Macroeconomic Fluctuations with HANK & SAM: an Analytical Approach

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PRELIMINARY

Abstract

New Keynesian models with unemployment and incomplete markets are rapidly becoming a new workhorse model in macroeconomics. Such models typically require heavy computational methods, which may obscure intuition and overlook equilibria. We present a highly tractable version which can be analyzed with pencil and paper. Exploiting the model's tractability, we explore global steady-state equilibria, their local determinacy properties, and the effects of shocks on key macro variables and financial risk premia. Our results highlight that– due the interaction between incomplete markets, sticky prices and endogenous unemployment risk– pessimistic beliefs may be self-fulfilling and move the economy into temporary episodes of low demand and high unemployment, as well as into a long-lasting "unemployment trap".

1 Introduction

The New Keynesian (NK) model has gained widespread use both in academic research and in policy circles. The crux of the NK model is that nominal frictions induce inefficient fluctuations in the economy that monetary and fiscal policy can be designed to address. A key concern regarding fluctuations in the economy concern the labor market implications especially fluctuations in unemployment and the impact on the poorer parts of society. Paradoxically, however, the standard NK model abstracts from both unemployment and from distributional issues. Blanchard and Gali (2010), Christiano, Eichenbaum and Trabandt (2016) and others have recently extended NK models with Diamond-Mortensen-Pissarides-style Search and Matching (SAM) frictions and studied the resulting fluctuations in unemployment and the policy implications thereof. This work, however, relies upon the existence of insurance markets which shield individual agents from idiosyncratic risk which therefore means that unemployment is not a major concern. Recently, a number of papers have introduced financial market incompleteness in the Bewley-Aiyagari tradition, generating wealth inequality. Kaplan, Moll and Violante (2016) have dubbed such models Heterogeneous Agents New Keynesian (HANK) models.[1] By giving centre stage to HANK and SAM, the new generation of macroeconomic models mark a clear break with the traditional "representative agent" assumption, facilitated by increasingly sophisticated numerical methods in the spirit of Krusell and Smith (1998).

An important advantage of the new generation of models with heterogeneous agents is that they allow for a more detailed confrontation with the data, as they offer a rich array of new empirical predictions in both the time and cross-sectional dimension. Moreover, the added frictions give rise to new macroeconomic mechanisms. Specifically, the joint presence of a precautionary savings motive, unemployment risk and nominal rigidities may lead to possibly large fluctuations in aggregate demand, providing a new perspective on the sources and effects of labor market fluctuations (Ravn and Sterk, 2012).

The richness of heterogenous-agents NK models, however, does come at an important cost: it has become increasingly difficult to provide a rigorous characterization of the equilibrium of the model. In particular, the new generation of heterogeneous-agents NK models typically relies

[1] Other papers in this new vein include Guerrieri and Lorenzoni (2016), Beaudry, Galizia and Portier (2014) , Bayer, Pham-Dao, Luetticke and Tjaden (2015), Auclert (2015), Berger, Dew-Becker, Schmidt and Takahasi (2016), Challe, Matheron, Ragot and Rubio-Ramirez (2016), den Haan, Rendahl and Riegler (2016), Gornemann, Kuester and Nakajima (2016), Kekre (2016), Luetticke (2015), McKay, Nakamura and Steinsson (2016), McKay and Reis (2016), and Ravn and Sterk (2012, 2016).

on numerical solutions which makes it hard to understand the underlying economic mechanisms in a clear fashion. Another serious problem is that the interactions between frictions may give rise to additional equilibria that may be overlooked in numerical procedures. This possibility is not a mere technical artefact, as fluctuations driven by "animal spirits" can arise naturally under incomplete markets and endogenous employment risk.[2]

This paper complements the new vintage of NK models with an analytically tractable counterpart that is as simple and intuitive as the basic NK model in Clarida, Gali and Gertler (2000), but nonetheless features search and matching frictions and incomplete markets. The model's tractability derives from a special assumption on households' borrowing limit, see also Krusell, Smith and Mukoyama (2009), Ravn and Sterk (2012) and Werning (2015).[3] The assumption gives rise to an equilibrium with limited cross-sectional wealth heterogeneity, while preserving an endogenous precautionary savings motive. In equilibrium, three groups of households emerge: borrowing-constrained unemployed households, unconstrained but asset-poor employed households, and asset-rich but liquidity-constrained households (see Kaplan, Violante and Weidner, 2014, for empirical evidence on the distribution of different types of assets cross households). The wealthy households face a relatively low degree of consumption risk, which weakens their desire to accumulate bonds for precautionary reasons, relative to the poor households. As a result, the wealthy households are unwilling to invest in bonds at prevailing market interest rates, causing them to be liquidity-constrained in equilibrium.

In this setting, two key wedges arise. The first is a standard sticky-price wedge in the labor demand equation (also called the "NK Phillips Curve"). The second wedge appears in the Euler equation, and results from financial market incompleteness. Werning (2015) highlights the emergence of this wedge in an analytically aggregated Euler equation, but does not model explicitly how the wedge is determined in equilibrium. We demonstrate how the incomplete markets wedge, in a model with search and matching frictions, is pinned down as a function of tightness of the labor market, how it interacts with the sticky-price wedge, and how it is affected by policy. We thereby also provide a micro-foundation for exogenous discount factor shocks that

[2] Intuitively, a wave of pessimism among households about their employment prospects could be self-fulfilling as the increased desire to build precautionary savings reduces aggregate demand, causing firms to hire fewer workers when prices are sticky and stabilization policy is insufficiently responsive.

[3] Krusell, Smith and Mukoyama (2009) study asset pricing in an endowment economy without nominal rigidities. Werning (2015) derives an aggregate Euler equation under incomplet markets, but does not explicitly model nominal rigidities or search and matching frictions. Ravn and Sterk (2012) study labor market shocks using numerical simulations.

are often introduced in NK models, either to better fit the data (see e.g. Smets and Wouters, 2007) or to drive down the interest rate to a point at which the Zero Lower Bound (ZLB) on the nominal interest rate becomes binding (see e.g. Christiano, Eichenbaum and Evans, 2013).

We systematically explore the model's implications for equilibrium determinacy, both globally and locally, and the macroeconomic effects of fundamental and non-fundamental shocks. The tractability of the model allows us to characterize explicitly how equilibrium outcomes emerge from interactions between market frictions, policy parameters and the ZLB on the nominal interest rate. We consider not only the implications for standard macro variables, such as output, inflation and the monetary policy rate, but also on the prices of risky assets, which in our setting may bear non-negligible risk premia which vary endogenously with the state of the macroeconomy.

We present four sets of results. The first concerns the steady-state properties of the model. As in the basic NK model there is an "intended" steady state as well as an unintended "liquidity trap". In the latter steady state, the ZLB binds and output is relatively low, as in Benhabib, Schmitt-Grohe and Uribe (2001, 2002). Unlike the standard NK model, however, the incomplete markets has an additional third steady state, which we label the "unemployment trap," where aggregate demand is depressed to a level at which it is no longer profitable for firms to invest in costly vacancies. Hence, hiring comes to a complete standstill which perpetuates high job uncertainty and hence low demand.

Next, we study the local determinacy properties of the three possible steady states, exploring the scope for belief-driven dynamics around the steady states (Section 3.2). We first present an analytical determinacy condition for the intended steady state. We show that local indeterminacy can arise even when the "Taylor principle" is satisfied (i.e. the interest rate rule coefficient on inflation is larger than one). This result is due to the presence of the incomplete markets wedge, but depends crucially on its interaction with the sticky-price wedge. Additionally, we show that the unemployment trap is determinate under a standard rule which responds more than one-for-one to inflation. Around this steady state, the monetary policy rule determines the rate of inflation, but has no grip on unemployment.

Our third set of results concerns the responses to fundamental and non-fundamental shocks (Section 4). We present an analytical formula for the local response to a productivity shock around the intended steady state and show that the presence of incomplete markets can create large amplification. When the steady state is locally indeterminate, pessimistic belief shocks generate joint declines in employment, inflation and the real interest rate. The persistence of the effects is endogenously determined, and is maximized at degrees of price stickiness and market incompleteness that are just strong enough to generate local indeterminacy, but are otherwise relatively moderate. We further study the effects of large shocks that may move the economy to make the ZLB binding, and/or move the economy to a different steady-state equilibrium. A key finding here is that under incomplete markets, ZLB episodes are not necessarily very deflationary. This happens as the real interest rate declines when unemployment increases, due to a heightened demand for precautionary savings. At the ZLB, a decline in the real interest rate implies an increase in inflation via the Fisher relation.

Finally, we study the determination of risk premia (Section 5). There is little existing research on this topic within the context of the NK model, since under complete markets the model does not generate first-order risk premia. At the same time, monetary policy is widely believed to have a profound impact on financial markets. We show that under incomplete markets, the NK model can generate substantial risk premia and we provide an analytical formula for their magnitudes. The formula reveals a close connection between risk premia, the business cycle, and monetary policy. This results from the fact that idiosyncratic unemployment risk comoves negatively with aggregate demand, causing households to dislike risky assets which pay off relatively little after an adverse shock hits the macroeconomy. Monetary policy has a dual effect on risk premia, since more stable fluctuations in aggregate demand reduce both fluctuations in asset payoffs and fluctuations in households' stochastic discount factors.

2 The Model

We construct a model which combines nominal rigidities in price setting as in the NK tradition, labor market matching frictions in the Diamond-Mortensen-Pissarides (DMP) tradition, and incomplete asset markets in the Aiyagari-Bewley tradition. The economy is made up of households who consume and work, firms which produce output, and a monetary authority in charge of monetary policy. We allow for both aggregate and idiosyncratic uncertainty and assume lack of household insurance against idiosyncratic income risk.

