section with b and γ replaced by b^T and $\hat{\gamma}$, respectively.[21]

[20] Lane and Shambaugh (2010) estimate approximately 90% of US external liabilities to be denominated in USD.

[21] If the Hegemon takes into consideration the welfare of private issuers net of entry costs, then the objective function of the Hegemon as a function of b^T is different and given by

$$V(b^T) = 2\gamma\sigma^2 b^T(w^* - b^T)) - \frac{\mu}{\xi}\frac{[2\gamma\sigma^2(w^* - b^T)]^2}{2}.$$

This model is consistent with the empirical regularity that the consolidated (private and public) external balance sheet of the Hegemon consists of low return safe and liquid liabilities and high return risky and illiquid assets as emphasized by Despres, Kindleberger and Salant (1966), Gourinchas and Rey (2007*a*). In particular, the model is consistent with the notion that it is the private sector, and not the government, that holds foreign risky assets while the government issues safe assets to finance current spending.

3 Limited Commitment and Triffin Dilemma

We first analyze the equilibria that occur for a given quantity of debt b being sold, then we study the optimal issuance of b from the perspective of the Hegemon.

At $t = 1$, if a disaster has occurred, the Hegemon decides whether to appreciate or depreciate by solving:

$$\max_{e \in \{1, e_L\}} \quad C_1 - \tau(1-e),$$
$$s.t. \quad sR^r_L - b\,R\,e = C_1.$$

The Hegemon chooses e_L if and only if

$$bR(1-e_L) > \tau(1-e_L).$$

Intuitively, the Hegemon depreciates if the gains on lower repayment of debt by choosing e_L rather than $e_H = 1$ are greater than the penalty $\tau(1-e_L)$. Depreciation is an effective way to reduce debt payments in real terms because we have stressed that b is the stock of debt held by RoW agents. The above condition further simplifies to a simple threshold property:

$$b\,R > \tau. \tag{8}$$

This threshold property plays a crucial role in generating multiple equilibria since it makes time-1 Hegemon decisions depend on time-0 chosen interest rate on the debt R.

If $bR > \tau$ the Hegemon depreciates in bad times at $t = 1$. RoW agents at time $t = 0^+$ anticipate that the Hegemon will depreciate and therefore treat Hegemon debt as a perfect substitute with the risky asset, requiring $R = R^r_H$ and then being willing to absorb any quantity of debt being sold at that price. This outcome, therefore, is possible for all $b > \underline{b}$, where $\underline{b} \equiv \frac{\tau}{R^r_H}$.

If $bR \leq \tau$ the Hegemon does not depreciate in bad times at $t = 1$ and therefore the Hegemon debt is safe. The interest rate then is $R = R^s(b)$. This outcome, therefore, is possible for all $b < \bar{b}$, where[22]

$$\bar{b} \equiv \frac{-\bar{R}^r + 2w^*\gamma\sigma^2 + \sqrt{(\bar{R}^r - 2w^*\gamma\sigma^2)^2 + 8\gamma\sigma^2\tau}}{4\gamma\sigma^2}. \tag{9}$$

We collect these results in the lemma below.

Lemma 2 ***(The Three Regions of the IMS)*** *For a given level of issuance b at $t = 0^-$, the structure of continuation equilibria for $t = 0^+$ onwards is as follows:*

1. *If $b \in [0, \underline{b}]$ (The Safety region) there is a unique equilibrium, the safe equilibrium, where the Hegemon does not depreciate in the disaster state at $t = 1$. The yield on Reserve currency debt is given by:*

$$R^s(b) = \bar{R}^r - 2\gamma(w^* - b)\sigma^2$$

   and is increasing in b.

2. *If $b \in (\underline{b}, \bar{b}]$ (The Instability region) there are two equilibria. The safe equilibrium described above, and the collapse equilibrium in which Reserve currency debt has no safety premium ($R = R^r_H$) and the Reserve currency depreciates conditional on a disaster.*

3. *If $b \in (\bar{b}, w^*]$ (The Collapse region) there is a unique equilibrium, the collapse equilibrium described above.*

3.0.1 Reserve Country Optimal Issuance of Debt

Given the possibility of multiple equilibria at $t = 0^+$ when issuance is in the Instability region, we need to select an equilibrium. Given our focus on strategic issuance rather than equilibrium selection, we adopt here the simplest possible selection device. We select the safe equilibrium if the realization of the sunspot is s and the collapse equilibrium otherwise. Accordingly, we define a function $\alpha(b) \in [0, 1]$ to denote the $t = 0^-$ probability that the continuation equilibrium for

[22] $\bar{b}$ is the only positive root of the quadratic equation corresponding to the inequality $b(\bar{R}^r - 2\gamma(w^* - b)\sigma^2) \leq \tau$. We focus in this paper on the interesting case $\bar{b} \leq w^*$, which requires the parameter restriction $\tau \leq \bar{R}^r w^*$, so that commitment is sufficiently imperfect that the Hegemon cannot provide the RoW with full insurance. Once this condition is imposed we have the following ordering $\underline{b} \leq \bar{b} \leq w^*$. The first inequality holds because $R^s(b) < \bar{R}^r \quad \forall b \in [0, \bar{b}]$ conditional on the debt being safe. Hence $\bar{b}\bar{R}^r > \tau$.

$t = 0^+$ onwards is the collapse equilibrium:[23]

$$\alpha(b) = \begin{cases} \alpha(b) = 0, & for \quad b \in [0, \underline{b}], \\ \alpha(b) = \alpha, & for \quad b \in (\underline{b}, \bar{b}], \\ \alpha(b) = 1, & for \quad b \in (\bar{b}, w^*]. \end{cases}$$

Hence we have two (potential) points of discontinuity for $\alpha(b)$ at the boundaries of the regions. By analogy with the full-commitment problem in equation (5), the Hegemon maximization problem is:

$$\max_{b \geq 0} \quad V(b) \equiv (1-\alpha(b))V^{FC}(b) - \alpha(b)\lambda\tau(1-e_L), \tag{10}$$

where recall that $V^{FC}(b) = b(\bar{R}^r - R^s(b))$ is the value function under full commitment. This formulation shows that utility maximization is equivalent to maximizing the expected wealth transfer from the RoW net of the expected cost of a possible depreciation.[24] The value function in equation (10) is discontinuous, if $\alpha \in (0,1)$, at $b = \{\underline{b}, \bar{b}\}$ and otherwise twice continuously differentiable. Hence we cannot apply entirely standard optimization methods. Notice that $V^{FC}(b) \geq V(b)$ and the equality holds only $\forall b \in [0, \underline{b}]$. This value function is illustrated in Figure 2 with the value function under full commitment plotted as a dotted line to ease comparison. We formalize the optimal issuance solution in the proposition below and then describe it intuitively using the illustration in Figure 2.

Proposition 2 ***Limited Commitment Equilibrium and the Triffin Dilemma.*** *Under limited commitment, the equilibrium issuance by the Hegemon is given by:*

1. *If $b^{FC} \leq \underline{b}$, then the Hegemon issues b^{FC} in the Safety region.*
2. *If $\bar{b} \geq b^{FC} > \underline{b}$, then the Hegemon issues $\underline{b}$ in the Safety region or it issues b^{FC} in the Instability region, whichever generates higher net monopoly rents.*
3. *If $b^{FC} > \bar{b}$, then the Hegemon either issues $\underline{b}$ in the Safety region or it issues $\bar{b}$ in the Instability region, whichever generates higher net monopoly rents.*

For all equilibria, the Hegemon enjoys an exorbitant privilege in the form of positive net expected monopoly rents.

[23] One could consider many alternative functions $\alpha(b)$ (continuous or discontinuous, monotonically increasing or not). Our constant formulation, which has the advantage of simplicity, is a benchmark in the literature (see for example Cole and Kehoe (2000)). One interesting alternative would be to consider a function $\alpha(b)$ that jumps up in the interior of the Instability region to capture the notion of neglected risk (Gennaioli, Shleifer and Vishny (2012, 2013)).

[24] See Online Appendix, Lemma A.1, for details.

Despite the limited commitment, the multiplicity of equilibria, and the jumps in the value function, the model remains very tractable and simple to analyze in closed form. Figure 2 illustrates some of the possible equilibrium outcomes in the above proposition. Panel A in the figure corresponds to case 1 in Proposition 2, in which the Hegemon finds it optimal to issue in the interior of the Safety region.

More interesting for us are cases 2 and 3 in Proposition 2 in which the Hegemon faces a meaningful tradeoff, or "dilemma", between issuing less debt but in the Safety region ($\underline{b}$) and issuing more debt but in the Instability region (either b^{FC} or $\bar{b}$). For example, Panel B in Figure 2 illustrates case 2 in Proposition 2 for a parametrization that leads the Hegemon to prefer issuing more debt at the risk of a collapse of the IMS. This tradeoff is our model's rationalization of the Triffin dilemma, which Kenen (1963) summarizes as:

> Triffin has dramatized the long-run problem as an ugly dilemma: If the present monetary system is to generate sufficient reserve assets to lubricate payments adjustment, the reserve currency countries must willingly run payments deficits enduring a deterioration of their net reserve positions that could erode foreign confidence in the reserve currencies. If, contrarily, the reserve currency countries are to maintain their net reserve positions, there must someday be a shortage of reserve assets and this will cause serious frictions in the process of payments adjustment.[25]

As we documented in the introduction, the history of the IMS is characterized by repeated collapses (e.g. the Gold-Exchange standard in 1931, Bretton Woods in 1973). One possible interpretation is that these collapses are caused by large unforeseen shocks. The Triffin dilemma offers the different interpretation that the Hegemon endogenously chooses to put the IMS at risk of collapse.

Whether a Triffin dilemma arises in our model (cases 2 and 3) or not (case 1) depends on the level of the demand for safe assets as captured by w^* compared to the safe debt capacity of the country as captured by τ. More precisely it depends on whether $b^{FC} = 1/2w^* > \tau/R_H^r = \underline{b}$. In cases 2 and 3 when $b^{FC} > \underline{b}$, there exists a threshold $\alpha_m^* \in (0,1)$ such that the Hegemon issues at the boundary of the Safety region $\underline{b}$ if and only if $\alpha > \alpha_m^*$, and otherwise issues either b^{FC} (case 2) or $\bar{b}$ (case 3). [26]

[25] In our model the motive for reserve accumulation is risk aversion and/or desire for liquidity by the RoW which provides a more general illustration of the demand for reserves than the original payments/defense of exchange rates reasons highlighted by Triffin (1963). This more general motive for reserve accumulation is consistent with the dramatic accumulation of reserves during the post-Asian-crisis global imbalances period under floating exchange rates, and with the resurgence of a Triffin-style dilemma in this environment.

[26] Indeed, the value function is independent of α at the boundary $\underline{b}$ of the Safety Region and is continuous and monotonically decreasing in α in the Instability region; with $\alpha = 1$, we always have $V(\underline{b}) > V(\min\{b^{FC}, \bar{b}\})$, and with $\alpha = 0$, we always have $V(\underline{b}) < V(\min\{b^{FC}, \bar{b}\})$.

All else equal, an increase in the world demand for safe assets, $\uparrow w^*$, or a decrease in the safe debt capacity $\downarrow \tau$, activates the Triffin margin since the Hegemon then faces a choice between a safe option with a low level of debt at the boundary of the Safety region and a risky option with a high level of debt $\min\{b^{FC}, \bar{b}\}$ in the Instability region. Indeed, policy concerns regarding Triffin-like phenomena have precisely arisen in periods when there was a perception that the global demand for reserve assets was outstripping the safe debt capacity of the Hegemon, such as during the Bretton Woods era and more recently during the post-Asian-crisis global imbalances era.

The above modeling also helps shed light on the historical intellectual debate between what are called the "consensus view", the view that US balance of payments deficits were ultimately unsustainable as articulated by Triffin (1963), and the "minority view", the view articulated by Despres, Kindleberger and Salant (1966) that US balance of payments deficits were sustainable with the US playing the role of a world banker with both gross liabilities and gross assets, with a positive liquidity premium between the two sides of its balance sheet.[27] Our model, while consistent with the minority view of the Hegemon acting as a financial intermediary that collects a safety/liquidity premium on its gross assets/liabilities, is also consistent with the concerns of the consensus view regarding the confidence in the US Dollar, but crucially ties these concerns to the gross (not the net) external debt position of the US. We share the view of banking as a fragile activity subject to self-fulfilling panics that can have macroeconomic consequences with the domestic macro literature (Gertler and Kiyotaki (2015)), but in our context the problem is exacerbated by the absence of a plausible Lender of Last Resort (LOLR) with a sufficient fiscal capacity to support the Hegemon world banker.

Risk-Sharing and LOLR Arrangements and the Triffin Dilemma. One avenue to mitigate the Triffin Dilemma and the associated instability of the IMS is to introduce policies that reduce the demand for reserve assets for any level of global savings w^*. Policies to this effect have often been proposed by economists looking to reform the IMS (Keynes (1943), Harrod (1961), Machlup (1963), Meade (1965), Rueff (1963), Farhi, Gourinchas and Rey (2011)).[28] In their most recent incarnation they have included swap lines amongst central banks, credit lines by the IMF as a LOLR, and international reserve sharing agreements such as the Chang Mai initiative. Our framework can capture the rationale behind these policies with a simple extension of the demand curve for reserve assets in equation (3). We assume that each of the many countries

[27] Despres, Kindleberger and Salant (1966) write: "such lack of confidence in the dollar as now exists has been generated by the attitudes of government officials, central bankers, academic economists, and journalists, and reflects their failure to understand the implications of this intermediary function."

[28] See Grubel (1963) for a reprint of the main policy proposals up to the 1960s.