2.1 Preferences and technologies

Preferences: There is a continuum of mass 1 of infinitely lived single-member households indexed by $i \in (0,1)$. Households maximize the expected discounted present value of their utility streams:

$$\mathcal{V}_{i,t} = \mathbb{E}_t \sum_{s=t}^{\infty} \beta^{s-t} \left(\frac{\mathbf{c}_{i,s}^{1-\mu} - 1}{1-\mu} - \zeta \mathbf{n}_{i,s} \right), \tag{1}$$

where $\mathbb{E}_t x_s = E(x_s|I_t)$ is the date t conditional expectation of x_s, $\beta \in (0,1)$ the subjective discount factor, $\mathbf{c}_{i,s}$ household i's consumption level in period s, $\mu > 0$ the measure of relative risk aversion, $\mathbf{n}_{i,s}$ the household's employment status, and $\zeta > 0$ is a parameter that measures the disutility of market work. An individual household is either employed and works full-time or works not at all:

$$\mathbf{n}_{i,s} = \begin{cases} 0 \text{ if not employed at date } s \\ 1 \text{ if employed at date } s \end{cases}. \tag{2}$$

The consumption level of an individual household is a constant elasticity of substitution aggregator of a basket of goods:

$$\mathbf{c}_{i,s} = \left(\int_j \left(c_{i,s}^j\right)^{1-1/\gamma} dj \right)^{1/(1-1/\gamma)}, \tag{3}$$

where c_i^j is household i's consumption of good j and $\gamma > 1$ is the elasticity of substitution between goods varieties. Workers who are not employed produce ϑ units of the aggregate consumption good at home.

Households decide on consumption, savings, on the financial portfolio, and on whether or not to participate in the labor force. A household not in the labor force cannot search for jobs in the market. Households who stand to lose on the net from employment declare themselves out of the labor force. We discuss the savings and portfolio problems later.

Production technology: Market goods are produced by a continuum of monopolistically competitive firms that each supply a differentiated good. The technology is:

$$y_{j,s} = \exp(A_s)\, n_{j,s}, \tag{4}$$

where y_j is firms j's output and n_j its employment. A_s is an *aggregate* stochastic productivity shock which follows a first-order autoregressive process:

$$A_s = \rho A_{s-1} + \sigma_A \varepsilon_s^A, \tag{5}$$

where $\rho \in (-1,1)$, $\sigma_A > 0$ and $\varepsilon_s^A \sim \mathcal{N}(0,1)$.

The law of motion of employment of firm j is:

$$n_{j,s} = (1-\omega)\, n_{j,s-1} + h_{j,s}, \tag{6}$$

where ω is a constant employment separation rate and h_j denotes hiring by firm j. Firms hire workers by posting and fill each posted job vacancy with probability q_s. We take firms to be sufficiently large that q_s is also the fraction of vacancies that are filled.[4] Thus, the total number of vacancies posted by firm j is given by h_j/q_s.

Matching technology: Agents receive information about current productivity shocks at the beginning of each period. Existing worker-firm relationships are resolved at the end of the period and new ones are formed at the beginning of the next period. Job separations are exogenous events that affect existing hires randomly so that employees perceive ω to be the risk that they lose their current job.

New hires are produced by a matching function which relates the measure of worker-firm matches to the aggregate measures of vacancies and job searchers as:

$$\mathbf{M}(e_s, v_s) = \psi e_s^{\alpha} v_s^{1-\alpha}, \tag{7}$$

where $\psi > 0$ indicates match efficiency, $\alpha \in (0,1)$, e_s is the measure of job searchers and $v_s = \int_j (v_{j,s} + \overline{v})\, dj$ is the aggregate measure of vacancies. Here, $v_{j,s}$ denotes the number of "formal" vacancies posted by firm j, which come at a flow cost $\exp(A_s)\kappa > 0$ per unit.[5] Further, $\overline{v} \geq 0$ is a fixed amount of "informal" vacancies, available to each firm, which come at no cost. The idea behind the latter type of vacancies is that even without devoting any resources to recruitment, firms would be able to hire some workers via word-of-mouth channels.[6]

The job filling probability, q_s, and the job finding rate, η_s,i.e. the probability that a jobless worker finds a new employer, are determined as:

$$q_s = \frac{\mathbf{M}(e_s, v_s)}{v_s} = \psi \boldsymbol{\theta}_s^{-\alpha}, \tag{8}$$

$$\eta_s = \frac{\mathbf{M}(e_s, v_s)}{e_s} = \psi \boldsymbol{\theta}_s^{1-\alpha}, \tag{9}$$

where $\boldsymbol{\theta}_s \equiv \frac{v_s}{e_s}$ is labor market tightness, the ratio of vacancies to job searchers. Note that the job filling rate and the job finding rate are related as $q_s = \psi^{\frac{1}{1-\alpha}} \eta_s^{\frac{\alpha}{\alpha-1}}$. It turns out that ψ and

[4] This is useful because we will later assume symmetry across firms and the large firm assumption avoids having to consider that the number of vacancies filled are stochastic.

[5] The scaling of the hiring cost by productivity ensures that productivity shocks do not mechanically alter the relative cost of hiring and production.

[6] In much of the analysis, the informal vacancies play no role since in most equilibria new workers are –at the margin– hired via costly vacancies. However, we will show that there may also be equilibria in which firms are unwilling to invest any resources in vacancies. In those cases, the informal vacancies become relevant as they avoid a complete collapse of employment.

κ enter the model equations in a way that is observationally equivalent for our purpose. Hence we normalize ψ to one from now on.

2.2 Price and Wage Setting

Prices: Firms set prices of their products, $P_{j,s}$, subject to a quadratic price adjustment cost as in Rotemberg (1982). The extent of nominal rigidities in price setting is parameterized by $\phi \geq 0$ which determines the size of the price adjustment costs. Let w_s denote the average real wage, y_s aggregate output, and P_s be the aggregate price level. We anticipate that in equilibrium wages are the same for all workers and hence exclude worker- and firm specific indices for the wage. Firms maximize:

$$\mathbb{E}_t \sum_{s=t}^{\infty} \mathbf{\Lambda}_{j,t,t+s} \left[\frac{P_{j,s}}{P_s} y_{j,s} - w_s n_{j,s} - \kappa v_{j,s} - \frac{\phi}{2} \left(\frac{P_{j,s} - P_{j,s-1}}{P_{j,s-1}} \right) y_s \right], \tag{10}$$

subject to (6) as well as to a demand constraint which derives from the consumers' decision problems:

$$y_{j,s} = \left(\frac{P_{j,s}}{P_s} \right)^{-\gamma} y_s, \tag{11}$$

where $y_s = \int_j y_{j,s} dj$ denotes aggregate output and $\mathbf{\Lambda}_{j,t,t+s}$ is the discount factor of the firm's owners (discussed below). We also impose that investment in formal vacancies cannot be negative, i.e.

$$v_{j,s} \geq 0. \tag{12}$$

Real marginal costs is the sum of the wage and hiring costs of a marginal worker (relative to productivity). To hire a marginal additional worker at date s, firms must spend $\kappa \exp(A_s)/q_s$ but since matches persist, hiring today brings about future hiring cost savings $(1-\omega)\exp(A_{s+1})\kappa/q_s$ (discounted at the appropriate rate). Real marginal costs are therefore:

$$\mathbf{mc}_{j,s} = \frac{w_s}{\exp(A_s)} + \frac{\kappa}{q_s} - \lambda_{v,j,s} - (1-\omega)\,\mathbb{E}_s \mathbf{\Lambda}_{j,s,s+1} \left\{ \frac{\kappa}{q_{s+1}} \frac{\exp(A_{s+1})}{\exp(A_s)} - \lambda_{v,j,s+1} \right\}, \tag{13}$$

where $\lambda_{v,j,s} \geq 0$ is the Kuhn-Tucker multiplier on Equation (12), which satisfies the complementary slackness condition $\lambda_{v,j,s} v_{j,s} = 0$. Exploiting symmetry across firms, marginal costs equalize across firms and hence we drop the firm subscript from now on. The firms' price-setting problems deliver the following first-order condition:[7]

$$1-\gamma+\gamma\mathbf{mc}_s=\phi\left(\Pi_s-1\right)\Pi_s-\mathbb{E}_s\mathbf{\Lambda}_{j,t+s}\left[\frac{y_{s+1}}{y_s}\left(\Pi_{s+1}-1\right)\Pi_{s+1}\right]. \tag{14}$$

Wages: Because of the matching friction, worker-firm matches produce surpluses which need to be divided between firms and workers. We assume that real wages are determined by Nash bargaining between workers and firms. As discussed by Krusell, Mukoyama and Sahin (2010), financial market incompleteness and risk aversion jointly imply that the surplus that households derive from employment generally depend on their wealth levels. Hence we label the households' value and surplus functions by i. Firms are symmetric and hence we do not include a firm index in the bargaining equations. The wage solves the following maximization problem:

$$\max\left(\mathbf{S}^e_{i,s}\right)^{\upsilon}\left(\mathbf{S}^f_s\right)^{1-\upsilon}, \tag{15}$$

where $\mathbf{S}^e_{i,s}$ is the surplus of the worker, $\mathbf{S}^f_s$ is the surplus of the firm and $\upsilon\in(0,1)$ is the bargaining weight of the worker. We assume that were negotiations to fall through, the worker becomes unemployed while the firm can attempt to hire a new worker in the same period. The employed worker's surplus ($\mathbf{S}^e_{i,s}$), the difference between the value of being employed ($\mathbf{V}^e_{i,s}$) and unemployed ($\mathbf{V}^u_{i,s}$), is then:

$$\begin{aligned}
\mathbf{S}^e_{i,s} &= \mathbf{V}^e_{i,s}-\mathbf{V}^u_{i,s},\\
\mathbf{V}^e_{i,s} &= \frac{c^{1-\mu}_{i,e,s}}{1-\mu}+\beta\mathbb{E}_s\omega\left(1-\eta_{s+1}\right)\mathbf{V}^u_{i,s+1}+\beta\mathbb{E}_s\left(1-\omega\left(1-\eta_{s+1}\right)\right)\mathbf{V}^e_{i,s+1},\\
\mathbf{V}^u_{i,s} &= \frac{c^{1-\mu}_{i,u,s}}{1-\mu}+\zeta+\beta\mathbb{E}_s\left(1-\eta_{s+1}\right)\mathbf{V}^u_{i,s+1}+\beta\mathbb{E}_s\eta_{s+1}\mathbf{V}^e_{i,s+1},
\end{aligned}$$

where $c_{i,e,s}$ ($c_{i,u,s}$) is the consumption level optimally chosen by the household in case of employment (unemployment).

Since the firm will post vacancies to hire a replacement worker should the current negotiations fail, the surplus of the match to the firm satisfies:

$$\mathbf{S}^f_s=\frac{\kappa}{q_s}. \tag{16}$$

[7] Note that in the absence of price rigidities and search and matching frictions, the marginal cost equals $mc_s=\frac{w_s}{\exp(A_s)}=\frac{\gamma-1}{\gamma}$. To avoid trivial equilibria in which market work can generate no surplus to workers, even without labor market and price setting frictions, we assume that $\frac{\vartheta^{1-\mu}-1}{1-\mu}+\zeta<\frac{\left(\frac{\gamma-1}{\gamma}\exp(A_s)\right)^{1-\mu}-1}{1-\mu}$. Strictly speaking, this requires a bound on the support of the stochastic productivity process.

2.3 Monetary policy

The monetary authority follows an interest rate rule. Specifically, the interest rate responds to inflation, given by $\Pi_s \equiv \frac{P_s}{P_{s-1}}$, and to labor market tightness. The latter variable naturally captures labor market slack. The interest rate rule is given by:

$$R_s = \max \left\{ \overline{R} \left(\frac{\Pi_s}{\overline{\Pi}} \right)^{\delta_\pi} \left(\frac{\theta_s}{\overline{\theta}} \right)^{\delta_\theta}, 1 \right\}, \tag{17}$$

where $\overline{R}$, $\overline{\Pi}$, $\overline{\theta}$, δ_π and δ_θ are policy parameters and the max operator captures the zero lower bound on the net nominal interest rate, $R_s - 1$.

2.4 Financial Markets

In NK models with unemployment it is typically assumed that individual households are insured against idiosyncratic earnings shocks within large diversified families or, alternatively, that households can purchase unemployment insurance contracts at actuarially fair prices. Whilst this conveniently allows one to use a representative agent framework, it also has the unfortunate consequence that individuals' consumption streams do not depend on their idiosyncratic earnings shocks - including unemployment - which questions the empirical relevance of the model.

Here we instead assume that households live in single-member families and cannot purchase unemployment insurance contracts, c.f. follow Challe and Ragot (2016) and Ravn and Sterk (2012, 2016). Households can attempt to self-insure against job uncertainty through savings in a zero-dividend one-period nominal bond purchased at price $1/R_s$ units of currency at date s. Let the household's purchases of bonds at date s be given by $b_{i,s}$. Households must observe a liquidity constraint:

$$b_{i,s} \geq \underline{b}. \tag{18}$$

A second asset that is available to households is firm equity. In Section 5 we discuss asset pricing implications and introduce additional risky and riskless assets.

2.5 Conditions for a tractable equilibrium

Without further assumptions, the model above can only be solved numerically. In this paper we aim at an analytical characterization of the equilibrium. It turns out that this can be attained by imposing two assumptions. First, we impose the following borrowing constraint:

$$\underline{b} = 0. \tag{19}$$

Second, we assume that only a fraction $\xi \in (0,1)$ of the households has the ability to invest in firm equity.

To appreciate why the model now simplifies very considerably, consider first the households who cannot invest in firm equity. Amongst these households, employed workers have an incentive to save while unemployed workers have an incentive to borrow. The borrowing constraint above implies that unemployed workers cannot borrow and therefore, in equilibrium, employed workers cannot save in bonds unless the households who can invest in equity also are active in bond markets. In equilibrium asset rich households will specialize in equity (see below) and asset-poor employed households will therefore be unable to save in equilibrium. Nonetheless, employed workers' Euler equations will still need to be satisfied since these agents have a savings motive and therefore are not directly constrained by (19). The consumption levels of employed and unemployed asset-poor households are, respectively, given by:

$$\begin{aligned} c_{e,i\geq\xi,s} &= w_s, \\ c_{u,i\geq\xi,s} &= \vartheta. \end{aligned}$$

where $c_{e,i\geq\xi}$ ($c_{u,i\geq\xi}$) is the consumption level of an employed (unemployed) asset-poor household.

Next, consider the households who can invest in firm equity and end up being asset-rich as they receive the monopoly profits of the firm. These households will typically receive higher levels of income than those who are unable to invest in equity, and therefore may be unwilling to work depending on the level of the disutility parameter ζ and the fraction of households that can invest (which determines the amount of profits per household). For simplicity we will assume that parameter values are such that investing households declare themselves out of the labor force, but this is not important for the key results. Consumption levels of the asset-rich, $c_{i<\xi,s}$, equalize across all agents $i \in (0,\xi)$:

$$c_{i<\xi,s} = \vartheta + \frac{1}{\upsilon}\left(y_s - \kappa v_s - w_s n_s - \frac{\phi}{2}\Pi_s^2 y_s\right).$$

It follows that firms discount profits at a common rate $\mathbf{\Lambda}_{t,t+s} = \beta\left(c_{i<\xi,t}/c_{i<\xi,t+s}\right)^{\mu}$.

All workers (asset-poor households) have the same wealth (zero) and therefore bargain the same wage because their outside options are identical. Similarly, all investors (asset-rich households) consume the same amounts. Since there is no heterogeneity across households conditional on their type and employment status, we drop the i-subscript and denote consumption levels as $c_{e,s} = c^{e}_{i\geq\xi,s}$, $c_{u,s} = c^{u}_{i\geq\xi,s}$, and $c_{r,s} = c^{u}_{i<\xi,s}$, where subscript r denotes the asset-rich households. The first-order condition for bonds delivers the following Euler equations for the three types of households:

$$c_{e,s}^{-\mu} = \beta \mathbb{E}_s \frac{R_s}{\Pi_{s+1}} \left(\omega \left(1 - \eta_{s+1} \right) c_{u,s+1}^{-\mu} + \left(1 - \omega \left(1 - \eta_{s+1} \right) \right) c_{e,s+1}^{-\mu} \right) + \lambda_{e,s}, \tag{20}$$

$$c_{u,s}^{-\mu} = \beta \mathbb{E}_s \frac{R_s}{\Pi_{s+1}} \left(\left(1 - \eta_{s+1} \right) c_{u,s+1}^{-\mu} + \eta_{s+1} c_{e,s+1}^{-\mu} \right) + \lambda_{u,s}, \tag{21}$$

$$c_{r,s}^{-\mu} = \beta \mathbb{E}_s \frac{R_s}{\Pi_{s+1}} c_{r,s+1}^{-\mu} + \lambda_{r,s}, \tag{22}$$

where $\lambda_{e,s}$, $\lambda_{u,s}$, and $\lambda_{r,s}$ are the Kuhn-Tucker multipliers on the liquidity constraints of the employed asset-poor households, the unemployed asset-poor households and the asset-rich households, respectively.

Since $w_s > \vartheta$ and $\omega > 0$, the liquidity constraint always binds for the asset-poor unemployed households, i.e. $\lambda_{u,s} > 0$ at all times. Further, it is straightforward to verify that in any steady state without aggregate uncertainty that (i) $\lambda_{r,s} > 0$, i.e. the asset rich households are at the liquidity constraint, and (ii) $\lambda_{e,s} = 0$, i.e. the employed asset-poor households are not at the liquidity constraint, (iii) and $\mathbb{E}_s \frac{R_s}{\Pi_{s+1}} < \beta$. Intuitively, the asset-rich are not exposed to idiosyncratic risk and hence are unwilling to take a positive position in bonds, which pay a real interest rate that lies below their subjective discount rate.[8] The employed asset-poor households, by contrast, are exposed to idiosyncratic risk. This gives rise to a precautionary savings motive which makes them willing to invest in bonds at a return that is lower than their subjective discount rate. Therefore, when analyzing steady-state equilibria or in their vicinity, we can drop Equations (21) and (22), as well the variables $\lambda_{u,s}$, and $\lambda_{r,s}$.

Finally, consider the equilibrium labor market flows. Provided that the asset-poor are unwilling to leave the labor force, the labor market participation rate is constant and given by $1 - \xi$. In that case, the aggregate unemployment rate is given by:

$$u_s = 1 - \mathbf{n}_s, \tag{23}$$

where $\mathbf{n}_s = \frac{1}{1-\xi} \int_j n_{j,s} dj$ is the aggregate employment rate, as a fraction of the labor force. The law of motion of unemployment is given as

$$u_s = u_{s-1} + \omega \mathbf{n}_{s-1} - \mathbf{h}_s, \tag{24}$$

where $\mathbf{h}_s = \frac{1}{1-\xi} \int_j h_{j,s} dj$ is the number of new hires as a fraction of the labor force. The aggregate number of job searchers is given by $e_s = (1 - \xi) \left(u_{s-1} + \omega \mathbf{n}_{s-1} \right)$.

[8] Note that even in scenarios in which the asset-rich would be willing to work, they would still be better insured against idiosyncratic income risk than the asset-poor households.

3 Non-stochastic equilibria

This section discusses the set of equilibria that can arise in absent aggregate shocks. We explore the nature of the steady-state equilibria that can arise and whether such equilibria are unique locally in the vicinity of those steady states.

3.1 Global Determinacy

Consider of a version of the model without aggregate productivity shocks ($\sigma_A = 0$) and without potential belief shocks. A crucial difference vis-a-vis the extant complete markets NK literature is that although the aggregate allocation (and prices) are constant in the steady-state, the incomplete markets model still features idiosyncratic risk due to lack of insurance against job uncertainty. This has fundamental consequences for the properties of the model which we now consider.