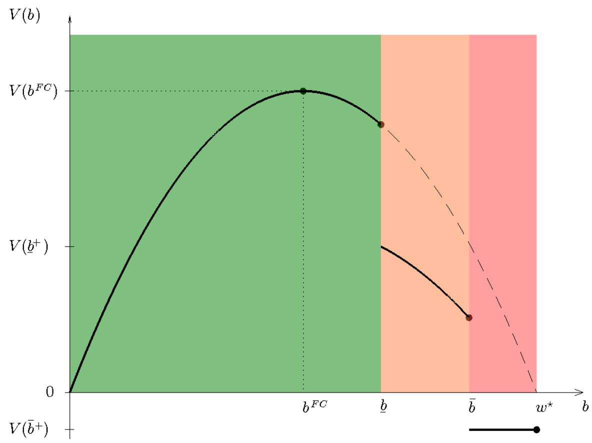
in the RoW is saddled with an idiosyncratic background endowment risk ω_i. We also assume that if in equilibrium variance of C_1^* is above a variance threshold then international investors penalize variance at the margin with "risk aversion" $\bar{\gamma}$ rather than with $\underline{\gamma} < \bar{\gamma}$. This is a simple reduced-form way of capturing a form of precautionary savings. We assume that the variance of ω_i is so big that even when the country invests all its savings in reserve assets the variance of future consumption is still above the variance threshold, but that in the absence of idiosyncratic background risk, it falls below the variance threshold even where there are no reserves assets. Then a sufficiently good idiosyncratic risk-sharing arrangement among RoW countries reduces the equilibrium demand for reserve assets by lowering marginal "risk aversion" to the lower level $\underline{\gamma}$. In a world with more idiosyncratic risk-sharing and lower "risk aversion", the Hegemon finds issuing in the Safety region relatively more attractive than issuing in the Instability region. Indeed, assuming that $\underline{b} < b^{FC} < \bar{b}(\gamma)$ for both values of γ, the profits from issuing b^{FC} are equal to $(1-\alpha)b^{FC}2\gamma\sigma^2(w^* - b^{FC}) - \alpha\lambda\tau(1-e_L)$ and the profits from issuing $\underline{b}$ are equal to $\underline{b}2\gamma\sigma^2(w^* - \underline{b})$. Hence, when γ drops from $\bar{\gamma}$ to $\underline{\gamma}$, the profits from issuing b^{FC} decrease more than the profits from issuing $\underline{b}$.

4 Welfare Consequences of the Triffin Dilemma

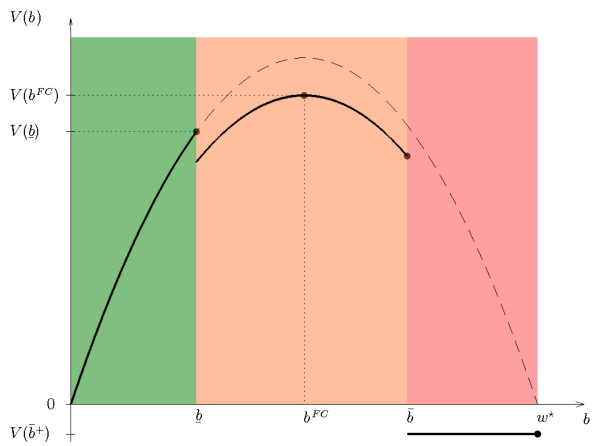
In the previous section we formalized the Triffin dilemma as the choice of a monopolistic Hegemon issuer of reserve assets between issuing fewer assets, but safe for sure, or more assets, but with the possibility that they might turn out to be risky. The Hegemon maximizes expected net monopoly rents (producer surplus) without taking into account RoW expected utility (consumer surplus). In this section we consider social welfare (social surplus) which adds expected net monopoly rents and RoW expected utility. We always evaluate welfare from the perspective of expected utility at time $t = 0^-$ before the sunspot is selected.

The interesting case to consider is when there is a meaningful tradeoff, the Triffin dilemma, between issuing in the Safety region or in the Instability region ($b^{FC} > \underline{b}$, cases 2 and 3 in Proposition 2). In this configuration, the Hegemon faces a choice between a safe option with low issuance at $\underline{b}$ and a risky option with higher issuance at $\bar{b}$. We compare the rankings of these two options from the perspective of the Hegemon, the RoW, and social welfare, respectively. If the Hegemon prefers the high-issuance risky-option to the low-issuance safe-option, but the RoW would have preferred the opposite, we say that there is over-issuance from the perspective of the RoW. Similarly, if the Hegemon prefers the low-issuance safe-option to high-issuance risky-option, but the RoW would have preferred the opposite, we say that there is under-issuance from the perspective of the RoW. Under- and over-issuance from the perspective of social welfare are

Figure 2: Reserve Country Optimal Debt Issuance



(a) Optimal Issuance in Safety Region



(b) Optimal Issuance in Instability Region

Note: Panel (a) illustrates a parameter configuration in which full-commitment issuance b^{FC} can be achieved in the Safety region. Panel (b) illustrates a parameter configuration in which full-commitment issuance b^{FC} can only be achieved in the Instability region. In both panels optimal issuance under limited commitment still occurs at the full commitment level.

defined analogously.

One might have conjectured, by analogy with standard monopoly problems, that there would always be under-issuance from a social welfare perspective. While this can certainly happen in our model, we also show that it is possible to get over-issuance. We trace this surprising result to the fact that the options faced by the Hegemon involve two inter-related dimensions: the traditional quantity dimension analyzed in standard monopoly problems and a novel risk dimension.

The crux of the argument hinges on the shape of the demand curve. Thus far we have restricted our attention to a linear demand curve, in the interest of tractability. When it comes to welfare, more insights can be gleaned by generalizing the demand function to allow for non-linearities since these govern the infra-marginal RoW surplus. In particular, we found that a tractable model that still captures these more general effects can be rendered via a concave, but piece-wise linear, demand curve with a single concave kink.[29] One way to obtain this type of demand curve is to augment the preferences of the RoW to include a "bond in the utility" function component as in Section 2.1 (equation (4)) but with the difference here that the satiation point for liquidity occurs at lower levels of bond holdings within the Safety region.

In the set-up of this section, the RoW solves the following maximization problem:

$$\max_{b} \quad \mathbb{E}^{+}[C_1^*] - \gamma Var^{+}(C_1^*) - \gamma_L \left(\hat{b} - \min(b, \hat{b}) \mathbf{1}_{\{\mathbb{E}^{+}[e]=1\}}\right)^2,$$

$$s.t. \quad w^* R^r + b(Re - R^r) = C_1^*, \qquad b \geq 0,$$

where $\gamma_L > 0$, $\hat{b}$ is an exogenous threshold, and $\mathbf{1}_{\{\mathbb{E}^{+}[e]=1\}}$ is the indicator function that takes value one if its argument is satisfied. If debt is safe (i.e. $\mathbb{E}^{+}[e] = 1$), then the extra utility (liquidity) value of owning bonds is $\gamma_L(\hat{b} - b)^2$ for $b < \hat{b}$ and zero otherwise. If debt is risky (i.e. $\mathbb{E}^{+}[e] < 1$), then the extra utility loss $\gamma_L \hat{b}^2$ is the one that would have occurred if the agent had chosen $b = 0$ in the presence of safe debt.

We assume, for simplicity, that $\hat{b} = \underline{b} = \frac{\tau}{R_H}$. This implies that if debt is expected to be safe, then the demand curve is given by[30]

$$R^s(b) = \bar{R}^r - 2\gamma(w^* - b)\sigma^2 - 2\gamma_L(\underline{b} - b)\mathbf{1}_{\{b \leq \hat{b}\}}. \tag{11}$$

If debt is expected to be risky, which can only happen for $b > \underline{b}$, then the result from Proposition A.1 applies and $R = R_H^r$, so that risky debt is a perfect substitute for the risky asset. Therefore, if the debt is safe, the demand function has an extra liquidity component for all $b \leq \underline{b}$ and is

[29] Concavity refers to the function $R^s(b)$, thus implying a convex demand curve $b(R^s)$.

[30] We impose the parameter restriction $\bar{R}^r - 2\gamma w^* \sigma^2 - 2\gamma_L \hat{b} > 0$, by analogy with the previous sections.

otherwise identical to the one considered in the previous sections.

This set-up lends itself to welfare evaluation as the “area under the demand curve", which conveniently allows for intuitive and graphical representation of welfare. RoW expected utility can be computed as:

$$V_{RoW}(b) = V_{RoW}(R^s(0)) + (1-\alpha(b)) \int_{R^s(0)}^{R^s(b)} b(\tilde{R}^s) d\tilde{R}^s, \tag{12}$$

where $b(R^s)$ is the demand curve that expresses the amount of debt being demanded as a function of the interest rate, as in equation (11).[31]

Figure 3 Panel B illustrates the piecewise-linear demand function in equation (11) and allows to visualize RoW expected utility as the area below the demand curve.[32] For example, RoW expected utility when the Hegemon issues $\underline{b}$ is represented by the green area. Similarly, RoW expected utility when the Hegemon issues of $\bar{b}$ is represented by the orange area. This latter area is shrunk, compared to the total area under the demand curve, in line with equation (12), to account for the fact that the equilibrium issuance $\bar{b}$ is safe only with probability $1-\alpha$.

The Hegemon net expected monopoly rents are given by

$$V(b) = (1-\alpha(b))b(\bar{R}^r - R^s(b)) - \alpha(b)\lambda\tau(1-e_L). \tag{13}$$

The green rectangle in Figure 3 Panel A represents the net expected monopoly rents that accrue from issuing $\underline{b}$. The orange rectangle represents the net expected monopoly rents that accrue from issuing $\bar{b}$. This latter area is shrunk, compared to the total area $\bar{b}(\bar{R}^r - R^s(\bar{b}))$, in line with equation (10), to account for the fact that the equilibrium issuance $\bar{b}$ is safe only with probability $1-\alpha$ and that there is an expected cost of depreciation $\alpha\lambda\tau(1-e_L)$.

Intuitively, higher values of liquidity ($\uparrow \gamma_L$) increase RoW expected utility in the green area in Figure 3 Panel B. This increases the (infra-marginal) RoW expected utility loss in case of a collapse of the IMS when the Hegemon issues $\bar{b}$ rather than $\underline{b}$. For a given probability of the collapse α, the higher the value of liquidity, the higher the RoW expected utility losses from issuance in the Instability region. However, the Hegemon does not internalize this loss when choosing issuance between $\bar{b}$ and $\underline{b}$. Indeed, Figure 3 Panel A illustrates that the comparison the Hegemon makes in choosing optimal issuance is independent of infra-marginal demand from the RoW for $b < \underline{b}$, as long as the Hegemon does not find it optimal to issue in the interior of the Safety region. This misalignment in the source of Hegemon and RoW welfare opens up the

[31]See Online Appendix for full details.

[32]Figure 3 plots $R^s(b)$, but expected utility is the area under the curve $b(R^s)$, hence in the figure this area is the horizontal space between the function $R^s(b)$ and the vertical axis.

possibility of socially inefficient issuance of reserve assets.

When the value of liquidity is low, and always in the limit of no liquidity value and linear-demand for safe debt, there is under-issuance from a social perspective, as in standard monopoly problems. More surprisingly, when the value of liquidity is sufficiently high, there is over-issuance from a social perspective. For some values of the probability of collapse α, the monopolist chooses to issue $\bar{b}$ but the RoW would have been better off with the safe issuance at $\underline{b}$, so much so that social welfare is higher at $\underline{b}$.

In order to formalize the above intuition, we focus on cases 2 and 3 in Proposition 2 in which $b^{FC} > \underline{b}$. It is convenient to define the following three thresholds: α^*_m, α^*_{RoW}, α^*_{TOT}. In Section 3.0.1 we have discussed α^*_m, the cutoff probability of the collapse outcome that makes the Hegemon indifferent between issuing at the upper bound of the Safety region ($\underline{b}$) or issuing at the local maximum in the Instability region $\min\{b^{FC}, \bar{b}\}$. We now similarly define α^*_{RoW} to be the cutoff probability that equalizes RoW expected utility at the boundary of the Safety region $\underline{b}$ and at $\min\{b^{FC}, \bar{b}\}$ in the Instability region. The analogous cutoff for social welfare is α^*_{TOT}. The proof of Proposition 3 in the Online Appendix shows that α^*_{RoW} and α^*_{TOT} are unique and in the interval $(0,1)$.

These thresholds have intuitive implications for over- and under-issuance of reserve assets. For example, if $\alpha^*_m > \alpha^*_{RoW}$, then for all probabilities $\alpha \in (\alpha^*_{RoW}, \alpha^*_m)$, the Hegemon over-issues from the perspective of RoW. Similarly, if $\alpha^*_m < \alpha^*_{RoW}$, then for all probabilities $\alpha \in (\alpha^*_m, \alpha^*_{RoW})$, the Hegemon under-issues from the perspective of RoW. Similar conclusions can be drawn from the ranking between α^*_m and α^*_{TOT}, but now from the perspective of social welfare.

Proposition 3 ***(Over-issuance by a Monopolist Hegemon)*** *If $\gamma_L = 0$, so that the demand curve is linear, then in equilibrium the cutoff probabilities are ranked as follows:*

$$\alpha^*_m < \alpha^*_{TOT} < \alpha^*_{RoW},$$

and the Hegemon under-issues for $\alpha \in (\alpha^*_m, \alpha^*_{RoW})$ from the perspective of RoW, and for $\alpha \in (\alpha^*_m, \alpha^*_{TOT})$ from a social perspective.

There exists $\bar{\gamma}_L(\tau) > 0$, which makes the demand curve sufficiently concave, such that for all $\eta \in (0,1]$, when τ is sufficiently small, and when $\gamma_L \in [\eta\bar{\gamma}_L(\tau), \bar{\gamma}_L(\tau)]$, the cutoff probabilities are ranked as follows:

$$\alpha^*_m > \alpha^*_{TOT} > \alpha^*_{RoW},$$

and the Hegemon over-issues for $\alpha \in (\alpha^*_{RoW}, \alpha^*_m)$ from the perspective of RoW and for $\alpha \in (\alpha^*_{TOT}, \alpha^*_m)$ from a social perspective.

Proof. In the interest of intuition and brevity we provide here the full proof of the first statement:

for linear demand the monopolist under-issues from a social perspective. The Online Appendix provides the proof of the second statement, that there can be over-issuance for sufficiently concave demand curves.

Assume $\gamma_L = 0$. Define $b^* \equiv \min\{b^{FC}, \bar{b}\}$ to be the optimal level of issuance that the Hegemon chooses conditional on issuing in the Instability region. RoW expected utility is equalized at issuance levels $\underline{b}$ and b^* for a threshold probability of the collapse α^*_{RoW}:

$$(1-\alpha^*_{RoW})b^{*2} = \underline{b}^2.$$

Indeed, these are the areas under the demand curve as described in equation (12). Similarly, Hegemon net expected monopoly rents are equalized at issuance levels $\underline{b}$ and b^* for a threshold probability of the collapse α^*_m:

$$(1-\alpha^*_m)2\gamma\sigma^2(w^*-b^*)b^* - \alpha^*_m\lambda\tau(1-e_L) = (w^*-\underline{b})2\gamma\sigma^2\underline{b},$$

where we recall that $R^s(w^*) = \bar{R}^r$. We conclude that:

$$1-\alpha^*_m = \frac{w^*-\underline{b}}{w^*-b^*}\frac{\underline{b}}{b^*} + \frac{\alpha^*_m\lambda\tau(1-e_L)}{2\gamma\sigma^2 b^*(w^*-b^*)} > \frac{\underline{b}}{b^*} > \left(\frac{\underline{b}}{b^*}\right)^2 = 1-\alpha^*_{RoW}.$$

Since α^*_{TOT} is a convex combination of α^*_{RoW} and α^*_m with interior non vanishing weights on each of the elements, we obtain the result in the Proposition.