We indicate steady-state values by removing time subscripts from variables. Define for convenience $\widetilde{R} \equiv \overline{R}\,\overline{\Pi}^{-\delta_\pi}\overline{\theta}^{-\delta_\theta}$. The solution to the steady-state wage can be expressed as function of the job finding rate, $\mathbf{w}(\eta)$. This function is derived in the Appendix, in which we also discuss some of its basic properties. Steady-state equilibria can be characterized by the solutions to:

$$\underbrace{\phi(1-\beta)(\Pi-1)\Pi}_{\text{sticky-price wedge}} = 1-\gamma+\gamma\left(\mathbf{w}(\eta)+\left(\kappa\eta^{\alpha/(1-\alpha)}-\lambda_f\right)(1-\beta(1-\omega))\right), \qquad (\mathbf{PC})$$

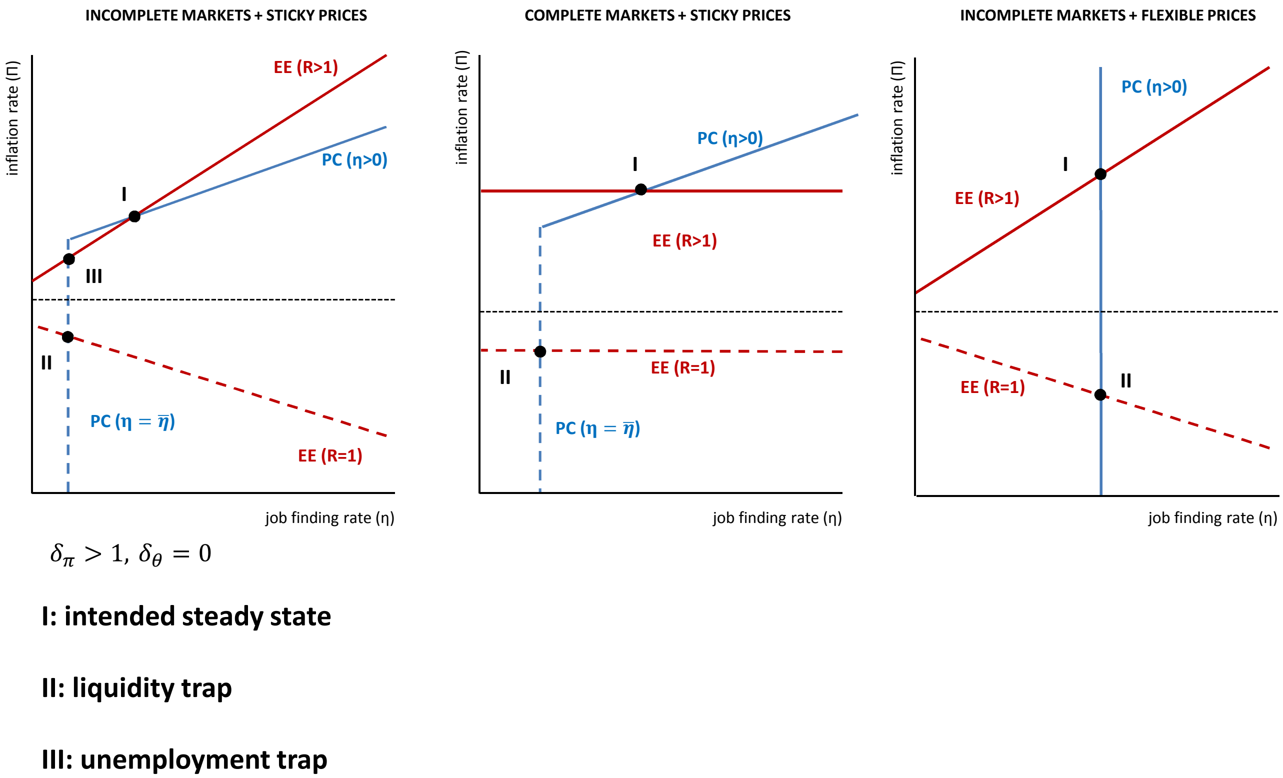
$$1 = \beta\frac{\max\{\widetilde{R}\Pi^{\delta_\pi}\eta^{\delta_\theta/(1-\alpha)},1\}}{\Pi}[\underbrace{\omega(1-\eta)(\vartheta/\mathbf{w}(\eta))^{-\mu}+1-\omega(1-\eta)}_{\text{incomplete-markets wedge}}], \qquad (\mathbf{EE})$$

where $\eta \geq \overline{\eta}$, $\lambda_f \geq 0$, and $\lambda_f(\eta-\overline{\eta}) = 0$. Here, $\overline{\eta}$ is the job finding rate that would prevail in a steady-state in which firms are unwilling to invest in formal vacancies thus only hire via costless informal channels. In such a steady-state, the job finding rate is given as $\overline{\eta} = \frac{\mathbf{M}(\overline{e},\overline{v})}{\overline{e}}$, where $\overline{e} = (1-\xi)\left(\frac{\omega(1-\overline{\eta})}{\omega(1-\overline{\eta})+\overline{\eta}}(1-\omega)+\omega\right)$.

Equation (**PC**) is the steady-state version of (14), the optimality condition for prices in the symmetric equilibrium ("the Phillips Curve"), and defines a relationship between inflation, Π, and the job finding rate, η, provided that $\eta > \overline{\eta}$. The left-hand side of the equation is a standard sticky-price wedge which vanishes in the absence of price adjustment costs ($\phi = 0$).

Equation (**EE**) is the steady-state version of the employed households' Euler equation (20) which also defines a relationship between Π and η. The term between square brackets arises due

Figure 1: Illustration of steady-state equilibria.



to incomplete markets, and would collapse to one when either $\vartheta = \mathbf{w}(\eta)$, i.e. when consumption is fully insulated against job loss, or when $\mu = 0$, i.e. when households are risk neutral. Note that the wedge is a function of the job finding rate and that the two schedules are non-linear. Two sources of non-linearity are particularly important. The first is the ZLB on the short term nominal interest rate paves the way for multiple steady-states. The second is the non-negativity constraint on the number of vacancies.

Solutions to the above system give the steady-state outcomes for inflation and the job finding rate. Figure 1 illustrates the steady-state schedules. For simplicity, we consider a case in which the interest rate rule only reacts to inflation (i.e. $\delta_\theta = 0$). There are two EE schedules, one at which the ZLB binds ($\mathbf{EE(R = 1)}$) and one at which the ZLB does not bind ($\mathbf{EE(R > 1)}$). Further, the PC schedule slopes upward as long as the job finding rate is positive ($\mathbf{PC(\boldsymbol{\eta} > \overline{\boldsymbol{\eta}})}$) but becomes vertical at the point the job finding rate hits zero and the non-negativity constraint on vacancies becomes binding ($\mathbf{PC(\boldsymbol{\eta} = \overline{\boldsymbol{\eta}})}$). Consider first the left panel of Figure 1, which illustrates a case with incomplete markets and sticky prices.[9] Three possible steady states

[9] The Figure assumes that $\delta_\theta = 0$, i.e. monetary policy only reacts to inflation, and a sticky real wage, i.e.

emerge:

I *Intended steady state.* This steady state occurs at the intersection of the $\mathbf{PC}(\boldsymbol{\eta} > \overline{\boldsymbol{\eta}})$ and the $\mathbf{EE}(\mathbf{R} > \mathbf{1})$ schedule. This is the "usual" steady state at which the ZLB does not bind and the job finding rate is relatively high.

II *Liquidity trap.* This steady state arises because of the ZLB on the nominal interest rate and occurs at the intersection of the $\mathbf{PC}(\boldsymbol{\eta} = \overline{\boldsymbol{\eta}})$ and the $\mathbf{EE}(\mathbf{R} = \mathbf{1})$ schedule. This is the "liquidity trap" examined by Benhabib, Schmitt-Grohe and Uribe (2001, 2002) and Mertens and Ravn (2013). This steady state features a lower rate of inflation than the intended steady state, as well as a lower job finding rate. In fact, the job finding rate is zero in the illustration.

III *Unemployment trap.* This steady state occurs at the intersection of the $\mathbf{PC}(\boldsymbol{\eta} = \overline{\boldsymbol{\eta}})$ and the $\mathbf{EE}(\mathbf{R} > \mathbf{1})$ schedule. In this equilibrium, investment in vacancies comes to a complete standstill despite the fact that the ZLB on the nominal interest rate does not bind. Thus, any hiring takes place occurs informally. Note that the inflation rate in this steady state lies in between those in the intended steady state and the liquidity trap.

The two first of these types of equilibria occur also in standard complete markets representative agent NK models. There are, however, important differences between the properties of the equilibria under complete and incomplete markets. With full insurance, the steady-state real interest rate needs to equal $1/\beta$ in order to be consistent with constant consumption. Without full insurance, the wedge in (**EE**) is positive, which reduces the equilibrium real interest rate below the inverse of the discount rate, $\frac{R}{\Pi} < \frac{1}{\beta}$. Intuitively, the consumption loss associated with job loss creates a precautionary savings motive. Since the net-supply of bonds is zero, the real interest rate adjusts downward to restore equilibrium. The equilibrium real interest rate in the incomplete markets economy depends on the job finding rate and on the consumption loss that a worker experiences in case of job loss. It therefore follows that the equilibrium long run real interest rate in the incomplete markets model depends on economic policy to the extent that policy choices influence the job finding rate and/or the consumption loss. Secondly, whilst the aggregate quantities and prices are constant in the steady states, the combination of unemployment risk and incomplete markets imply that individual households are subject to idiosyncratic

$\upsilon = 0$.

risk in the steady state in the incomplete markets model. The liquidity trap generated by the model have very interesting properties which we will discuss in detail in Section 5.

The possible emergence of a third steady state depends critically on the interaction between sticky prices and incomplete markets. This is illustrated by the middle and right panels if Figure 1 which illustrate, respectively, a case with complete markets (but sticky prices) and a limit case with flexible prices (but incomplete markets). Under complete markets, the **EE** schedules become horizontal, because the steady-state real interest rate equals the households' subjective discount rates. This rule out a third steady state.[10] In the limit case with flexible prices, the $\mathbf{PC}(\boldsymbol{\eta} > \mathbf{0})$ schedule becomes vertical, as inflation no longer affects firms' marginal costs. As a result, the $\mathbf{PC}(\boldsymbol{\eta} = \overline{\boldsymbol{\eta}})$ schedule vanishes, thus allowing for only two steady states. Thus, without the interaction between sticky prices and incomplete markets, the third steady state cannot not exist.[11]

The unemployment trap is an intriguing outcome. First note that when the job finding rate is zero, Nash bargaining implies that employed workers' values are driven down to equalize unemployed workers values. As a result, jobless workers become precisely indifferent between participating in the labor force and dropping out, which is interesting in light of the decline in labor force participation since the Great Recession. Moreover, the slow recovery after the Great Recession and the very protracted nature of the surge in unemployment observed in the U.S. (and many other OECD economies) have spurred a renewed interest in "secular stagnation," equilibrium outcomes consistent with long periods of low activity and high unemployment. Hansen (1939) argued that such outcomes (with negative natural real interest rates) were most likely produced by a combination of low rate of technological progress and population ageing implying high savings rates and low investment rates. Recently, Eggertsson and Mehrotra (2014) have argued that deleveraging may lead to secular stagnation and exacerbate the problems that follow from an ageing population and falling investment goods prices.