Note that in this derivation, the shape of the demand curve only enters through the sufficient statistics b^* and $\frac{\tau}{2\gamma\sigma^2}$. The ranking $\alpha^*_{RoW} > \alpha^*_m$ does not depend on the precise choice of b^* or on the precise value of $\frac{\tau}{2\gamma\sigma^2}$. This clarifies why changes in the slope of the demand curve are not sufficient to overturn this ranking. However, changes in the degree of concavity of the demand curve are sufficient as proved in the continuation of this proof in the Online Appendix. ■

We can relate our notion of over- and under-issuance, as a choice between the safe and the risky option in the Triffin dilemma, to another connected notion. We define $b^*_m(\alpha)$, $b^*_{RoW}(\alpha)$, and $b^*_{TOT}(\alpha)$ as the levels of issuance that maximize Hegemon net expected monopoly rents, RoW expected utility, and social welfare, respectively. We say that there is over-issuance from the perspective of RoW if $b^*_{RoW} < b^*_m$, and under-issuance from the perspective of RoW if $b^*_{RoW} > b^*_m$. The concept of over- and under-issuance from the perspective of social welfare is defined analogously.

A consequence of the above proposition is that: If $\gamma_L = 0$, then $b^*_m(\alpha) < b^*_{TOT}(\alpha) < b^*_{RoW}(\alpha)$ for every $\alpha \in [0,1]$, so that there is under-issuance from the perspective of RoW and of social welfare; There exists $\gamma_L(\tau) > 0$ such that for τ sufficiently small, then $\alpha \in (\alpha^*_{RoW}, \alpha^*_m)$, $b^*_m(\alpha) >$

$b^*_{RoW}(\alpha)$, so that there is over-issuance from the perspective of RoW, and for $\alpha \in (\alpha^*_{TOT}, \alpha^*_m)$, $b^*_m(\alpha) > b^*_{TOT}(\alpha)$ so that there is over-issuance from the perspective of social welfare.[33]

5 Gold-Exchange Standard and ZLB Recessions

The scarcity of reserve assets in the IMS has often be associated with recessionary pressures. Most famously, Keynes (1923) argued against the return by all countries to a gold standard at pre-WWI parities on the grounds that these would have required a policy of tight money, i.e. high interest rates, leading to an increase in demand for reserve assets that if unaccommodated by an increase in the supply of these assets would have lead to a recession. Furthermore, he argued that a peg to gold would have left interest rate policy to be determined by fluctuations (“vagaries") in the demand for reserve assets rather than focusing on domestic macroeconomic stabilization.

In this section we show that this argument can be rationalized within the context of our model. We then extend this logic to show that the argument would prevail also in a regime of floating exchange rates when the Reserve country monetary policy is constrained by the ZLB. This duality between Gold-Exchange standard and floating regimes with the ZLB shows that the key features behind the recessionary effect of the demand for reserve assets lies in the inability (ZLB) or unwillingness (peg) of the monetary authority to lower the interest in response to increases in demand for reserve assets coupled with the inability, due to limited commitment, of the Reserve country to expand the supply of reserve assets. Indeed, we show that under commitment the Reserve country finds it optimal to always issue enough reserve assets to reach full employment, because the demand curve for reserve assets endogenously becomes perfectly inelastic when there is slack in the economy. However, with limited commitment the Reserve country might be unable to credibly issue enough reserve assets to attain full employment.

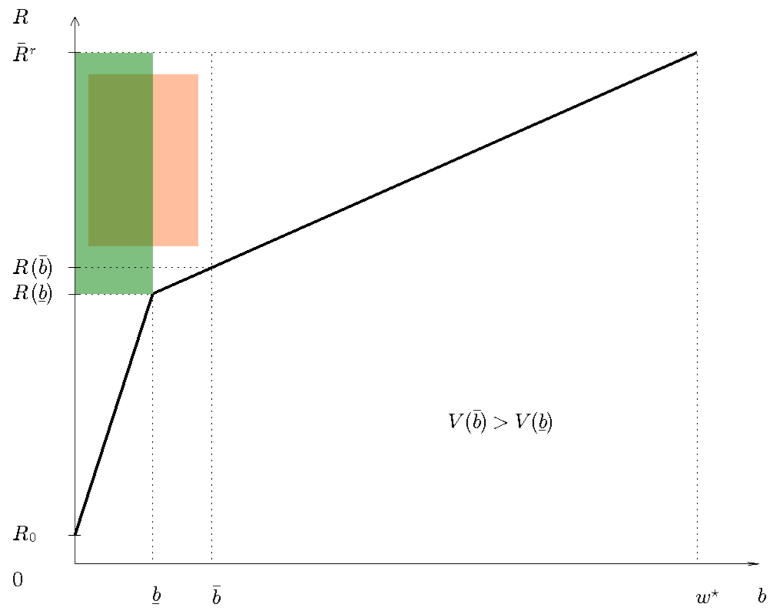
5.0.1 Floating Exchange Rates and ZLB

To formalize the above logic, we modify the production structure from the previous sections to allow for production within each period. We assume that a unit mass of competitive firms at time $t = 0$ and at $t = 1$ in the RoW have access to a linear one-for-one production technology from domestic labor. In both periods, labor is supplied without disutility up to a level $\bar{L}$ and with a large disutility for any amount of labor in excess of this level.[34] Output is produced instantaneously at $t = 0^+$ (and at $t = 1$), so that the decision to produce at $t = 0$ takes place after the equilibrium

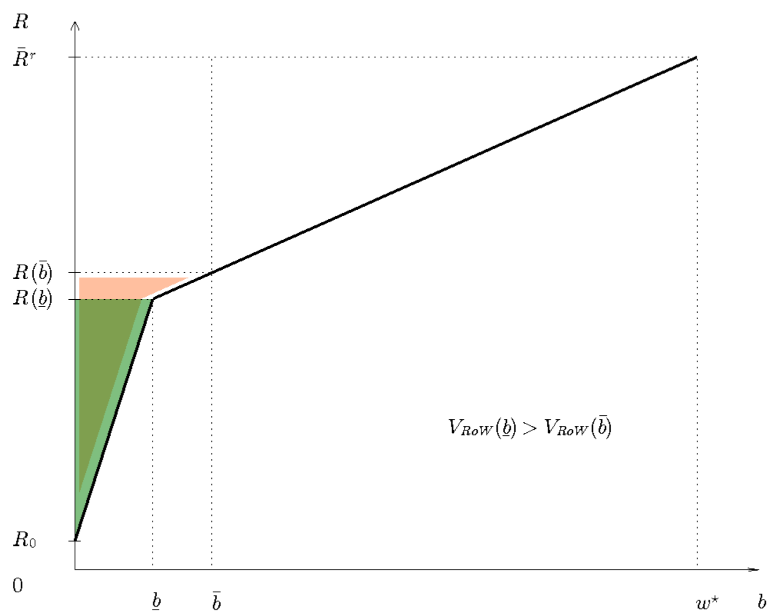
[33]While our results in Sections 3 and 4 depend on our specific choice of $\alpha(b)$, the results are robust to alternative specifications of $\alpha(b)$ (for example, continuously increasing).

[34]We assume that the disutility is sufficiently large that $\bar{L}$ is the natural level of output.

Figure 3: Welfare Consequences of the Triffin Dilemma



(a) Hegemon Net Expected Monopoly Rents



(b) RoW Expected Utility

Note: Panel (a) illustrates net expected monopoly rents for the Hegemon issuance of either $\underline{b}$ (green) or $\bar{b}$ (orange). A parameter configuration is chosen such that the Hegemon finds it optimal to issue $\bar{b}$. Panel (b) illustrates RoW expected utility resulting from the Hegemon decision to issue either $\underline{b}$ (green) or $\bar{b}$ (orange). Under the parameter configuration, RoW would have preferred issuance to be $\underline{b}$.

sunspot has been selected. Wages are completely rigid in RoW currency in both periods while prices are fully flexible. We focus on an equilibrium, consistent with limited commitment, in which period 1 output is at full employment and focus on endogenous output determination at $t = 0^+$ and, consequently, omit time subscripts. Firms are competitive and take world prices for their output as given. Firms solve $\max_{\ell^*}(p^* - \bar{w}^*)\ell^*$. Optimality requires $p^* = \bar{w}^*$, so that output is demand determined. For simplicity we take $\bar{w}^* = 1$. Wage income is $w^{*\ell} \equiv \bar{w}^*\ell^*$. We extend the previous notation and denote (endowment) wealth by w^{*e}. Total investable wealth by RoW agents at time $t = 0^+$ is $w^* \equiv w^{*e} + w^{*\ell}$.

We denote the nominal RoW interest rate as $\tilde{R}^*_t$. We assume that the RoW central bank uses a truncated Taylor rule:

$$\tilde{R}^* = 1 + \max(-\phi(\bar{L} - \ell), 0), \tag{14}$$

where we take the limit as $\phi \uparrow \infty$ so that the central bank sets the RoW nominal interest rate at a level consistent with full employment or at zero: either $\ell = \bar{L}$ and $\tilde{R}^* \geq 1$ or $\ell < \bar{L}$ and $\tilde{R}^* = 1$.

If the debt is safe, no arbitrage implies that the interest rate on dollar debt is $R^s(b) = \tilde{R}^*$. The natural rate consistent with full employment, taking b as given, is: $\tilde{R}^{*n}(b) \equiv \bar{R}^r - 2\gamma\sigma^2(w^{*e} + \bar{w}\bar{L} - b)$. The RoW central bank sets $\tilde{R}^*(b) = max(\tilde{R}^{*n}(b), 1)$. The resulting RoW demand for safe debt is:

$$R^s(b) = max(\bar{R}^r - 2\gamma\sigma^2(w^{*e} + \bar{w}\bar{L} - b), 1). \tag{15}$$

If the debt is risky, the risky interest rate is determined as before to be $R = R^r_H$. The RoW central bank sets the RoW nominal interest rate equal to $\tilde{R}^*(0) = R^s(0) = 1$.

We assume that $\bar{R}^r - 2\gamma\sigma^2(w^{*e} + \bar{w}\bar{L}) < 1$ (or equivalently $\tilde{R}^*(0) = R^s(0) = 1$). The ZLB binds if $\bar{R}^r - 2\gamma\sigma^2(w^{*e} + \bar{w}\bar{L} - b) < 1$, which occurs for $b < b_{ZLB}$, where

$$b_{ZLB} \equiv \frac{1 - \bar{R}^r + 2\gamma\sigma^2(w^{*e} + \bar{w}\bar{L})}{2\gamma\sigma^2} > 0.$$

A crucial property of the demand curve for reserve assets in the presence of the ZLB, as shown in equation (15), is that is perfectly elastic at $R^s = 1$ for $b \in [0, b_{ZLB}]$. An immediate consequence of this property is that a Hegemon with full commitment always optimally chooses to supply enough safe assets $b > b_{ZLB}$ that the ZLB never binds and there is full employment.

Under limited commitment, the regions of the IMS are analogous to those under flexible prices in Lemma 2, with the only difference being that $\bar{b}$ is now potentially affected by the ZLB such that:[35]

$$\bar{b}_{ZLB} \equiv \min\left(\frac{-\bar{R}^r + 2(w^{*e} + \bar{w}\bar{L})\gamma\sigma^2 + \sqrt{(\bar{R}^r - 2(w^{*e} + \bar{w}\bar{L})\gamma\sigma^2)^2 + 8\gamma\sigma^2\tau}}{4\gamma\sigma^2}, \tau\right).$$

If debt is safe and $b < b_{ZLB}$ or if debt is risky, then the ZLB binds. With $\tilde{R}^* = R^s = 1$ and at full employment, the reserve asset market is in disequilibrium: there is shortage of (excess demand for) reserve assets. This disequilibrium cannot be resolved by a reduction in interest rates. Instead, output endogenously drops below potential, reducing investable wealth, the demand for reserve assets, and bringing the reserve asset market back to equilibrium. The equilibrium value of utilized labor l is the solution of the following implicit equation:[36,37]

$$\bar{R}^r - 2\gamma\sigma^2\left(w^{*e} + \bar{w}\ell - b\mathbf{1}_{\{\mathbb{E}^+[e]=1\}}\right) = 1. \tag{16}$$

We analyze two polar cases where b_{ZLB} is either very low $b_{ZLB} < \underline{b}$ or very high $b_{ZLB} > \bar{b}_{ZLB}$ compared to safe debt capacity.

If $b_{ZLB} < \underline{b}$, then if the Hegemon finds it optimal to issue in the Safety region, it chooses $b \in (b_{ZLB}, \underline{b})$, thus achieving full employment as in the full commitment case. However, if the Hegemon finds it optimal to issue in the Instability region, then, while the safe debt outcome has $R^s \geq 1$ and full employment, the collapse of the IMS makes the ZLB bind $\tilde{R}^*(0) = R^s(0) = 1$ and triggers a severe recession.

If $b_{ZLB} > \bar{b}_{ZLB}$ and the probability of a collapse α is zero, then the Hegemon finds it optimal to issue $\bar{b}_{ZLB}$ since it faces a perfectly elastic demand for its debt. In contrast with the full commitment case, this issuance is not enough, even in the absence of the possibility of a collapse, to exit the ZLB and achieve full employment.

We collect the above results in the proposition below.[38]

[35]Once we enrich the model with the ZLB, we see that all else equal a binding ZLB decreases the levels of debt that can be issued and still be safe since it increases the real interest rate charged on debt compared to the case of fully flexible prices.

[36]We assume throughout that the solution is $\ell > 0$.

[37]Intuitively, the equilibrium determination of utilized labor l and output $w^{*e} + \bar{w}l$ can be understood as a Keynesian cross AS-AD diagram $AS(l) = AD(l)$ with

$$AS(l) \equiv w^{*e} + \bar{w}l,$$

$$AD(l) \equiv \frac{\bar{R}^r - R^s}{2\gamma\sigma^2} + b\mathbf{1}_{\{\mathbb{E}^+[e]=1\}},$$

with $R^s = \tilde{R}^* = 1$. Here $\frac{\bar{R}^r - R^s}{2\gamma\sigma^2}$ is investment in the risky technology by RoW agents and b is consumption and investment by Hegemon agents.Crucially, the supply of reserve assets acts as a positive AD shifter. Reductions in the supply of reserve assets b that cannot be accommodated by a reduction in interest rates $R^s = \tilde{R}^*$ at the ZLB where $R^s = \tilde{R}^* = 1$, lead to a reduction in utilized labor l and output $w^{*e} + \bar{w}l$.

[38]The results in Proposition 4 also apply to an extension in which production also takes place in the Hegemon

Proposition 4 ***(Floating Exchange Rates and ZLB with a Hegemon)*** *If $b_{ZLB} < \underline{b}$, then if the Hegemon finds it optimal to issue in the Safety region, it chooses $b \in (b_{ZLB}, \underline{b})$, the ZLB does not bind ($\tilde{R}^*(b) = R^s(b) \geq 1$), and the economy is at full employment ($\ell = \bar{L}$). If the Hegemon finds it optimal to issue in the Instability region, then, either debt is selected to be safe ($\tilde{R}^*(b) = R^s(b) \geq 1$) and there is full employment, or debt is selected to be risky and the ZLB binds ($\tilde{R}^*(0) = R^s(0) = 1$) and output is below potential ($\ell < \bar{L}$).*

If $b_{ZLB} > \bar{b}_{ZLB}$, then the Hegemon either issues $\bar{b}_{ZLB}$ or issue $\underline{b}$, whichever generates the highest net expected monopoly rents. In both cases the ZLB binds ($\tilde{R}^*(b) = R^s(b) = 1$) and output is below potential ($\ell < \bar{L}$). If the debt is selected to be risky there is a more severe recession ($\tilde{R}^*(0) = R^s(0) = 1$).

Expenditure Switching Effects and the Incentives to Devalue. In the model the incentive of the Hegemon to devalue at $t = 1$ is the fiscal benefit of lower real debt repayment. We now consider an extension of the model that captures an additional incentive to devalue to stimulate domestic (Hegemon) output.

We introduce production of a non-traded good in the Hegemon country at both time $t = 0^+$ and $t = 1$ in a manner entirely analogous to the production considered in this Section for the RoW. The good is produced with a linear one-for-one technology by a unit mass of competitive firms. Firms hire local labor at a rigid wage $\bar{w}$ in Hegemon currency. Profit maximization for the firms implies that $p_{NT} = \bar{w}$.

Hegemon agents supply labor with no disutility up to a maximum $\bar{L}$ and have a large disutility for any amount beyond that level.[39] We extend Hegemon agents preferences to include a (potentially time and state dependent) separable utility value of non-tradable consumption, such that the per-period utility function is now $C_t + \nu_t(C_{NT,t})$.[40] Hegemon consumption plan optimality implies that in each period:

$$\frac{\bar{w}}{\bar{w}^*} e_t = \nu_t'(C_{NT,t}),$$

where $e_0 = 1$ and $e_1 = \{1, e_L\}$. If a disaster has occurred at $t = 1$, this condition shows that a depreciation increases the consumption, and therefore the production, of non-tradables. We define the decreasing function

$$C_{NT,t}(e) \equiv \nu_t'^{-1}\left(e\frac{\bar{w}}{\bar{w}^*}\right).$$

economy in a set-up entirely analogous to that of the RoW. The Hegemon production reinforces its incentives to issue as much debt as possible to avoid a recession. To highlight that this element is not necessary for our result, we have omitted it from the main text, but include it here for realism.

[39] We assume that the disutility is sufficiently large that $\bar{L}$ is the natural level of output.

[40] For generality, we allow the function ν_t depends on the realization of R^r, which allows to capture variations in the natural exchange rate over time and across states.

In equilibrium we have $Y_{NT,t} = C_{NT,t} = C_{NT,t}(e_t)$.

If output is below potential at $t = 1$, so much so that $C_{NT,1}(e_L) \leq \bar{L}$, then there is an additional benefit $\nu(C_{NT,1}(e_L)) - \nu(C_{NT,1}(1))$ to the Hegemon from depreciating its currency at $t = 1$ because it stimulates domestic output. The model is then isomorphic to the basic one but with an adjusted value of τ now given by:[41,42]

$$\tilde{\tau} \equiv \tau - \frac{\nu(C_{NT,1}(e_L)) - \nu(C_{NT,1}(1))}{1 - e_L} < \tau.$$

5.0.2 Gold-Exchange Standard

We introduce gold in the model as an asset that both pays a dividend D for sure at time $t = 1$ and provides a convenience yield in the form of an extra perceived dividend at $t = 1$. We maintain the production structure from the previous subsection. One can think of the dividend as a liquidity or hedonic service out of holding gold that materializes independently from the state of the economy. We assume that the asset is in infinitesimal supply for tractability. Since gold is safe it is discounted at the same rate of risk-free debt $p_G = \frac{D}{R^s}$. Where p_G is expressed in foreign currency units (D is also expressed in nominal foreign currency).

The world economy operates under a Gold-Exchange standard in which the price of gold p_G is constant at $\bar{p}_G$ in all currencies. The RoW monetary policy is no longer described by a Taylor rule, instead monetary policy is dictated by the imperative of maintaining gold parity:

$$\tilde{R}^*(b) = \bar{R}^G > 1 \qquad \text{with} \quad \bar{R}^G \equiv \frac{D}{\bar{p}_G}. \tag{17}$$

If Reserve country debt is safe, no arbitrage implies that:

$$R^s(b) = \bar{R}^G.$$

If Reserve country debt is risky, its rate of return is determined as before to be the same as the risky asset $R = R^r_H$.

Under the Gold-Exchange standard, the demand for reserve assets is perfectly elastic at $\bar{R}^G$, so that changes in the supply of reserve assets b do not get accommodated by changes in the

[41]The only difference is that if the domestic recession at $t = 1$ in case of a disaster is severe enough, the Hegemon might be better off not trying to commit not to depreciate its currency. In this case, the Hegemon issues risky debt, there is no commitment problem, and the equilibrium is trivial. We place ourselves in the alternative case where under limited commitment, the Hegemon chooses to try to commit not to depreciate, and only fails to do so in equilibrium when it issues in the Instability region and expectations are unfavorable.

[42]Note that under flexible wages, then there is no further benefit from depreciating $\tilde{\tau} = \tau$. Output is always at potential $Y_{NT,t} = C_{NT,t} = \bar{L}$ and the condition $\frac{\bar{w}_t}{w^*_t} e_t = \nu'_t(\bar{L})$ simply pins down the relative wage $\frac{\bar{w}_t}{w^*_t}$. The model is then completely isomorphic to the basic model.

interest rate R^s but instead induce variation in output:

$$\bar{R}^r - 2\gamma\sigma^2(w^{*e} + \bar{w}^*\ell - b\mathbf{1}_{\{safe\}}) = \bar{R}_G.$$

This is similar to output determination at the ZLB, as in equation (16), but with the interest rate fixed at $\bar{R}_G > 1$ rather than 1. By analogy with the ZLB analysis, we define b_G to be the amount of reserve assets that are consistent with both maintaining the gold parity and full employment:

$$b_G = \frac{\bar{R}_G - \bar{R}^r + 2\gamma\sigma^2(w^{*e} + \bar{w}\bar{L})}{2\gamma\sigma^2},$$

and we assume $b_G > 0$. We define $\bar{b}_G$ the highest safe debt level that the Reserve country can sustain under the Gold-Exchange standard:

$$\bar{b}_G = \min\left(b_G, \frac{\tau}{\bar{R}^G}\right).$$

As in the previous subsection with the ZLB, we analyze two polar cases where b_G is either very low $b_G < \underline{b}$ or very high $b_G > \bar{b}_G$ compared to safe debt capacity.

Proposition 5 ***(Gold Exchange Standard with a Hegemon)*** *Since the demand curve for reserve assets is perfectly elastic, the Hegemon chooses to issue either $\underline{b}$ or $\bar{b}_G$. If the Hegemon issues $\underline{b}$, a recession (output below potential) occurs if $b_G > \underline{b}$, and otherwise there is a boom (output above potential). If the Hegemon issues $\bar{b}_G$ and the debt is safe, a recession occurs if $b_G > \bar{b}_G$, and otherwise there is a boom. If the Hegemon issues $\bar{b}_G$ and the debt is risky, a recession occurs independently of b_G. In all three cases the recession is more severe or the boom more shallow, the higher b_G.*

There are potentially additional benefits to the Hegemon from not only depreciating against the current price of gold, but also against other currencies. For example, the Hegemon could go off-gold unilaterally while the RoW stays on gold. We can capture these benefits by extending the model to allow for expenditure switching effects of exchange rate movements along the lines of the previous subsection with Hegemon production of non-tradable goods. As in the previous subsection, the model is isomorphic to the one considered here with the change of τ in $\tilde{\tau} < \tau$ to incorporate the extra incentives to depreciate.

The model helps rationalize the collapse of the Gold-Exchange standard in the 1930s and the Bretton Woods system in the 1970s. In all these historical episodes the decision by the Hegemon(s) to depreciate was both the result of external factors in the form of a confidence crisis and internal factors to bolster the domestic economy. For example, the British economy in 1931 was severely depressed and the British pound was under pressure due to France's (and other countries) demand to covert their pound holdings into gold. The British unilateral and unexpected decision to devalue and go off gold caused major losses for international reserve holders (the Banque de France went bankrupt) and contributed to alleviating the U.K. recession.[43] Following the pound depreciation, the rest of the world attempted to liquidate their remaining dollar reserves ultimately causing a confidence crisis in the US resulting in the US going off gold in 1933. Similarly, the US decision to go off gold and devalue the dollar in 1971-73 was the result of domestic recessionary pressures (the 1969 recession) and foreign demand to liquidate their dollar balances into gold.

If all countries depreciate against gold by the same amount, $\bar{p}'_G > \bar{p}_G$, the resulting monetary accommodation $\bar{R}^{G'} < \bar{R}^G$ stimulates the economy at a given level of reserve asset issuance ($b'_G < b_G$). If all countries decide to float their currencies, then the only potential remaining obstacle to achieving full employment is the ZLB, as in the previous subsection.

We can also formalize the concerns of Keynes (1923) that gold is an unsuitable asset for a monetary standard since it ties monetary policy to non-monetary shocks to the demand and supply of gold:

> If we restore the gold standard, are we to return also to the pre-war conceptions of bank-rate, allowing the tides of gold to play what tricks they like with the internal price-level, and abandoning the attempt to moderate the disastrous influence of the credit-cycle on the stability of prices and employment? In truth, the gold standard is already a barbarous relic. **Keynes, 1923, A Tract on Monetary Reform**

One way to capture non-monetary shocks to the supply and demand for gold is via one-time unexpected shocks to D. Under the Gold-Exchange Standard these shocks are accommodated one-for-one by changes in $R^G = D/\bar{p}_G$ which in turn result in fluctuations in b_G and output.

6 The Multipolar World Model

We have so far focused on an IMS dominated by a Hegemon which has a monopoly over issuance of reserve assets. Of course, this is an idealization and the real world which, while currently dominated by the US issuance of reserve assets, features other competing issuers as illustrated in Figure 6. Indeed, the Euro and the Yen already play a partial role as reserve currencies and there are speculations that the future of the IMS might involve other reserve currencies, such as the

[43] Eichengreen and Sachs (1985), Bemanke and James (1991) document that countries that went off gold earlier recovered faster than those who stayed on gold longer.

Chinese Renminbi.[44] In this section we explore the equilibrium consequences of the presence of multiple issuers of reserve assets for the total quantity of reserves assets and the stability of the IMS. We characterize the conditions under which the emergence of a multipolar world is likely to be beneficial by increasing the total quantity of reserve assets, as predicted by Eichengreen (2011) among others, or detrimental by fostering more instability, as warned by Nurkse (1944). The thrust of our analysis is that the benefits of a more multipolar world are U-shaped in the number of reserves issuers: a lot of competition is good but a little competition might be worse than monopoly.

6.1 The Benefits of a Multipolar World

In this subsection we allow for multiple symmetric issuers to compete in the provision of reserve assets in their own currencies. The issuers engage in competition in quantities à la Cournot before any potential sunspot is selected. Each Country can at $t = 1$ decide its exchange rate depreciation rate with $e = \{1, e_L\}$. So that all safe and risky currencies are perfect substitutes within each group.

We focus first on the case of full commitment. The RoW demand function for safe debt of country i is:

$$R^s(b_i, b_{-i}) = \bar{R}^r - 2\gamma\sigma^2(w^* - b_i - b_{-i}),$$

where b_i is the quantity of country i debt and b_{-i} is the total quantity of safe debt issued by all other $n-1$ countries. The slope of this demand function is still given by $\frac{\partial R^s(b_i,b_{-i})}{\partial b_i} = 2\gamma\sigma^2$ as in the monopolist case and optimal issuance is still given by $b_i = \frac{\bar{R}^r - R^s(b_i,b_{-i})}{2\gamma\sigma^2} > 0$ or $b_i = 0$. Of course, the safety premium now depends on total issuance by all countries. The best-response supply of reserve assets by country i given issuance b_{-i} is:

$$b_i = \frac{1}{2}(w^* - b_{-i}) > 0 \quad or \quad b_i = 0.$$

There exists a unique equilibrium and it is symmetric. Individual issuance is given by:

$$b_{i,n}^{FC} = \frac{1}{(n+1)} w^*.$$

Total issuance is $B_n^{FC} = n b_{i,n}^{FC}$ and the equilibrium interest rate on safe debt is:

$$R^s\left(B_n^{FC}\right) = \bar{R}^r - \frac{2}{n+1}\gamma\sigma^2 w^*.$$

[44]For example Portes and Rey (1998), Cooper et al. (2009).

Individual and total monopoly rents are given respectively by

$$\frac{2}{(n+1)^2}\gamma\sigma^2 w^{*2}, \quad \text{and} \quad \frac{2n}{(n+1)^2}\gamma\sigma^2 w^{*2}.$$

In the limit as $n\uparrow\infty$ we converge to perfect competition with: $\lim_{n\to\infty} B_n^{FC} = w^*$ and $\lim_{n\to\infty} R^s\left(B_n^{FC}\right) = \bar{R}^r$. As we mentioned in Lemma 1 in this equilibrium the exorbitant privilege is completely dissipated, no monopoly rents, and the RoW obtains full insurance.

Under limited commitment, the size of the Safety region (the interval $[0, \bar{b} = \tau/R_H^r]$) does not depend on the equilibrium interest rate on reserve assets and is therefore unaffected by competition. With a sufficiently large number of countries n issuing reserve currencies, each country finds it optimal to issue debt within its Safety region; the equilibrium is then identical to that of full commitment. This provides one possible rationalization of the common support, among academics and policymakers, for a multipolar IMS. Sufficient competition both reduces monopoly rents and makes the IMS stable.