The unemployment trap that can arise in the incomplete markets NK model offers an alternative perspective of secular stagnation which ties together low real interest rates, high unemployment and low activity. In this steady-state, hiring is at a minimum and unemployment therefore potentially very high. Moreover, because of the low job finding rate, there is a strong

[10] In the left and middle panel of Figure 1, the liquidity trap does not feature any hiring. For different parameter configurations, the liquidity trap can occur at a positive job finding rate. This, however, would rule out the third steady state.

[11] Note further that this combination is a necessary but not sufficient condition for the emergence of the third steady state, since the complete markets version is the limit of the incomplete markets version.

incentive for precautionary savings which drives down the real interest rate. Intriguingly, the unemployment trap can occur in our model purely becuase of expectations and thus does not rely on sudden changes in popluation growth, technological progress or financial tightening. Furthermore, while the nominal interest rate may be low in the unemployment trap, its root cause does not derive from the ZLB on nominal interest rates.

3.2 Local determinacy

The log-linearized model: We now log-linearize the model in order to study the local stability properties of the equilibria. Let a hat denotes a log deviation from the intended steady state, i.e. $\widehat{x}_s = \ln x_s - \ln \overline{x}^I$, where $\overline{x}^I$ denotes the value of x_s in the intended steady-state (discussed above). We assume that policy parameters are such that monetary policy parameters are such that $\overline{R}$, $\overline{\theta}$ and $\overline{\Pi}$ correspond to the steady-state levels of, respectively, R, θ and Π.

Noting that $\widehat{w}_s = \widehat{c}_{e,s}$, the log-linearized Euler equation of the employed households, (20), can be expressed as (see the Appendix for details):

$$\begin{aligned} -\mu\widehat{w}_s + \mu\beta\overline{R}\mathbb{E}_s\widehat{w}_{s+1} &= \widehat{R}_s - \mathbb{E}_s\widehat{\Pi}_{s+1} - \underbrace{\beta\overline{R}\Theta\mathbb{E}_s\widehat{\eta}_{s+1}}_{\text{incomplete-markets wedge}}, \\ \Theta &\equiv \omega\eta\left((\vartheta/w)^{-\mu} - 1\right) - \chi\mu\omega\left(1-\eta\right) \end{aligned} \tag{25}$$

$\widehat{R}_s - \mathbb{E}_s\widehat{\Pi}_{s+1}$ is the real interest rate while the last term on the right-hand side is the incomplete-markets wedge, which fluctuates proportionally with the expected job finding rate. Its strength is determined by Θ, a convolution of parameters which consists of two parts. The first part, $\omega\eta\left((\vartheta/w)^{-\mu} - 1\right) > 0$, represents the impact of job loss on the marginal utility of consumption. If home production equals the steady-state wage, i.e. if $\vartheta = w$, or if the household is risk neutral ($\mu = 0$), this part of the wedge collapses to zero. An expected decline in the job finding rate increases the probability of unemployment and strengthens the households precautionary savings motive. The second part, $-\chi\mu\omega\left(1-\eta\right) < 0$, derives from the fact that, under incomplete markets, expected wage growth transmits only partially to expected consumption growth, since the worker may be unemployed in the next period. This part of the wedge vanishes if wages are fully sticky ($\chi = 0$) or when households are risk neutral ($\mu = 0$).

Next, we log-linearize the firms price-setting condition, Equation (14), around the intended steady state:

$$\underbrace{\frac{\phi}{\gamma}\widehat{\Pi}_s - \beta\frac{\phi}{\gamma}\mathbb{E}_s\widehat{\Pi}_{s+1}}_{\text{sticky-price wedge}} = w\left(\widehat{w}_s - A_s\right) + \frac{\kappa}{\overline{q}}\frac{\alpha}{1-\alpha}\widehat{\eta}_s - \beta\left(1-\omega\right)\frac{\kappa}{\overline{q}}\left(\frac{\alpha}{1-\alpha}\mathbb{E}_s\widehat{\eta}_{s+1} + \widehat{\Lambda}_{s,s+1} - \left(1-\rho\right)A_s\right), \tag{26}$$

where we have exploited that $q_s = \eta^{-\frac{\alpha}{1-\alpha}}$. For now, we abstract from productivity shock, setting $A_s = 0$ at any date s. The left-hand side of the above equation is the sticky-price wedge, which vanishes in the absence of price adjustment costs ($\phi = 0$) or in the limit with perfect competition ($\gamma \to \infty$). The right-hand side is the log-linearized marginal cost, which is standard given the presence of search and matching frictions.

The policy rule reads, log-linearized around the intended steady state, reads:

$$\widehat{R}_s = \delta_\pi\widehat{\Pi}_s + \delta_\theta\widehat{\theta}_s. \tag{27}$$

In the Appendix, we further show that the log-linearized bargaining equations imply that:

$$\widehat{w}_s = \chi\widehat{\eta}_s, \tag{28}$$

where χ is a convolution of the model's deep parameters, which captures the sensitivity of the wage to fluctuations the job finding rate and depends critically on the bargaining parameter υ. Finally, note that in equilibrium the employed households consume their wage, i.e. $\widehat{c}_{e,s} = \widehat{w}_s$.

Reducing the model to a single equation: For maximal tractability, we introduce two further assumptions which allow us to reduce the model to a single equation. First, we set the monetary policy coefficient equal to $\delta_\pi = \frac{1}{\beta} > 1$. This is inconsequential, since the coefficient on tightness, δ_θ, is left unrestricted.[12] Second, we assume that the households who can invest in equity (i.e. those with index $i < \xi$) are risk neutral. In this case, the log-linearized model has no endogenous state variables. In the appendix, we relax this assumption and show that allowing for risk-averse equity investors introduces an additional source of amplification to the model.

The log-linearized model can now be reduced just one dynamic equation for the job finding rate (see the Appendix for a derivation):

$$\begin{aligned} \mathbb{E}_s\widehat{\eta}_{s+1} &= \Psi\widehat{\eta}_s \\ \Psi &\equiv \frac{\phi\gamma^{-1}\mu\chi\beta + \phi\gamma^{-1}\frac{\beta\delta_\theta}{1-\alpha} + w\chi + \frac{\kappa}{q}\frac{\alpha}{1-\alpha}}{\frac{\kappa}{q}\frac{\alpha\beta(1-\omega)}{1-\alpha} + \phi\gamma^{-1}\mu\beta^2\overline{R}\chi + \phi\gamma^{-1}\beta^2\overline{R}\Theta} \end{aligned} \tag{29}$$

[12] The log-linearized model contains no endogenous state variables and hence for any desire pair of values δ_π and δ_θ one can find a value δ_θ^* such that the same solution is obtained under the restriction that $\delta_\pi = \frac{1}{\beta}$.

Under conventional parameter values, both the numerator and the denominator of Ψ are positive, and we will proceed under the assumption that this is the case. While the expression for Ψ seems complicated at a first glance, it turns out to deliver very intuitive results, which we present below.

Determinacy around the intended steady state: rigid real wages: How does the presence of incomplete markets impact on the possibility of local self-fulfilling equilibria? Intuitively, an increase in job uncertainty reduces aggregate demand, which in turn reduces the incentives to post vacancies. The reduction in vacancies in turn reduces the job finding rate, further increasing unemployment risk. It is precisely this feedback spiral that opens up the possibility that exogenous changes in beliefs, or "sunspot fluctuations" , are a source of macroeconomic fluctuations, as the equilibrium is no longer uniquely determined.

The model formalizes the condition under which such fluctuations can occur. For simplicity, we start with a version with sticky wages ($\chi = 0$). Since market tightness is not a state variable, the equilibrium is locally determinate if and only if $\Psi \leq 1$, i.e. if and only if:

$$\phi\gamma^{-1}\left(\beta^2\overline{R}\Theta - \frac{\beta\delta_\theta}{1-\alpha}\right) \leq \frac{\kappa}{q}\frac{\alpha}{1-\alpha}\left(1-\beta\left(1-\omega\right)\right).$$

The above equation makes clear how the occurrence of local indeterminacy depends on five types of market frictions present in the model, as well as on monetary policy:

(i) *Price rigidity.* If prices are fully flexible ($\phi = 0$) the equilibrium is always determinate since the left-hand side collapses to zero and the right-hand side is strictly positive.

(ii) *Imperfect competition.* Under perfect competition ($\gamma \to \infty$) the equilibrium is always determinate, for the same reason as above.

(iii) *Incomplete markets.* Under sticky wages, the incomplete-markets parameter collapses to $\Theta = \omega\eta\left((\vartheta/w)^{-\mu} - 1\right)$. The case $\vartheta = w$ corresponds to full insurance, in which case the incomplete markets term collapses to zero. In this case, the equilibrium is always determinate. The same is true when households are risk neutral ($\mu = 0$).

(iv) *Monetary policy.* The more aggressively monetary policy responds to tightness, i.e. the higher δ_θ, the less likely indeterminacy is to occur.

(v) *Labor adjustment cost.* The term $\frac{\kappa}{q}\frac{\alpha}{1-\alpha}\left(1-\beta\left(1-\omega\right)\right)$ denotes the steady-state marginal cost of hiring a worker today rather than tomorrow, so we can think of it as a labor adjustment cost, i.e. a real labor rigidity. Note that this cost is proportional to the steady-state hiring cost $\frac{\kappa}{q}$.

There are two main differences between the incomplete markets model and the standard model with insurance against idiosyncratic risk. The first is simply that the conditions for determinacy are more stringent under incomplete markets. With complete markets, a sufficient conditions for local determinacy is that $\delta_\pi > 1$ as we have assumed (notice that the left hand side of the inequality is negative when $\Theta = 0$). Under incomplete markets this is no longer a sufficient condition.