Under full commitment competition is always desirable. Under limited commitment this is only true when there are a large number of issuers, but, as we show in the next subsections, competition can have adverse effects when there are only a few issuers (e.g. the US, China and Europe).

6.2 Third Party Issuance

We consider an equilibrium where the Hegemon issues safe debt b and does not depreciate its currency in bad times.[45] We introduce a small issuer with time 1 utility function U who must raise real resources κ at date 0 to finance consumption at date 0.

We assume that the small issuer can either denominate its debt in Reserve currency or in a risky currency that depreciates by $(1-e_L)$ in bad times and that this issuer is too small to influence the equilibrium. The small issuer decides to issue in the Reserve currency if and only if

$$-\kappa R^s(b) > \mathbb{CE}^-[-\kappa R^r], \tag{18}$$

where we define $\mathbb{CE}^-[-\kappa R^r] = U^{-1}\left(\mathbb{E}^-[U(-\kappa R^r)]\right)$. This condition makes clear that the small issuer is more likely to issue in the Reserve currency, the lower $R^s(b)$, the higher and the more volatile R^r, and the higher risk aversion embedded in the utility function U of the small issuer.

This helps rationalize the evidence in Eichengreen, Chitu and Mehl (2014) reproduced in Figure 7 showing that third party issuance was predominantly denominated in pounds during the

[45] In this subsection we assume that equilibrium prices are free of arbitrage as in Proposition A.3.

1920s, when the British pound was the main reserve currency, and has subsequently switched to being denominated in dollars as the US Dollar emerged as the main reserve currency. This also helps understand why countries that suffer from "original sin", so that they cannot issue in their own currency, predominately issue in the reserve currency. Relatedly, Du and Schreger (2015) and Bruno and Shin (2015) show that emerging market corporations predominantly borrow in US dollar.

We now turn to a small issuer who is risk-neutral, so that it chooses to issue safe debt in a reserve currency (its own or that of the Hegemon). In this case the issuer, while having a small debt capacity, is at least sufficiently big to affect the equilibrium (for example, Switzerland). This third party issuance, by absorbing some of the RoW savings w^*, lowers the residual demand for reserve assets faced by the Hegemon. Indeed, from the perspective of the Hegemon, it acts exactly as a reduction in w^*. The Hegemon therefore cuts back its own reserve asset issuance, as is standard in monopoly problems. It is relatively straightforward that the Hegemon reduces issuance within the Safety region or within the Instability region. It is less immediate that the Hegemon might find it optimal to jump from issuing in the Instability region to the Safety region, but not the other way around. While there are many cases in which third party issuance increases the total supply of reserve assets, this result illustrates the possibility that a small increase in third party issuance might dramatically reduce the total supply of reserve assets. This is a leading example of a more general theme that we develop further in subsection 7.2, whereby a little competition in reserve asset issuance can backfire and reduce total issuance in the presence of limited commitment.

We formalize the above intuition by considering the Hegemon model in the configuration when $\underline{b} < b^{FC} < \bar{b}$ and set the probability of the collapse sunspot to α_m^* so that the Hegemon is indifferent between issuing $\underline{b}$ and b^{FC}. We then consider a third party issuer with very low debt capacity. We assume that this issuer is always confronted with the most unfavorable investor expectations such that it chooses to issue the arbitrarily small amount $\varepsilon > 0$ of safe debt in its own currency. Since the third party issuance is safe, it is a perfect substitute for the Hegemon issuance whenever this latter issuance is also safe.

We now consider the response of the Hegemon to the entrance of the third party issuer. The Hegemon expected profits from issuing $\underline{b}$ have now been lowered by (comparative static):

$$\frac{\partial V(\underline{b})}{\partial \varepsilon} = \underline{b}\frac{\partial R^s(\underline{b})}{\partial \varepsilon} = -2\gamma\sigma^2\underline{b}.$$

The Hegemon expected profits from issuing b^{FC} have now been lowered by (comparative static):

$$\frac{\partial V(b^{FC})}{\partial \varepsilon} = (1-\alpha_m^*) b^{FC} \frac{\partial R^s(b^{FC})}{\partial \varepsilon} = -(1-\alpha_m^*) 2\gamma\sigma^2 b^{FC},$$

where the second equality follows from the envelope theorem because b^{FC} is an interior local maximum. Since the Hegemon was originally indifferent between b^{FC} and $\underline{b}$, one has that $\underline{b} < (1-\alpha_m^*) b^{FC}$, and consequently the Hegemon now jumps down to optimally issuing $\underline{b}$.[46]

6.3 Nurkse Instability under Oligopoly

We formalize the warning by Nurkse (1944) that a disadvantage of the presence of multiple competing reserve issuers is that it introduces coordination problems across a priori substitutable reserve currencies. Nurkse famously pointed to the instability of the IMS during the interregnum of Dollar and Sterling as reserve currencies in the 1920s. He diagnosed the increased difficulty to coordinate on the ultimate reserve asset by noticing the frequent switches of rest of the world reserve holdings between the two currencies. Eventually, the IMS of the time collapsed with the UK devaluing first in 1931 and the US devaluing in 1933.

To capture this additional instability arising from coordination, we propose two stylized configurations of the model under a duopoly of issuers of reserve assets indexed by $i \in \{1,2\}$. These configurations correspond to two different coordinations of investors' expectations.

In the first configuration one country faces the most favorable expectations regarding the stability of its currency $\alpha_i = 0$, while the other one faces the least favorable expectations $\alpha_{-i} = 1$. This configuration boils down to Cournot competition of firms under heterogenous fixed capacity constraints; here the fixed capacity constraints refer to the two boundaries $\bar{b}_i$ and $\underline{b}_{-i}$. Country i issues more than country $-i$. We interpret the switches in RoW reserve holdings between Dollar and Sterling as unexpected inversions in which countries face the favorable or unfavorable expectations.

Nurkse's conjecture that it is easier to coordinate expectations towards a favorable outcome when there is a Hegemon issuer compared to a duopoly of issuers can be rendered in our model by assuming that a Hegemon would have faced $\alpha = 0$. Under this configuration, coordination problems reduce the benefits of competition (less total issuance) compared to an ideal situation in which both duopoly issuers would have faced favorable expectation $\alpha_i = \alpha_{-i} = 0$.

In the second configuration exactly one country $\tilde{i}$ out of the two is selected at random at $t = 0^+$ to face the most favorable expectations, while the other country $-\tilde{i}$ faces the least favorable

[46] These results depend on the non-concavity of the objective function of the Hegemon. In our set-up this occurs more generally when $\alpha(b)$ increases sufficiently fast with b.

expectations. Each country i now optimally behaves as a Hegemon with $\alpha_i = 0.5$. Like above, we assume that a true Hegemon would have faced the most favorable expectations $\alpha = 0$.

For this second configuration, we focus on two interesting subcases. The first case arises when the demand for reserve assets is so high that a true Hegemon (under monopoly) would have chosen $\bar{b}$ even when facing $\alpha = 0.5$. Under duopoly, there can be multiple equilibria, but we show that it is always an equilibrium for both issuers to issue $\bar{b}$, and we focus on that case.[47] Then, both under monopoly and under duopoly, each issuer chooses to issue $\bar{b}$, so that total issuance of reserve assets is twice as high under duopoly than under monopoly. This occurs because going from monopoly to duopoly: the boundaries $\bar{b}$ and $\underline{b}$ are unchanged; the (equilibrium) expected payoff to each issuer from issuing $\bar{b}$ is unchanged, because when they issue in the Instability region the competing issuers under duopoly do not actually compete since one is safe when the other is risky and vice versa; the (out of equilibrium) expected payoff to each issuer from issuing $\underline{b}$ is lower since that issuer competes with the other issuer who issues at $\bar{b}$ with probability 0.5. However under duopoly, in equilibrium, each unit of reserve asset is safe only with probability 0.5, so that the total effective supply of safe assets is the same as under monopoly. In addition, the duopoly world features instability with the collapse of one of the currencies occurring for sure, while the monopoly world is stable.

The second case arises when the demand for reserve assets is intermediate, so that a true Hegemon issues $\bar{b}$ when $\alpha = 0$ but $\underline{b}$ when $\alpha = 0.5$. In this case, going from monopoly to duopoly can (but does not always) reduce the total effective supply of safe assets because under duopoly, individual issuance might jump to the Safety region below $\underline{b}$, in which case total issuance might go down if $\underline{b} < \frac{1}{2}\bar{b}$. In Section 7.2 we characterize in closed form a related mechanism whereby going from monopoly to unstable duopoly erodes the future monopoly rents of each issuer, thus lowering commitment, and prove analytically this force to be so strong to reduce effective total issuance.

We emphasize that in mapping the model to Nurkse's facts about the 1920s Gold-Exchange Standard, the quantity b refers not to the total stock of debt but to the part of this stock held abroad. The instability, therefore, can manifest itself in debt switching hands between foreign and domestic residents and not necessarily in the total amount being issued.

[47] The only other possible equilibrium in this case is one where both issuers issue in the Safety region below $\underline{b}$. This may or may not be an equilibrium. We either focus on cases where it is not, or when it is, we select the other equilibrium.

6.4 Endogenous Emergence of a Hegemon in a Multipolar World

In this section we consider configurations of the Multipolar model that lead to asymmetric equilibria with a large and a small issuer of reserve assets. Such an asymmetric equilibrium can be interpreted as the natural emergence of a Hegemon.

First, we consider a duopoly $i \in \{1,2\}$ and assume that prices are fully rigid in one of the two reserve currencies, say $i = 1$, rather than in RoW currency as assumed in Section 5. This captures the empirical regularity that prices are disproportionately quoted in the dominant reserve currency, in US dollars at present and in British sterling in the 1920s (Gopinath (2015)).

In this case the real return of debt denominated in reserve currency 1, in which the goods are priced, is always safe. The crucial consequence is that country 1 endogenously acquires de facto full commitment, while country 2 still faces limited commitment as in our analysis so far.[48] We solve for an illustrative equilibrium by assuming that country 2 faces the least favorable expectations with $\alpha_2 = 1$. This is isomorphic to a standard Cournot model with two firms, one of which has a fixed capacity constraint while the other is unconstrained, where $\underline{b}$ plays the role of the fixed capacity constraint. In equilibrium, country 1 issues more, potentially much more, than country 2. This offers one rationalization for the association in the data between currency of pricing in the goods market and currency denomination of reserve assets.

Second, we consider a different scenario where in a duopoly $i \in \{1,2\}$ issuers differ in their fiscal capacity. We model fiscal capacity as the social cost of public finds whereby repaying bR actually requires resources $bR\phi$ with $\phi > 1$. To capture increasing costs of public funds we assume that the cost is zero for all levels of repayments below $\underline{b}R^s(\underline{b})$ and ϕ for each incremental unit of repayment after that.

We consider a starting duopoly equilibrium in which $\phi > 1$ and identical for both issuers and choose the probability of the collapse equilibrium, perfectly correlated among the two issuers, such that when one issuer issues $\underline{b}$ then the other issuer is indifferent between issuing $\underline{b}$ or issuing in the Instability region. In this case there are three possible equilibria: either both issuers issue $\underline{b}$, or one of them issues $\underline{b}$ and the other issues in the Instability region.

We then consider a comparative static in which we decrease the fiscal capacity of one of the two issuers and increase the fiscal capacity of the other: that is we set $\phi_i < \phi < \phi_j$, with $\phi_j - \phi_i < \varepsilon$ and ε arbitrarily small. As a result there exists a unique asymmetric equilibrium in which issuer i with the largest fiscal capacity (ϕ_i) issues in the Instability region and issuer j issues at the upper bound $\underline{b}$ of the Safety region. This emphasizes that fiscal capacity is a crucial

[48] In practice debt reductions could be engineered either through an exchange rate depreciation or through an outright default. The pricing of goods in the reserve currency reduces the ex-post incentives to depreciate. While the incentives to default are unchanged, such defaults are rarer in practice perhaps because of higher true or perceived associated costs.

aspect of reserve currencies, and that even small differences in fiscal capacity can lead to large differences in issuance and the endogenous emergence of a Hegemon.

7 Endogenous Reputation, Coordination, and Competition

In this section we present an infinite horizon extension of the basic model. This extension achieves two distinct objectives. First, it provides a foundation via reputation for the cost τ assumed in most of this paper. Second, it allows for further characterization of the interaction between competition and commitment. We derive a sharp result according to which competition never increases the supply of reserve assets beyond the maximum that a Hegemon could have credibly issued, even as the number of competing issuers approaches infinity. This is because what disciplines issuers is the expectation of future monopoly rents. By reducing these rents, competition decreases commitment. Each issuer cuts issuance so much that total issuance does not increase with the number of issuers.

Time is discrete and the horizon is infinite. Reserve countries issue one period bonds in each period. The issuers are infinitely lived, risk neutral, and have rate of time preference $\delta \in (0,1)$. We maintain the assumption that $\delta^{-1} = \bar{R}^r$. The RoW is populated by overlapping generations with each generation alive for 1 period. The young are born at period t with constant endowment w^* and invest in the bonds and the risky technology. The young have mean variance preferences over consumption at the end of their lives at $t+1$ and consume all proceeds of investment at that time.

The timing of decisions within each date is identical to the one period model. At each date the issuers choose the depreciation of the exchange rate between two gross growth rates $e = \{1, e_L\}$ with $e_L < 1$. That is $e_{t+1} = e\ e_t$ with $e \in \{1, e_L\}$. The probability of disasters is constant over time (they are i.i.d.) and equal to λ.

Consider first this model with a Hegemon under full commitment. The Hegemon decides to not depreciate in bad times, the debt is safe, and the equilibrium is characterized by exactly the same equations as in Proposition 1. Similarly, the equilibrium with n issuers, who compete in quantities à la Cournot under full commitment, is a repeated version of that in Subsection 6.1 and also converges to perfect competition as the number of issuers increases to infinity.

Under limited commitment, we remove exogenous fixed costs of depreciation (i.e. $\tau = 0$). We assume that if an issuer chooses to depreciate in bad times at time t when ex-ante facing an interest rate consistent with expectations of no depreciation ($R^s_{t-1}(b_i) < R^r_H$), then it is punished forever by a bad continuation equilibrium in which RoW agents expect a depreciation of the currency conditional on disaster, which indeed occurs in equilibrium. In that bad continuation equilibrium, RoW demand for this issuer's debt is perfectly elastic at $R^s_z(b_i) = R^r_H$ for $z > t$. There is, instead, no punishment going forward for depreciations by an issuer who is currently facing the interest rate R^r_H and has not previously depreciated as described in the previous case. While we are allowing for non-Markovian strategies to depend on interest rates for safe debt R and past default, we are not allowing the strategies to depend on the history of issuances.[49]

7.1 The Hegemon Model with Endogenous Reputation

By analogy with Section 3, we first analyze the equilibrium for a given amount of debt issued by a single player. Since we have no fixed cost of default ($\tau = 0$) and we assumed that the trigger strategies do not punish a depreciation following a period in which $R = R^r_H$, the issuer always depreciates ex-post (if a disaster occurs) when ex-ante facing $R = R^r_H$. We assume that this equilibrium outcome, which can occur for all levels of b, is selected with probability $\alpha \in (0,1)$ for levels of debt when a safe debt equilibrium also exists, and otherwise with probability 1. In analogy with the previous sections we abuse the notation and denote this criterion by a function $\alpha(b)$.