Secondly, there is an important interaction between market incompleteness, sticky prices, and risk aversion due to the multiplicative nature of the coefficient on the left hand side. Specifically, price rigidities only make indeterminacy more likely if the incomplete markets effect dominates the monetary policy effect, i.e. if $\Theta > \beta\delta_\theta$. Moreover, less complete financial markets, i.e. higher Θ, make indeterminacy more likely, but only if prices are sticky and the goods market is imperfectly competitive. However, if $\Theta > \beta\delta_\theta$, market incompleteness, nominal rigidities and risk aversion are complements making local indeterminacy increasingly likely in combination.

An intriguing insights regards the impact of labor market frictions. According to the condition above, the higher is the labor adjustment cost, the *less* likely it is for indeterminacy to happen. Thus, less flexible labor markets imply less amplification. The reason for this is that when it is costly for firms to adjust on the labor margin, they are more likely to adjust prices which neutralizes the feedback mechanism.

Determinacy around the intended steady state: flexible real wages: The determinacy condition becomes somewhat more involved when we introduce wage flexibility ($\chi > 0$):

$$\phi\gamma^{-1}\left(\beta^2\overline{R}\Theta - \frac{\beta\delta_\theta}{1-\alpha}\right) - w\chi - \phi\gamma^{-1}\mu\beta\left(1-\beta\overline{R}\right)\chi \leq \frac{\kappa}{q}\frac{\alpha}{1-\alpha}\left(1-\beta\left(1-\omega\right)\right).$$

Wage flexibility affects determinacy via three channels. First, it does so via an *incomplete markets channel*. Recall that $\Theta \equiv \omega\eta\left(\left(\vartheta/w\right)^{-\mu} - 1\right) - \chi\mu\omega\left(1-\eta\right)$, so wage flexibility reduces the incomplete markets wedge. Second, wage flexibility creates a *marginal cost channel*, as it pushes down wage costs during times of low market tightness, pushing up vacancy posting. This channel comes in via the term $-w\chi$. Finally, wage flexibility generates an *intertemporal substitution* channel, as a decline in wages reduces employed households' incentives to save. This channel enters via the term $-\phi\gamma^{-1}\mu\beta\left(1-\beta\overline{R}\right)\chi$. Finally, note that through all three channels wage flexibility pushes the model towards the determinacy region of the parameter space. In conclusion, real wage flexibility is stabilizing in the vicinity of the intended steady-state.

Determinacy around the unemployment trap: To analyze the determinacy properties around the unemployment trap we exploit that the non-negativity constraint on vacancies binds.

Hence, we can drop Equation (26) and set η_s equal to zero. Thus, the job finding rate is trivially determined. Consider the case in which the interest rate rule only responds to inflation (i.e. $\delta_\theta = 0$). The Euler equation, log-linearized around the unemployment trap, is given by:

$$0 = \delta_\pi \widehat{\Pi}_s - \mathbb{E}_s \widehat{\Pi}_{s+1}.$$

It follows immediately that the equilibrium is unique if and only if $\delta_\pi > 1$, i.e. the interest rate elasticity with respect to inflation exceeds unity. In that case, the solution is given by $\widehat{\Pi}_s = 0$ at all times. Thus, the unemployment trap may be determinate under a standard Taylor rule which responds more than one-for-one to inflation.[13]

4 Fluctuations

4.1 Local shocks

Belief shocks: We now explicitly solve for the local dynamics in the vicinity of the intended steady state in response to shocks. We first focus on "belief shocks" starting with a version of the model without productivity shocks. From Equation (29) it follows that if the equilibrium is locally determinate ($\Psi > 1$), then the only stable solution is given by $\widehat{\eta}_s = 0$ at all times. When equilibria are locally indeterminate, the solution is given by

$$\widehat{\eta}_{s+1} = \Psi \widehat{\eta}_s + \varepsilon^B_{s+1},$$

where ε^B_s is an i.i.d. belief shock with mean zero and a standard deviation normalized to one. Thus, tightness follows an AR(1) process. The persistence of the effects of belief shocks on the job finding rate is captured by Ψ, and thus endogenously determined. Persistence is maximal at $\Psi = 1$, i.e. at the border between the determinacy and indeterminacy region of the parameter space.

Productivity shocks: In the Appendix, we show that the model with productivity shocks can

[13] If the rule also responds to market tightness, there cannot be a determinate no-hiring trap. To see this, not that when $\eta = 0$ then $\theta = 0$. The monetary policy rule then (mechanically) implies that the ZLB on the nominal interest rate binds. As a result, the log-linearized Euler equation would become $0 = \mathbb{E}_s \widehat{\Pi}_{s+1}$, which implies that current inflation is not uniquely determined.

be written as:

$$\begin{aligned}
\mathbb{E}_s\widehat{\eta}_{s+1} &= \Psi\widehat{\eta}_s - \Omega A_s, \\
A_s &= \rho A_{s-1} + \sigma_A \varepsilon_s^A, \\
\Omega &\equiv \frac{w - \frac{\kappa}{q}\beta\left(1-\omega\right)\left(1-\rho\right)}{\frac{\kappa}{q}\frac{\alpha\beta(1-\omega)}{1-\alpha} + \frac{\phi}{\gamma}\mu\beta^2\overline{R}\chi + \frac{\phi}{\gamma}\beta^2\overline{R}\Theta}.
\end{aligned}$$

Consider first the determinate case ($\Psi > 1$). We apply the method of undetermined coefficients and guess a solution of the form $\widehat{\eta}_s = \Gamma_\eta A_s$. Plugging this guess into Equation (29) yields the following solution:

$$\Gamma_\eta = \frac{\Omega}{\Psi - \rho}. \tag{30}$$

It can now be shown that, in the determinacy region of the parameter space, the job finding rate response positively to a productivity shock, if and only if $\frac{\kappa}{q}\beta\left(1-\omega\right)\left(1-\rho\right) < w$. Note that the condition is satisfied if the average cost of recruiting a worker, $\frac{\kappa}{q}$, is smaller than the wage w, a condition that is typically satisfied in the literature.[14] To see why under this condition it holds that $\Gamma_\eta > 0$, first note that it implies that $\Omega > 0$. Second, note that in the determinacy region it holds that $\Psi - \rho > 0$ since determinacy requires $\Psi > 1$ and it further holds that $\rho < 1$.

Writing out the solution for Γ_η explicitly gives:

$$\Gamma_\eta = \frac{w - \frac{\kappa}{q}\beta\left(1-\omega\right)\left(1-\rho\right)}{\phi\gamma^{-1}\beta\left(\frac{\delta_\theta}{1-\alpha} - \rho\beta\overline{R}\Theta\right) + \frac{\kappa}{q}\frac{\alpha(1-\rho\beta(1-\omega))}{1-\alpha} + \left(w + \phi\gamma^{-1}\mu\beta\left(1-\beta\overline{R}\rho\right)\right)\chi}.$$

Note that $\frac{\partial\Gamma}{\partial\Theta} \geq 0$, i.e. a higher value of the market incompleteness parameter Θ amplifies the impact of productivity on the job finding rate. The amount of amplification, however, depends critically on the amount of price stickiness, since $\frac{\partial\Gamma}{\partial\Theta} = 0$ when $\phi = 0$. Similarly, more aggressive monetary policy dampens the response, since $\frac{\partial\Gamma}{\partial\delta_\theta} \leq 0$, but only when prices are sticky. Finally, note that the flexible-price response is given by $\Gamma_\eta = \frac{w}{\frac{\kappa}{q}\frac{\alpha}{1-\alpha}(1-\rho\beta(1-\omega))}$. Real wage flexibility dampens the response of the job finding rate to productivity shocks, since $\beta\overline{R} \leq 1$ $\rho \in (0,1)$ and $\frac{\partial\Theta}{\partial\chi} < 0$.

We can now solve for the inflation rate, guessing a solution of the form $\widehat{\Pi}_s = \Gamma_\Pi A_s$. Plugging this guess into the log-linearized Euler equation gives:

$$\Gamma_\Pi = \frac{\beta^2\overline{R}\Theta\rho - \frac{\beta\delta_\theta}{1-\alpha} - \mu\chi\beta\left(1-\rho\beta\overline{R}\right)}{1-\beta\rho}\Gamma_\eta.$$

[14] Silva and Toledo (2009) estimate that the average search cost to a firm is about 13.5 hours per new hire, or about 4 percent of the quarterly wage.

It follows that inflation increases following a positive technology shock (i.e. $\Lambda > 0$) if and only if $\beta^2 \overline{R} \Theta \rho > \frac{\beta \delta_\theta}{1-\alpha} + \mu \chi \beta \left(1 - \rho \beta \overline{R}\right)$. Thus, unlike the response of the job finding rate, the sign of the inflation response is ambiguous. Without the incomplete markets wedge ($\Theta = 0$), inflation declines following positive technology shocks, as long as either $\delta_\theta > 0$ or $\chi > 0$. The reason why prices may increase when the incomplete markets wedge is active comes from a demand channel: the increase in vacancy posting pushes up job finding rates, reducing the precautionary savings motive. This creates a boom in demand which pushes up prices, which may more than offset the direct effect of the technology shock, which is to reduce prices.

Finally, consider the local responses to productivity shocks in case the model parameters are in the indeterminacy region ($\Psi \leq 1$). This is fundamentally complicated by the fact that, within the model, it is not pinned down to what extent fundamental shocks change beliefs. Many different assumptions on this are possible. To understand the issue at hand, let us express Equation (29) as:

$$\widehat{\eta}_{s+1} = \Psi \widehat{\eta}_s - \Omega A_s + \Upsilon^A \varepsilon^A_{s+1} + \Upsilon^B \varepsilon^B_{s+1}.$$

It is straightforward to verify that in the determinacy region ($\Psi > 1$) it holds that $\Upsilon^A = \Gamma_\eta$ and $\Upsilon^B = 0$. In the indeterminacy region, however, Υ^A and Υ^B are not pinned down.

Lubik and Schorfheide (2004) suggest that to assume that the responses to fundamental shocks do not "jump" as the parameters are pushed from the determinacy region into the indeterminacy region. In our case, this boils down to assuming $\Upsilon^A = \Gamma_\eta$. Equation (30) makes clear that under this assumption, the response of tightness to a productivity shock may be either positive or negative, since $\Psi - \rho$ can have either sign when $\Psi < 1$. A singularity occurs at $\Psi = \rho$. Letting Ψ approach ρ from the left, the response approaches infinity. Thus, in the indeterminacy region amplification is possibly infinitely large. Figure 2 illustrates the amplification.

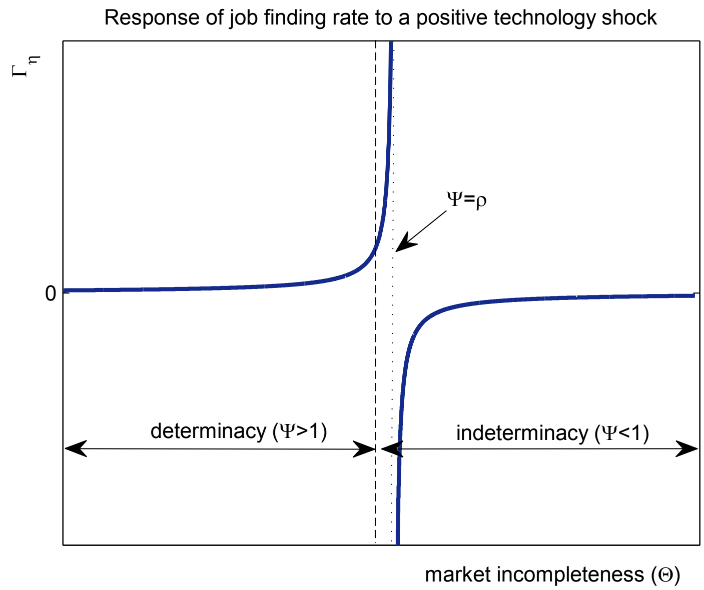
5 Implications for the Zero Lower Bound

5.1 Understanding missing disinflation

The Great Recession spurred a renewed interest in liquidity traps and in the impact of the ZLB on short-term nominal interest rates. Two issues arise with respect to the properties of the standard NK model: Its implications for inflation during ZLB episodes and the source of the liquidity trap.

In the standard NK model, the Euler equation implies that the gross real interest rate must

Figure 2: Illustration: amplification and determinacy.



equal $1/\beta$ in any steady-state since, in the presence of complete markets, any deterministic steady-state must satisfy:

$$\begin{aligned} u'(c) &= \beta \left(\frac{\overline{R}}{\overline{\pi}}\right)^{CM} u'(c) \Rightarrow \\ \left(\frac{\overline{R}}{\overline{\pi}}\right)^{CM} &= 1/\beta \end{aligned}$$

This implies that when the zero lower bound binds in a steady-state, the gross inflation rate must equal β. Since $\beta < 1$ this implies that liquidity traps must be deflationary. Temporary episodes at the ZLB will be even more deflationary than this since the stochastic Euler equation in that case will only be satisfied as long as $\pi < \beta$ during the ZLB regime.[15] It is important to notice that these implications are independent of the arguments that enter the interest rate rule. Although inflation has been moderate in the aftermath of the financial crisis, no country has experienced persistent deflation.

The incomplete markets NK model has different implications. As explained earlier, the

[15]Suppose that the ZLB regime persists with probability **p** while the intended steady-state is absorbing. In that case, the inflation rate during the ZLB episode is determined as $\overline{\pi}^{LT} = \gamma\beta / \left(1 - (1-\gamma) u'\left(\overline{c}^{I}\right) / u'\left(\overline{c}^{LT}\right)\right)$ where $\overline{\pi}^{LT}$ is the inflation rate during the liquidity trap, $\overline{c}^{I}$ is consumption in the intended steady-state and $\overline{c}^{LT}$ is consumption in the liquidity trap. This condition implies $\overline{\pi}^{LT} < \beta$ as long as $\overline{c}^{I} > \overline{c}^{LT}$.

relevant steady-state condition for the real interest rate in this model implies that:

$$\begin{aligned} u'(\overline{c}_e) &= \beta \left(\frac{\overline{R}}{\overline{\pi}}\right)^{IM} \left[(1-\omega(1-\overline{\eta}))\, u'(\overline{c}_e) + \omega(1-\overline{\eta})\, u'(\overline{c}_u)\right] \Rightarrow \\ \left(\frac{\overline{R}}{\overline{\pi}}\right)^{IM} &= \frac{1}{\beta}\frac{1}{(1-\omega(1-\overline{\eta})) + \omega(1-\overline{\eta})\, u'(\overline{c}_u)/u'(\overline{c}_e)} < \frac{1}{\beta} \end{aligned}$$

This model does not necessarily mean that a permanent liquidity trap triggers deflation. Suppose that the ZLB binds, so that inflation is given as:

$$\overline{\pi}^{LT} = \beta\left[\left(1-\omega\left(1-\overline{\eta}^{LT}\right)\right) + \omega\left(1-\overline{\eta}^{LT}\right) u'\left(\overline{c}_u^{LT}\right)/u'\left(\overline{c}_e^{LT}\right)\right]$$

This condition may be consistent with positive or negative inflation in the liquidity trap depending on $\overline{\eta}^{LT}$, $\overline{c}_u^{LT}$ and $\overline{c}_e^{LT}$.[16] Suppose for example that $\overline{c}_e^{LT} < \overline{c}_e^{I}$ while $\overline{c}_u^{LT} = \overline{c}_u^{I}$. In this case, a sufficiently large drop in the job finding rate would imply that inflation falls little in the liquidity trap. Intuitively, if the liquidity trap brings about a large increase in the amount of idiosyncratic risk, the real interest rate will remain low in the liquidity trap due to the precautionary savings motive.

One can relate this to the source of the liquidity trap. In the recent literature, several explanations have been put forward

5.2 Puzzles at the Zero Lower Bound

It is well known that at the ZLB, the representative-agent NK model has some puzzling properties, which cast doubt on the plausibility of the model. Two paradoxes have gained special attention. The first is a "supply shock paradox": at the ZLB, positive shocks to the supply side of the economy can be contractionary. The second is a "paradox of flexibility", and is associated to the finding that, at the ZLB, a higher degree of price flexibility creates a larger drop in output.[17]

[16] In this model, it is even possible that inflation is identical in the intended steady-state and in the liquidity trap. This would be the case when:

$$\overline{R} = \frac{\left[\left(1-\omega\left(1-\overline{\eta}^{LT}\right)\right) + \omega\left(1-\overline{\eta}^{LT}\right) u'\left(\overline{c}_u^{LT}\right)/u'\left(\overline{c}_e^{LT}\right)\right]}{\left[\left(1-\omega\left(1-\overline{\eta}^{I}\right)\right) + \omega\left(1-\overline{\eta}^{I}\right) u'\left(\overline{c}_u^{I}\right)/u'\left(\overline{c}_e^{I}\right)\right]}$$

[17] Throughout this subsection, we consider equilibria which ultimately lead to the intended steady state. Properties of equilibria leading to the liquidity trap steady state can be very different, see e.g. Mertens and Ravn (2014).

Both paradoxes operate via a reduction in expected inflation at the ZLB. The flexibility paradox originates from the fact that at the ZLB, firms are cutting prices, i.e. inflation is negative. The lower price adjustment costs, the more willing firms are to cut prices and hence the lower is the rate of inflation. The supply shock paradox arises from the fact that a positive supply shock pushes down production costs and hence inflation.

The effect of a decline in expected inflation, at the ZLB, is that the real interest rate increases. This, however, is incompatible with an increase in output, when markets are complete. Intuitively, a transitory increase in income would encourage households to save, which implies excess demand for bonds, if at the same time the real interest rate increases. Thus, given that expected inflation declines, only a decline in output can be consistent with equilibrium in the bonds market. Hence, both positive supply shocks and a higher degree of price flexibility reduce output at the ZLB, in contrast to conventional intuition.

The joint presence of incomplete markets and search and matching frictions, however, can mitigate or even overturn these results. Intuitively, an increase in output implies an increase in hiring, which reduces the precautionary savings motive. The reduced desire to save makes an expansion in output compatible with an increase in the real interest rate.

We now formalize these arguments. In order for the ZLB to become binding, we need sufficiently large shocks. At the same time, we wish to analyze the effects of small shocks once the ZLB binds. For analytical tractability, we introduce a productivity process with a discrete and a continuous component. The discrete component fluctuates between a "depressed" state and a "normal" state. We assume that the depressed state is such that the ZLB binds. The continuous component adds additional shocks, if the discrete component is in the depressed state. We assume that these additional shocks are small and do not affect whether the ZLB binds or not.

Formally, we assume productivity is given by $\exp(I_s^{ZLB}\left(A_s - \sigma^{ZLB}\right))$. Here, A_s evolves as in Equation (5) and I_s^{ZLB} is an indicator variable, where $I_s^{ZLB} = 1$ indicates the depressed state and $I_s^{ZLB} = 0$ the normal state. The transition probability from $I_s^{ZLB} = 0$ to $I_s^{ZLB} = 1$ is given by $\mathbf{p} > 0$, whereas the normal state is absorbing. Finally, $\sigma^{ZLB} > 0$ is a parameter which denotes the reduction in average log productivity in the depressed state, relative to the normal state.

Consider an economy in which productivity is initially in the depressed state and suppose parameter values are such that there is a unique non-explosive solution which ultimately brings the economy back to the intended steady state, once productivity has reverted back to its normal state. It is straightforward to verify that the solution is two-part linear, i.e. linear conditional on I_s^{ZLB}. As derived above, the solution in the normal state ($I_s^{ZLB} = 0$) is given by $\widehat{\eta}_s = 0$ and $\widehat{\Pi}_s = 0$. The log-linearized Euler equation, conditional on $I_s^{ZLB} = 1$, can be written as:

$$-\mu w \chi \widehat{\eta}_s^{ZLB} + \beta \overline{R} (\mu w \chi + \Theta) \mathbf{p} \mathbb{E}_s \widehat{\eta}_{s+1}^{ZLB} = -\mathbf{p} \mathbb{E}_s \widehat{\Pi}_{s+1}^{ZLB},$$

where a superscript ZLB indicates that $I_s^{ZLB} = 1$. From the linearity of the solution it follows that $\mathbb{E}_s \widehat{\eta}_{s+1}^{ZLB} = \rho \widehat{\eta}_s^{ZLB}$. Plugging this result in the above equation and totally differentiating gives an expression for the elasticity of the job finding rate with respect to expected inflation, conditional on the ZLB:

$$\frac{d\widehat{\eta}_s^{ZLB}}{d\mathbb{E}_s \widehat{\Pi}_{s+1}^{ZLB}} = \frac{\mathbf{p}}{\mu w \chi \left(1 - \beta \overline{R} \mathbf{p} \rho\right) - \beta \overline{R} \Theta \mathbf{p} \rho}$$

Under complete markets ($\Theta = 0$) we get: $\frac{d\widehat{\eta}_s}{d\mathbb{E}_s \widehat{\Pi}_{s+1}} > 0$, since $\beta \overline{R} \mathbf{p} \rho \in (0, 1)$. Thus, any shock which reduces expected inflation creates a labor market contraction. For $\Theta > \mu w \chi \left(\frac{1}{\beta \overline{R} \mathbf{p} \rho} - 1 \right)$, however, $\frac{d\widehat{\eta}_s}{d\mathbb{E}_s \widehat{\Pi}_{s+1}} < 0$, i.e. the decline in expected inflation is expansionary if markets are sufficiently incomplete.