The expected value to the issuer from issuing debt b forever and not depreciating, unless faced with interest rate R^r_H, is :

$$V(b) = \sum_{z>t} \delta^{(z-t)} b(1-\alpha(b)) E^s_t \left[\bar{R}^r - R_z e\right] = b(1-\alpha(b)) \frac{\bar{R}^r - R^s(b)}{\bar{R}^r - 1}.$$

A depreciation at time t, when facing the favorable interest rate ($R^s_{t-1}(b_i) < R^r_H$), causes this real expected value to be lost since the trigger strategy would then impose $\alpha(b) = 1$ in the continuation equilibrium for all levels of b and hence the continuation value is zero. Hence the long-term cost of a depreciation is

$$V(b) = b(1-\alpha(b)) \frac{\bar{R}^r - R^s(b)}{\bar{R}^r - 1}.$$

The one-off short-term benefit of a depreciation is

$$bR^s_t(b) \frac{e_{t-1} - e_{L,t}}{e_{t-1}} = bR^s(b)(1-e_L).$$

[49] The Nixon shock of 1971 and the float of the US Dollar in 1973 did not cause a major drop in the use of the Dollar as an international reserve currency (see Figure 6). While for simplicity we have made our trigger strategies very stark, so that a depreciation in a disaster looses the privilege forever, one could study more lenient punishments that occur only with some probability and with finite duration.

The issuer therefore decides not to depreciate if and only if

$$b(1-\alpha(b))\frac{\bar{R}^r - R^s(b)}{\bar{R}^r - 1} \geq bR^s(b)(1-e_L).$$

Substituting in the condition above the demand for safe debt $R^s(b) = \bar{R}^r - 2\gamma\sigma^2(w^* - b)$, we obtain the upper bound for the issuance of safe debt:

$$\bar{b}^{\infty}_{\alpha} \equiv w^* - \frac{\bar{R}^r(1-e_L)(\bar{R}^r - 1)}{2\gamma\sigma^2(1-\alpha+(1-e_L)(\bar{R}^r-1))}. \tag{19}$$

We use the superscript ∞ to distinguish the variables in this infinite horizon model from the analogous concepts in the one period model. Note that $b^{\infty}_{\alpha=0} > 0$ and finite, $b^{\infty}_{\alpha=1} = 0$, and the upper boundary decreases in the probability of the collapse equilibrium selection: $\frac{\partial \bar{b}^{\infty}_{\alpha}}{\partial \alpha} < 0$.

The problem of Hegemon is:

$$\max_{b \in [0, \bar{b}^{\infty}_{\alpha}]} \quad (1-\alpha)b\frac{\bar{R}^r - R^s(b)}{\bar{R}^r - 1} = (1-\alpha)V^{FC}(b),$$

$$s.t. \quad R^s(b) = \bar{R}^r - 2\gamma\sigma^2(w^* - b).$$

The Hegemon chooses to issue $b^{FC} = \frac{1}{2}w^*$, if it is credible, or $\bar{b}_{\infty,\alpha}$, if it is not. Optimal issuance of a Hegemon is given by

$$\min\{b^{FC}, \bar{b}^{\infty}_{\alpha}\}.$$

7.2 The Multipolar Model with Endogenous Reputation

We now analyze the multipolar world with n competing issuers in this infinite horizon set-up and set $\alpha = 0$ for simplicity. By analogy with the above analysis of the Hegemon, issuer i's best response to total issuance b_{-i} from other issuers is to issue the minimum between what it would have issued in best response under full commitment and the maximum credible amount that it can issue

$$b_i = \min\left\{b_i^{FC}(b_{-i}), \bar{b}^{\infty}(b_{-i})\right\}.$$

Crucially the upper bound of credible issuance depends on the other players' total issuance:

$$\bar{b}^{\infty}(b_{-i}) = w^* - b_{-i} - \frac{\bar{R}^r(1-e_L)(\bar{R}^r-1)}{2\gamma\sigma^2(1+(1-e_L)(\bar{R}^r-1))}.$$

The upper boundary decreases faster than the full commitment best response issuance: $\frac{\partial \bar{b}^{\infty}(b_{-i})}{\partial b_{-i}} = -1 < -\frac{1}{2} = \frac{\partial b_i^{FC}(b_{-i})}{\partial b_{-i}}$.

We construct and analyze a symmetric equilibrium in which all issuers issue at their upper bound.[50] We denote the symmetric issuance at the upper bound by

$$\bar{b}_n^{\infty} \equiv \frac{1}{n}\left(w^* - \frac{\bar{R}^r(1-e_L)(\bar{R}^r-1)}{2\gamma\sigma^2(1+(1-e_L)(\bar{R}^r-1))}\right).$$

and restrict parameters such that $\bar{b}_1^{\infty} < \frac{1}{2}w^* = b^{FC}$ so that the Hegemon would have issued the maximum credible amount $\bar{b}_1^{\infty}$. We emphasize that $\bar{b}_n^{\infty} = \frac{\bar{b}_1^{\infty}}{n}$ and conclude that as the number of issuers increases ($n \to \infty$) the total supply of the reserve assets remains constant at the level $\bar{b}_1^{\infty}$ that Hegemon would have issued alone. We collect the result in the proposition below.

Proposition 6 *(**The Failure of Competition to Increase Reserve Asset Stocks.**) Assume that debt is always safe ($\alpha = 0$), then if the Hegemon would have chosen to issue the maximum credible amount of reserve assets $\bar{b}_1^{\infty}$, competition never increases the total amount of safe assets. As the number of competitor issuers increases to infinity the equilibrium does not converge to perfect competition and instead total issuance stays constant at the level optimally chosen by a Hegemon: $\bar{b}_n^{\infty} = \frac{\bar{b}_1^{\infty}}{n}$. All issuers share equally the equilibrium monopoly rents.*

The key intuition for this proposition is that equilibrium issuance and per-period profits of a given issuer are inversely proportional to the number of issuers. To see why this is indeed an equilibrium, note that the short-term benefits of depreciating are proportional to equilibrium issuance, and that the long-term costs of depreciating are proportional to per-period profits. As a result, as the number of issuers increases, both the benefits and costs of depreciating decrease proportionately along the equilibrium path.[51]

To highlight the interaction between competition and coordination we extend the modeling of Nurkse instability from Section 6.3 to the repeated model set-up of this section. We reintroduce the assumption from Section 6.3 that in a duopoly exactly one country $\tilde{i}$ out of the two is selected at random at $t = t^+$ to face the most favorable expectations for that period, while the other country $-\tilde{i}$ faces the least favorable expectations. The selection of which country faces which expectations is i.i.d. over time. Each country i now optimally behaves as a Hegemon with $\alpha_i = 0.5$. As in Section 6.3, we assume that a true Hegemon would have faced the most favorable expectations $\alpha = 0$.

[50] Asymmetric equilibria exist but all feature the same amount of total issuance. Since the emphasis of this section is on total issuance, we focus on the symmetric equilibrium.

[51] We could have captured this effect in the one-period model of Section 2 in reduced form by assuming the cost of depreciation to scale inversely with the number of issuers $\tau = \frac{\tau^*}{n}$ for some invariant constant $\tau^* > 0$. The infinite horizon model shows how this functional form can arise naturally in a reputation equilibrium with limited commitment.

In each period the issuer that faces the unfavorable expectations depreciates ex-post if a disaster occurs since there is no punishment in this case. In each period the issuer that faces the favorable expectations does not depreciate ex-post, conditional on a disaster, if and only if:

$$\frac{1}{2}b\frac{\bar{R}^r - R^s(b)}{\bar{R}^r - 1} \geq bR^s(b)(1-e_L).$$

This leads to an upper boundary on the amount of credible debt equivalent to that of a true Hegemon facing the most favorable investors expectations with 50% probability: $\bar{b}^{\infty}_{\alpha=.5}$, as defined in equation (19).

In each period, each issuer decides how much debt to issue before knowing which investors expectations it will face. Each issuer i, therefore, anticipates that either it will face the perfectly elastic demand at R^r_H and make no expected profits for that period, or it will face the demand $R^s(b_i) = \bar{R} - 2\gamma\sigma^2(w^* - b_i)$. Each issuer solves the problem given below

$$\max_{b_i \in [0, \bar{b}^{\infty}_{\alpha=.5}]} \quad \frac{1}{2}b_i\frac{\bar{R}^r - R^s(b_i)}{\bar{R}^r - 1} = \frac{1}{2}V^{FC}(b_i)$$

$$s.t. \quad R^s = \bar{R}^r - 2\gamma\sigma^2(w^* - b)$$

The optimal issuance is $\min\{b^{FC}, \bar{b}^{\infty}_{\alpha=.5}\}$. We collect the result in the Proposition below.

Proposition 7 *Assume that a true Hegemon faces in each period the most favorable investor expectations* ($\alpha = 0$)*, but that in a duopoly exactly one country* $\tilde{i}$ *out of the two is selected at at random at* $t = t^+$ *to face the most favorable expectations for that period, while the other country* $-\tilde{i}$ *faces the least favorable expectations. The selection of which country faces which expectations is iid over time. Optimal issuance for each issuer in the duopoly is given by* $\min\{b^{FC}, \bar{b}^{\infty}_{\alpha=.5}\}$*. The effective total stock of reserve assets decreases going from a true Hegemon to a duopoly if* $\bar{b}^{\infty}_{\alpha=.5} < b^{FC}$.

Coordination undercuts commitment by reducing the expected future monopoly rents for each issuer. In this case, since each issuer only expects monopoly rents in 50% of the periods, the present value of future monopoly rents is cut by exactly 50%. Each issuer, therefore, behaves as a true Hegemon who faces the favorable expectations only half of the time. In a world of high demand for reserves ($\bar{b}^{\infty}_{\alpha=.5} < b^{FC}$), a true Hegemon that always faces the most favorable expectation would still have chosen to issue the maximal credible amount. The entrance of a second issuer and the emergence of coordination problems then reduces the total effective supply of reserve assets.

8 Conclusions

We have provided a simple and tractable framework to understand the International Monetary System. The framework helps to rationalize a number of historical episodes such as: the emergence and collapse of the Gold-Exchange standard in the 1920s, the recessionary forces associated with gold parities, the emergence and collapse of the Bretton Woods system, the duality of reserve currencies as saving and funding vehicles, and the dual use of reserve currencies as goods pricing currencies. The framework provides foundations for prominent conjectures regarding the workings and stability of the IMS such as: the Triffin Dilemma, the Nurkse Instability, and the beneficial nature of multipolar systems. Novel elements emerge from our analysis such as: the possibility that a Hegemon issuer of reserve assets might over- or under- issue from a social welfare perspective, the duality between recessionary forces in the IMS under a Gold-Exchange standard and a Floating standard at the ZLB, and the perverse effects that competition among countries in reserve asset issuance can have on the total supply of reserve assets.

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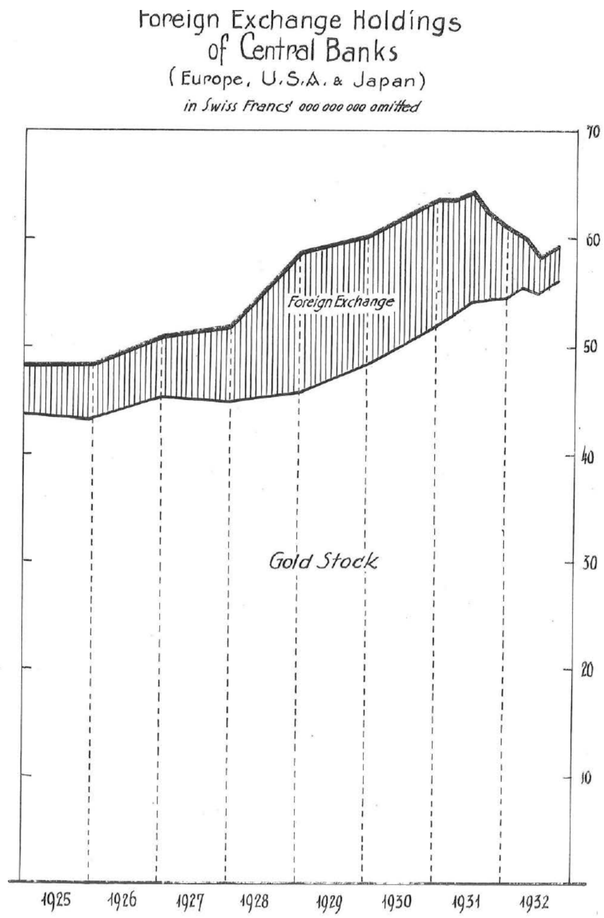
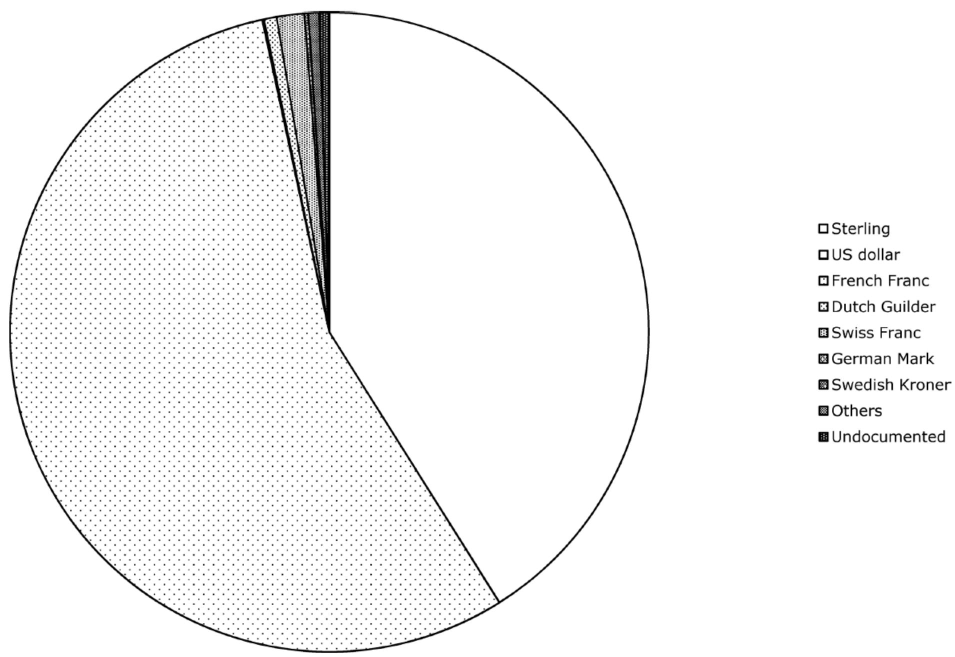
Figures

Figure 4: The Gold-Exchange Standard in the 1920s: Monetary Reserves and Gold Stock



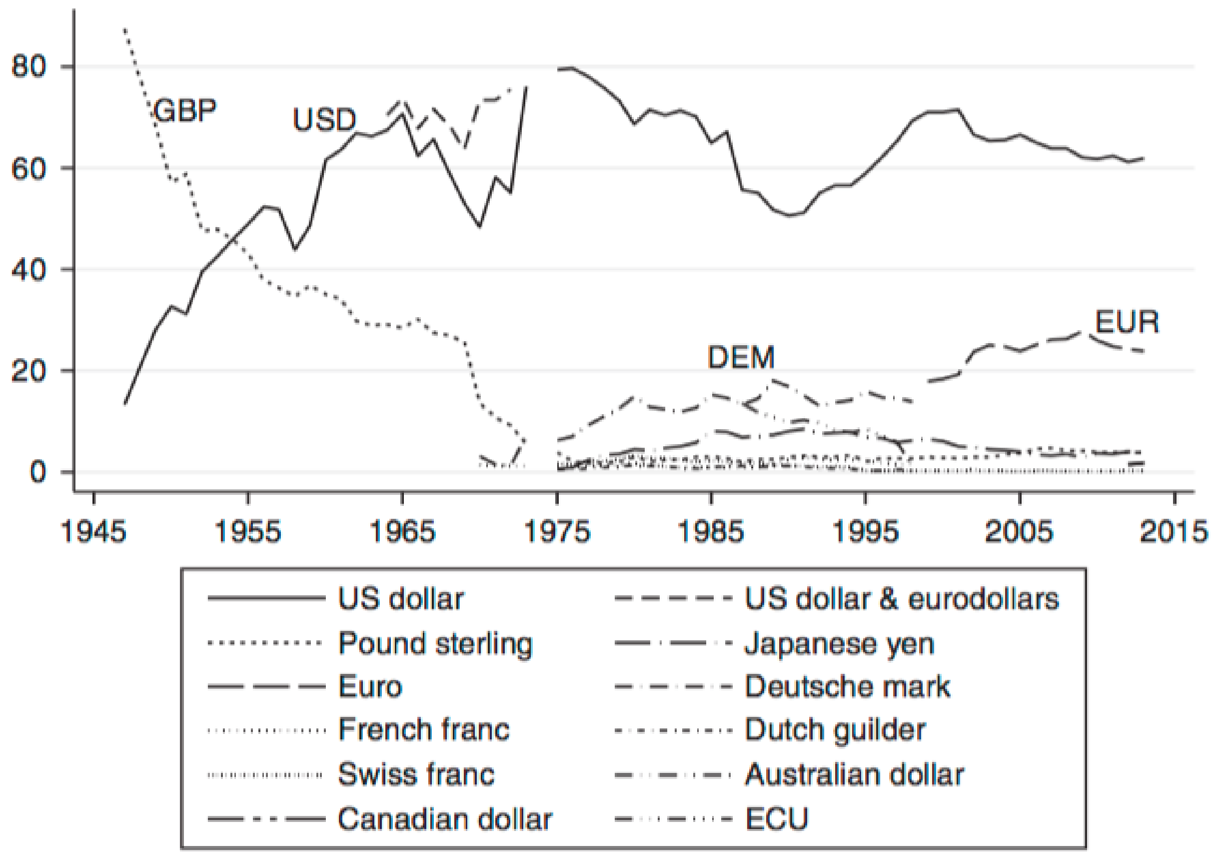
Note: Source: BIS.

Figure 5: The Gold-Exchange Standard in the 1920s: Currency Composition of Monetary Reserves



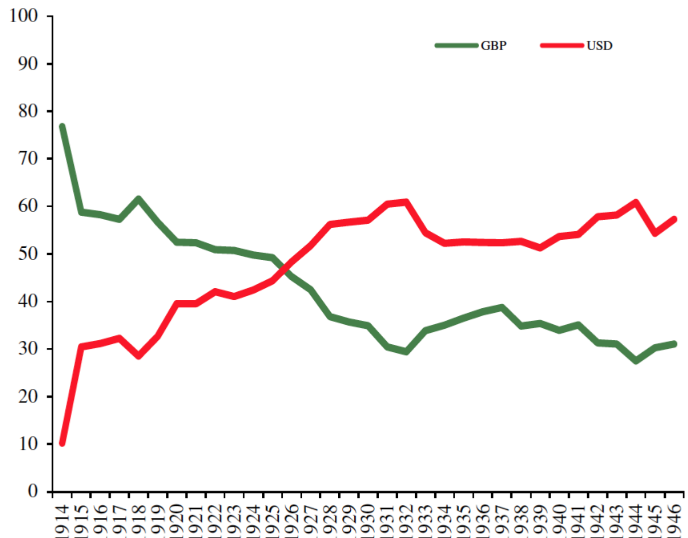
Note: Source: Eichengreen and Flandreau (2009). Currency composition of central banks' monetary reserves in 1929, see original source for details.

Figure 6: Currency Composition of Monetary Reserves 1947-2013



Note: Source: Eichengreen, Chitu and Mehl (2014).

Figure 7: Third Party Issuance in Reserve Currencies



Note: Source: Eichengreen, Chitu and Mehl (2014). The figure plots the percentage of sovereign debt issued in pounds or dollars as a fraction of all sovereign debt issued in foreign currency by the rest of the world. See original source for details.

Appendix to "A Model of the International Monetary System"

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A.1 Further Details for the Main Body of the Paper

We provide here full details of the derivation of the RoW demand function for Reserve currency debt in Equation (3).

Proposition A.1 *Focusing only on demand functions for debt that depend positively on its expected return, we conclude that either RoW agents are expecting debt to be safe and the demand function is:*

$$R^s(b) = \bar{R}^r - 2\gamma(w^* - b)\sigma^2,$$

or, if the agents are expecting the debt to be risky, it is priced identically to the risky technology and demand is indeterminate.

Proof. We start with the the generic maximization problem:

$$\max_b \quad \mathbb{E}^+[C_1^*] - \gamma Var^+(C_1^*),$$

$$w^* R^r + b(Re - R^r) = C_1^*, \quad b \geq 0.$$

The optimality condition is:

$$R\bar{e} - \bar{R}^r = \gamma[2b(R^2\sigma_e^2 + \sigma^2 - 2R\ \sigma\ \sigma_e) + 2w^*\ R\ \sigma\ \sigma_e - 2w^*\ \sigma^2], \tag{A.1}$$

where $\bar{e} = \mathbb{E}^+[e]$ and $\sigma_e = Var^+(e)$. Suppose agents were expecting debt to be safe, then $\bar{e} = 1$ and $\sigma_e = 0$, so we have:

$$R^s(b) = \bar{R}^r + \gamma[2b\sigma^2 - 2w^*\ \sigma^2] = \bar{R}^r - 2\gamma(w^* - b)\sigma^2.$$

This proves the first part of the proposition.

Suppose agents were expecting US debt to be risky, then $\bar{e} = \bar{R}^r/R_H^r$ and $\sigma_e = \sigma/R_H^r$ since we assumed $e_L = \frac{R_L^r}{R_H^r}$. Substituting these expressions into (A.1) and solving for R as a function of b, we have two roots:

$$R_- = R_H^r; \qquad R_+ = R_H^r\left(1 + \frac{\frac{\bar{R}^r}{2\gamma\sigma^2} - w^*}{b}\right).$$

The first root, which we will select, implies that the risky bond is now a perfect substitute for the risky asset and demand for the bond is therefore indeterminate. The second root we exclude on economic grounds (and by assumption in this proposition) since it generates a backward bending demand function: higher expected rates of returns on debt lower the demand for debt. ■

We provide here full details for the convenient representation of the Reserve country maximization problem adopted in the main text in Sections 2.2 and 3.

Lemma A.1 *The Reserve country full maximization problem*

$$\max_{s,b} \quad \mathbb{E}^-[C_0 + \delta(C_1 - \tau(1-e))],$$
$$s.t. \;\; w - C_0 = s - b,$$
$$s.t. \;\; sR^r - bR(b)\; e = C_1,$$
$$s.t. \;\; b \geq 0 \quad s \geq 0.$$

is equivalent to

$$\max_{b \geq 0} \quad b(\bar{R}^r - \mathbb{E}^-[R(b)e]) - \mathbb{E}^-[\tau(1-e)]. \tag{A.2}$$

Proof. Substituting the budget constraints in the objective function we have:

$$\max_{s,b} \quad \mathbb{E}^-[w + s(R^r\delta - 1) + b(1 - R(b)e\delta) - \delta\tau(1-e)],$$

The result is then obtained by recalling $\delta^{-1} = E^-[R^r]$. ■

We provide below conditions under which the full commitment equilibrium prices in Proposition 1 are free of arbitrage.

Proposition A.2 ***(Absence of Arbitrage in Full Commitment).*** *The full commitment equilibrium prices are arbitrage free iff $R^r_H > R^s(b^{FC}) > R^r_L$, which requires: $\gamma w^* \sigma^2 < (R^r_H - R^r_L)(1-\lambda)$.*

Proof. Let M be a valid SDF in this economy. We have two states and two linearly independent securities, so markets are complete, hence M is unique. Absence of arbitrage is equivalent to M being strictly positive. Requiring that M prices the two assets we have:

$$\mathbb{E}[M]R^s(b^{FC}) = 1,$$
$$\mathbb{E}[MR^r] = 1.$$

These are two equations in two unknowns. Solving for M we obtain:

$$M_H = \frac{1}{1-\lambda} \frac{R^s(b^{FC}) - R^r_L}{R^s(b^{FC})(R^r_H - R^r_L)},$$
$$M_L = \frac{1}{\lambda} \frac{R^r_H - R^s(b^{FC})}{R^s(b^{FC})(R^r_H - R^r_L)}.$$

Therefore $M_L > 0$. $M_H > 0$ iff $R^s(b^{FC}) > R^r_L$ which requires $\gamma w^* \sigma^2 < (R^r_H - R^r_L)(1-\lambda)$. ■

We note that condition $\bar{R}^r - 2\gamma w^* \sigma^2 > 0$, imposed in the main text, is not sufficient to guarantee the absence of arbitrage, but the stronger condition $\bar{R}^r - 2\gamma w^* \sigma^2 > 1$ is sufficient to guarantee the absence of arbitrage.

Proof of Proposition 2 We proceed by proving some useful claims:

Claim 1 *The Reserve country will never choose to issue so much debt as to lose the safety premium for sure.*

Proof. Note that $V(0) = 0$ and $V(b) = -\lambda\tau(1-e_L) < 0 \quad \forall b \in (\bar{b}, w^*]$.$\square$

Claim 2 *If the full-commitment equilibrium level of debt (b^{FC}) lies in the safety region, then the Reserve country will issue that level of debt and the only equilibrium is the safe equilibrium.*

Proof. Recall $b^{FC} = argmax(V^{FC}(b))$. If $b^{FC} \leq \underline{b}$ then $sup(V^{FC}(b)) = \max(V(b))$ since $V^{FC}(b) \geq V(b)$ and equality holds only for $b \in [0, \underline{b}]$.$\square$

Let us create a pseudo value function $\tilde{V}(b) \equiv (1-\alpha)V^{FC}(b) - \alpha\lambda\tau(1-e_L)$. Notice that $\tilde{V}(b) = V(b)$ $\forall b \in (\underline{b}, \bar{b}]$. If $b^{FC} > \underline{b}$ we could have several cases that are summarized below.

Claim 3 *Assume $b^{FC} > \underline{b}$, then the Reserve country issues either $b = \underline{b}$ or $\min\{b^{FC}, \bar{b}\}$, whichever generates higher expected profits. If the Reserve country issues $\underline{b}$ there is a unique safe equilibrium. If the Reserve country issues $\min\{b^{FC}, \bar{b}\}$ there are multiple equilibria: the safe and the collapse equilibria.*

Proof. In the region of debt issuance when only the safe equilibrium is possible ($b \in [0, \underline{b}]$), the local maximum of V is achieved at the upper boundary for $b = \underline{b}$. To verify this claim recall the assumption $b^{FC} = argmax(V^{FC}(b)) > \underline{b}$, the fact that $V(b) = V^{FC}(b)$ $\forall b \in [0, \underline{b}]$, and that $V^{FC}(b)$ is a strictly concave function.

The Reserve country therefore issues $b = \underline{b}$ iff this local maximum is also the global maximum, i.e. when $V^{FC}(\underline{b}) \geq \max_{b \in (\underline{b}, \bar{b}]} V$. Note that by claim 1, we can ignore the last region of the state space since $argmax(V(b)) \in (0, \bar{b}]$.

Suppose $V^{FC}(\underline{b}) < \max_{b \in (\underline{b}, \bar{b}]} V$, then the Reserve country issues b^{FC} if $b^{FC} \in (\underline{b}, \bar{b}]$ and otherwise issue $\bar{b}$. To verify this claim notice that globally $argmax(\tilde{V}(b)) = argmax(V^{FC}(b))$, since $\tilde{V}(b) = aV^{FC}(b) + c$ with constants $a > 0$ and $c < 0$. Furthermore $\tilde{V}(b)$ is a strictly concave function. Therefore, $argmax_{b \in (\underline{b}, \bar{b}]} V(b)$ takes value b^{FC} if $\bar{b} \geq b^{FC}$ or equals the upper bound $\bar{b}$.$\square$

The claims prove items 1,2,3 of the Proposition. The presence of an ex-ante safety premium in all equilibria follows from the expected return on debt:

$$\mathbb{E}^-[R(b)e] = (1-\alpha(b))R^s(b) + \alpha(b)\bar{R}^r$$

noticing that the optimal issuance level is always below w^*, so that at the optimal issuance one has: $R^s(b) < \bar{R}^r$, and $\alpha(b) < 1$. We conclude that $\mathbb{E}^-[Re] < \bar{R}^r$ and there is an exorbitant privilege.$\square$

The next proposition verifies under which conditions equilibrium prices in the model with limited commitment are arbitrage free.

Proposition A.3 ***(Absence of Arbitrage under Limited Commitment).*** *The equilibrium prices at time $t = 0^+$, conditional on debt being safe, are arbitrage free if and only if $R_H^r > R^s(b^*) > R_L^r$, where b^* is the equilibrium issuance. This condition requires $2\gamma\sigma^2(w^* - b^*) < (R_H^r - R_L^r)(1-\lambda)$. If issuance takes place at $b^* = b^{FC}$ then this condition is the same as that of Proposition A.2. If issuance takes place at $b^* = \underline{b}$ then this condition is less stringent than the requirement in Proposition A.2. Conversely, if issuance takes place at $b^* = \bar{b}$ then this condition is more stringent than the requirement in Proposition A.2.*

Proof. The proof is entirely analogous to that of Proposition A.2. ■

We provide here details on Section 4. We first introduce a Lemma that proves equation (12).

Lemma A.2 ***Welfare as the Area Under the Demand Curve.*** *RoW welfare can be computed according to:*

$$V_{RoW}(b) = V_{RoW}(R^s(0)) + (1-\alpha(b))\int_{R^s(0)}^{R^s(b)} b(\tilde{R}^s)d\tilde{R}^s,$$

where $b(R^s)$ *is the demand curve for safe debt given by*

$$b(R^s) = \frac{R^s - \bar{R}^r + 2\gamma\sigma^2 w^* + 2\gamma_L \underline{b}\mathbf{1}_{\{b\leq\underline{b}\}}}{2\gamma\sigma^2 + 2\gamma_L\underline{b}\mathbf{1}_{\{b\leq\underline{b}\}}},$$

and

$$V_{RoW}(0) = w^*\bar{R}^r - \gamma\sigma^2 w^{*2} - \gamma_L \underline{b}^2.$$

Proof. The maximization problem of the RoW is

$$\max_{b,s^*} \quad \mathbb{E}^+[C_1^*] - \gamma Var^+(C_1^*) - \gamma_L(\underline{b} - \min(b,\underline{b})\mathbf{1}_{\{\mathbb{E}^+[e]=1\}})^2,$$

$$s.t. \quad s^*R^r + bRe = C_1^*, \qquad b + s^* = w^*, \qquad b \geq 0.$$

Assume the debt is safe, then we can write the problem as:

$$\max_{b,s^*} \quad bR^s + s^*\bar{R}^r - \gamma s^{*2}\sigma^2 - \gamma_L(\underline{b} - w^* + s^*)^2\mathbf{1}_{\{w^*-s^*\leq\underline{b}\}} = bR^s + \nu(s^*) \quad s.t. \quad s^* + b = w^*.$$

This problem leads to optimality conditions that describe demand functions $b(R^s)$ and $s^*(R^s)$. In particular, optimality requires:

$$R^s = \nu'(s^*). \tag{A.3}$$

We then write $V^s_{RoW}(R^s) = b(R^s)R^s + \nu(w^* - b(R^s))$, and take the partial derivative w.r.t. R^s:

$$V'_{RoW}(R^s) = b(R^s) + b'(R^s)R^s + \nu'_b(w^* - b(R^s))b'(R_s).$$

Substituting in the above equation the optimality condition in equation (A.3), we obtain $V'(R^s) = b(R^s)$. Integrating over both sides we obtain:

$$V^s_{RoW}(R^s) = V_{RoW}(R^s_0) + \int_{R^s_0}^{R^s} b(\tilde{R}^s)d\tilde{R}^s,$$

where $R^s_0 = \bar{R}^r - 2\gamma\sigma^2 w^* - 2\gamma_L\underline{b}$, and $V_{RoW}(R^s_0) = w^*\bar{R}^r - \gamma\sigma^2 w^{*2} - \gamma_L\underline{b}^2$.

If instead we assume that debt is risky, then RoW welfare is given by:

$$V^r_{RoW} = w^*\bar{R}^r - \gamma\sigma^2 w^{*2} - \gamma_L\underline{b}^2.$$

Note that $V^r_{RoW} = V^s_{RoW}(R^s_0)$.

We define RoW welfare from an ex-ante perspective, before the equilibrium sunspot is selected, to be:

$$V_{RoW}(b) = (1-\alpha(b))V^s_{RoW}(R^s(b)) + \alpha(b)V^r_{RoW},$$

where we have found it convenient to write $V_{RoW}(b)$ as a function of b and $V^s_{RoW}(R^s)$ as a function of R^s. We conclude that:

$$V_{RoW}(b) = V_{RoW}(R^s(0)) + (1-\alpha(b))\int_{R^s(0)}^{R^s(b)} b(\tilde{R}^s)d\tilde{R}^s.$$

■

Continuation of Proof of Proposition 3

We continue here the proof initiated in the main text. We prove the second statement of the proposition: for a demand curve that is sufficiently concave one can have over-issuance by the Hegemon.

We start by deriving a bound on $\bar{\gamma}_L(\tau)$ such that the Hegemon does not want to issue in the interior of the Safety region for $b^{FC} > \underline{b}(\tau)$. Recall that the value function within the Safety region is: $V(b) = (\bar{R}^r - R^s(b))b$ for $b \in [0, \underline{b}]$. Hence, in that region $V'(b) = \bar{R}^r - R^s(b) - bR^{'s}(b)$. Since $V'(0) > 0$, and $V(b)$ is concave, then to have that $V'(b) > 0$ for $b \in [0, \underline{b}]$, it is sufficient to have $V'(\bar{b}) > 0$ which imposes the bound:

$$\gamma_L < \gamma\sigma^2\left(\frac{w^*}{\underline{b}(\tau)} - 2\right).$$

We define the function $\gamma_L(\tau)$ to be the highest value that γ_L can take in the above bound as a function of τ:

$$\bar{\gamma}_L(\tau) = \gamma\sigma^2\left(\frac{w^* R^r_H}{\tau} - 2\right).$$

In what follows, we assume $\gamma_L \in [\eta\bar{\gamma}_L(\tau), \bar{\gamma}_L(\tau)]$ for $\eta \in (0,1]$. We take the limit as $\tau \downarrow 0$, so that $b^{FC} = \frac{1}{2}w^* > \bar{b}(\tau)$ since $\lim_{\tau\downarrow 0} \bar{b}(\tau) = 0$. In this limit, and as described in the text more generally in Section 3.0.1, there exists $\alpha^*_m \in (0,1)$ s.t. the Hegemon issues $\bar{b}(0)$ for all $\alpha \leq \alpha^*_m$ and issues $\underline{b}(0)$ for all $\alpha > \alpha^*_m$. Below we prove that in this limit we have:

$$\lim_{\tau\downarrow 0} \alpha^*_m(\tau) = \frac{\frac{2\gamma\sigma^2 w^*}{\bar{R}^r - 2w^*\gamma\sigma^2} - \frac{2\gamma\sigma^2 w^*}{R^r_H}}{\frac{2\gamma\sigma^2 w^*}{\bar{R}^r - 2w^*\gamma\sigma^2} + \lambda(1 - e_L)} \in (0,1). \tag{A.4}$$

Similarly, we can compute a threshold $\alpha^*_{row}(\tau)$ s.t. the RoW investors would have preferred the equilibrium issuance $\bar{b}(\tau)$ for all lower αs and otherwise would have preferred the lower issuance $\underline{b}(\tau)$.

We change the notation slightly from Lemma A.2 and define the welfare of RoW investors to be the function $V_{RoW}(b, \alpha)$, to make the dependence on α more explicit. At issuance level $\underline{b}(\tau)$, we have:

$$V_{RoW}(\underline{b}, 0) = \underline{b}R^s(\underline{b}) + (w^* - \underline{b})\bar{R}^r - \gamma(w^* - \underline{b})^2\sigma^2.$$

Similarly welfare of RoW at issuance level $\bar{b}$ is given by:

$$\begin{aligned} V_{RoW}(\bar{b}, \alpha) &= (1-\alpha)\left(\bar{b}R^s(\bar{b}) + (w^* - \bar{b})\bar{R}^r - \gamma(w^* - \bar{b})^2\sigma^2\right) + \alpha(w^*\bar{R}^r - \gamma w^{*2}\sigma^2 - \gamma_L\underline{b}^2) \\ &= V_{RoW}(\bar{b}, 0) - \alpha(V_{RoW}(\bar{b}, 0) - V_{RoW}(0, 0)). \end{aligned}$$

Notice that $V_{RoW}(\underline{b}, 0)$ is independent of α and $V_{RoW}(\bar{b}, \alpha)$ is continuous and decreasing in α. Furthermore, $V_{RoW}(\bar{b}, 0) > V_{RoW}(\underline{b}, 0)$ and $V_{RoW}(\bar{b}, 1) < V_{RoW}(\underline{b}, 0)$. So that we conclude $V_{RoW}(\underline{b}, 0) = V_{RoW}(\bar{b}, \alpha^*_{RoW})$, with:

$$\alpha^*_{RoW} = \frac{V_{RoW}(\bar{b}, 0) - V_{RoW}(\underline{b}, 0)}{V_{RoW}(\bar{b}, 0) - V_{RoW}(0, 0)}.$$

Below we prove that in the limit $\tau \downarrow 0$, we have:

$$\lim_{\tau\downarrow 0} \alpha^*_{RoW}(\tau) = 0. \tag{A.5}$$

We conclude that for $\eta \in (0,1]$ and $\gamma_L \in [\eta\bar{\gamma}_L(\tau), \bar{\gamma}_L(\tau)]$, in the limit at $\tau \downarrow 0$ one has:

$$\lim_{\tau\downarrow 0} \alpha^*_{RoW}(\tau) = 0 < \frac{\frac{2\gamma\sigma^2 w^*}{\bar{R}^r - 2w^*\gamma\sigma^2} - \frac{2\gamma\sigma^2 w^*}{R^r_H}}{\frac{2\gamma\sigma^2 w^*}{\bar{R}^r - 2w^*\gamma\sigma^2} + \lambda(1-e_L)} = \lim_{\tau\downarrow 0} \alpha^*_m(\tau).$$

Since $\alpha^*_{TOT}(\tau)$ is a convex combination of $\alpha^*_{RoW}(\tau)$ and $\alpha^*_m(\tau)$ with interior non vanishing weights on each of the elements, we obtain the result in the Proposition.

We now prove the limits in equations (A.4) and (A.5). We prove the results only for $\eta = 1$. The generalization is straightforward. We start by proving that $\lim_{\tau\downarrow 0} \alpha^*_{RoW}(\tau) = 0$. For small τ, we have

$$\bar{\gamma}_L(\tau) = \frac{\gamma\sigma^2 w^* R^r_H}{\tau} - 2\gamma\sigma^2,$$

$$\underline{b}(\tau) = \frac{\tau}{R^r_H},$$

$$\bar{b}(\tau) = \frac{\tau}{\bar{R}^r - 2w^*\gamma\sigma^2} + O(\tau^2),$$

$$R^s(0) = \bar{R}^r - 4\gamma\sigma^2 w^* + 4\gamma\sigma^2 \frac{\tau}{R^r_H},$$

$$R^s(\underline{b}(\tau)) = R^s(0) + 2\gamma\sigma^2 w^* - 2\gamma\sigma^2 \frac{\tau}{R^r_H},$$

$$R^s(\bar{b}(\tau)) = R^s(\underline{b}(\tau)) + 2\gamma\sigma^2 \left[\frac{\tau}{\bar{R}^r - 2\gamma\sigma^2 w^*} - \frac{\tau}{R^r_H}\right] + O(\tau^2).$$

We can now compute consumer welfare using the area under the demand curve formula

$$V_{RoW}(\underline{b}(\tau), \alpha) = V_{RoW}(0, \alpha) + \int_{R^s(0)}^{R^s(\underline{b}(\tau))} b(R^s)dR^s.$$

We get

$$V_{RoW}(\underline{b}(\tau), \alpha) = V_{RoW}(0, \alpha) + \frac{2\gamma\sigma^2 w^* + 2\bar{\gamma}_L(\tau)\underline{b}(\tau) - \bar{R}^r}{2\gamma\sigma^2 + 2\bar{\gamma}_L(\tau)}[R^s(\underline{b}(\tau)) - R^s(0)] + \frac{1}{2}\frac{(R^s(\underline{b}(\tau)))^2 - (R^s(0))^2}{2\gamma\sigma^2 + 2\bar{\gamma}_L(\tau)},$$

which yields

$$V_{RoW}(\underline{b}(\tau), \alpha) = V_{RoW}(0, \alpha) + \frac{\gamma\sigma^2 w^*}{R^r_H}\tau + O(\tau^2).$$

We use

$$V_{RoW}(\bar{b}(\tau), \alpha) = V_{RoW}(0, \alpha) + (1-\alpha)[V_{RoW}(\underline{b}(\tau), \alpha) - V_{RoW}(0, \alpha)] + (1-\alpha)\int_{R^s(\underline{b}(\tau))}^{R^s(\bar{b}(\tau))} b(R^s)dR^s.$$

We get

$$V_{RoW}(\bar{b}(\tau), \alpha) = V_{RoW}(0, \alpha) + (1-\alpha)[V_{RoW}(\underline{b}(\tau), \alpha) - V_{RoW}(0, \alpha)] + O(\tau^2).$$

This immediately implies that

$$\alpha^*_{RoW}(\tau) = O(\tau).$$

We can also compute Hegemon welfare

$$V(\underline{b}(\tau), \alpha) = \frac{2\gamma\sigma^2 w^*}{R_H^r}\tau,$$

$$V(\bar{b}(\tau), \alpha) = (1-\alpha)\frac{R_H^r}{\bar{R}^r - 2w^*\gamma\sigma^2}V(\underline{b}(\tau), \alpha) - \alpha\lambda(1-e_L)\tau + O(\tau^2).$$

This implies that

$$\alpha_m^*(\tau) = \frac{\frac{2\gamma\sigma^2 w^*}{\bar{R}^r - 2w^*\gamma\sigma^2} - \frac{2\gamma\sigma^2 w^*}{R_H^r}}{\frac{2\gamma\sigma^2 w^*}{\bar{R}^r - 2w^*\gamma\sigma^2} + \lambda(1-e_L)} + O(\tau),$$

where

$$\frac{\frac{2\gamma\sigma^2 w^*}{\bar{R}^r - 2w^*\gamma\sigma^2} - \frac{2\gamma\sigma^2 w^*}{R_H^r}}{\frac{2\gamma\sigma^2 w^*}{\bar{R}^r - 2w^*\gamma\sigma^2} + \lambda(1-e_L)} \in (0,1).$$

□