6 Asset pricing

This section explores asset pricing implications of the model. We show that the model generates a positive and countercyclical equity premium, but only if markets are incomplete. For simplicity, consider the model with sticky wages ($\chi = 0$) and no sunspots. The stochastic discount factor of an employed household is given by:

$$\Lambda_{e,s,s+1} = \beta \omega \left(1 - \eta_{s+1}\right) (\vartheta / w)^{-\mu} + \beta \left(1 - \omega \left(1 - \eta_{s+1}\right)\right)$$

Note that $\Lambda_{s,s+1}$ perfectly correlates with η_{s+1} and hence with and A_{s+1}. Further, note that η_{s+1} is linear in η_{s+1}. Given the solution, the job finding rate is –up to a first-order approximation– given by $\eta_s = \eta + \eta \Gamma_\eta A_s$. We exploit this to write the period$-s$ conditional expectation and variance of $\Lambda_{e,s,s+1}$, respectively,as:

$$\begin{aligned}
\mathbb{E}_s \Lambda_{e,s,s+1} &= \beta \omega \left(1 - \mathbb{E}_s \eta_{s+1}\right) (\vartheta / w)^{-\mu} + \beta \left(1 - \omega \left(1 - \mathbb{E}_s \eta_{s+1}\right)\right) \\
&= \beta \omega \left(1 - \eta - \rho \eta \Gamma_\eta A_s\right) (\vartheta / w)^{-\mu} + \beta \left(1 - \omega \left(1 - \eta - \rho \eta \Gamma_\eta A_s\right)\right)
\end{aligned}$$

and

$$\begin{aligned}
Var_s \left\{\Lambda_{e,s,s+1}\right\} &= \beta^2 \omega^2 \left(1 - \vartheta^{-\mu}\right)^2 Var_s \left\{\eta_{s+1}\right\}, \\
&= \beta^2 \omega^2 \left(1 - \vartheta^{-\mu}\right)^2 \eta^2 \Gamma_\eta^2 Var_s \left\{\rho A_s + \sigma_A \varepsilon_{s+1}^A\right\}, \\
&= \beta^2 \omega^2 \left(1 - \vartheta^{-\mu}\right)^2 \eta^2 \Gamma_\eta^2 \sigma_A^2.
\end{aligned}$$

Note that under complete markets ($\vartheta/w = 1$) we obtain $Var_s\left\{\Lambda_{e,s,s+1}\right\} = 0$, i.e. the stochastic discount factor is constant. This is precisely why standard NK models generate no first-order risk premia. Further, note that under incomplete markets $\mathbb{E}_s\Lambda_{e,s,s+1}$ is increasing in A_s, reflecting the precautionary savings motive, which weakens during a boom.

Exogenous payoffs: We now use the model to price risky assets with simple payoff structures. First, consider a risky asset that pays off $1 + A_{s+1} - \rho A_s$ in period $s+1$. Suppose that this asset is in zero net supply and households cannot go short in it. Let z_s denote the price of the asset and note that the expected payoff of the asset equals 1. The pricing equation for the asset reads:

$$\begin{aligned} z_s &= \mathbb{E}_s\left\{\Lambda_{e,s,s+1}\left(1 + A_{s+1} - \rho A_s\right)\right\} \\ &= \mathbb{E}_s\Lambda_{e,s,s+1} - \sqrt{Var_s\left\{\Lambda_{e,s,s+1}\right\} Var_s\left\{1 + A_{s+1} - \rho A_s\right\}} \\ &= \mathbb{E}_s\Lambda_{e,s,s+1} - \beta\omega\left(\vartheta^{-\mu} - 1\right)\eta\Gamma_\eta\sigma_A^2 \end{aligned}$$

where we exploit the fact that the correlation between $\Lambda_{s,s+1}$ and A_{s+1} is perfect. Now consider a riskless asset that pays out 1 in the next period. The value of the riskless asset is:

$$\widetilde{z}_s = \mathbb{E}_s\Lambda_{e,s,s+1}.$$

Note that whenever markets are incomplete ($\vartheta < 1$) we obtain $\widetilde{z}_s > z$, i.e. the price of the risky asset is lower than the price of the riskless asset, i.e. there is an equity premium.

Endogenous payoffs: Consider now another risky asset with an payoff equal to $1 + \eta\widehat{\eta}_{s+1} - \rho\eta\widehat{\eta}_s$. Note that, again, the expected payoff is one and that the payoff is increasing in next period's job finding rate. The price of the risky asset is now given by:

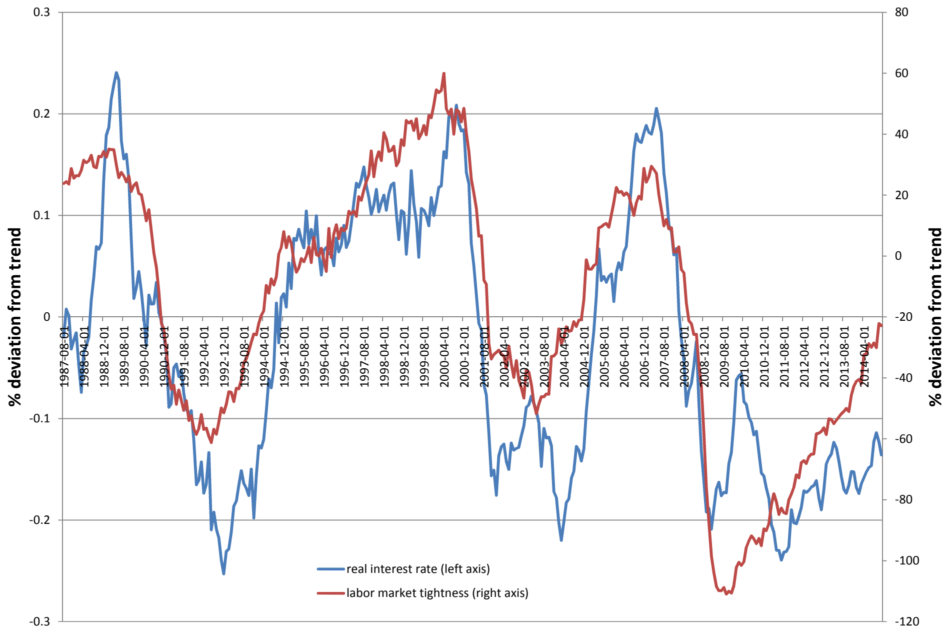
$$z_s = \mathbb{E}_s\Lambda_{e,s,s+1} - \beta\omega\left(\vartheta^{-\mu} - 1\right)\eta\Gamma_\eta^2\sigma_A^2$$

Note that in the return of the risky asset we now observe Γ_η^2 rather than Γ_η. This reflects the fact that the payoff of the asset is now endogenous. As a result, market frictions and monetary policy affect the equity premium via two channels: through he households' stochastic discount factor (via their unemployment risk) and through the asset payoff (via the equilibrium effects of household demand).

7 An empirical perspective

Using the log-linearized Euler equation, we can obtain the following expression for the real interest rate,

Figure 3: Real interest rate (R^r) and labor market tightness (v/u) in the data.



$$\widehat{R}_s^r = (1-\alpha)\left(-\mu\chi + (\mu\chi + \Theta)\,\beta\overline{R}\right)\rho\widehat{\theta}_s$$

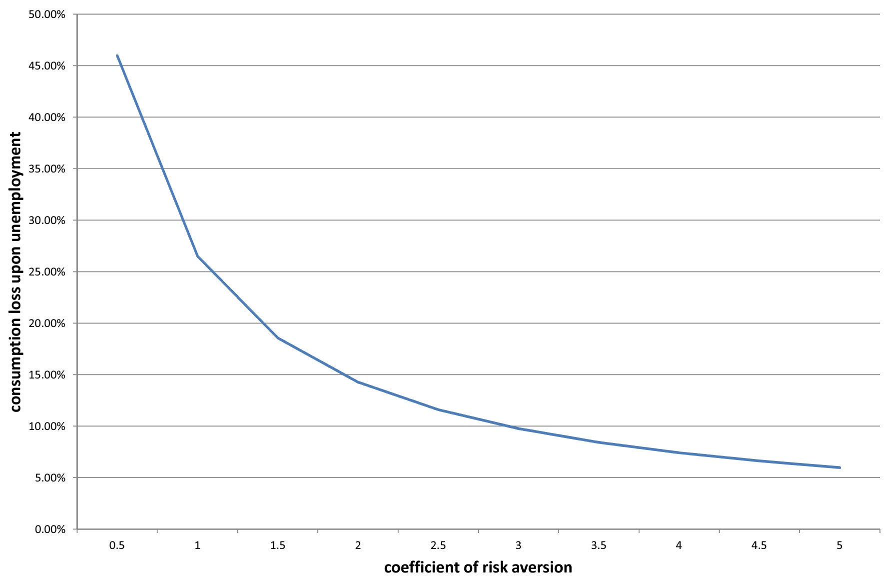
where $\widehat{R}_s^r \equiv \widehat{R}_s - \mathbb{E}_s\widehat{\Pi}_{s+1}$. The above equation provides a direct relation between market tightness and the real interest rate, which we can confront with the data.

Note that under complete markets ($\Theta = 0$) we obtain $\frac{d\widehat{R}_s^r}{d\widehat{\theta}_s} = (1-\alpha)\,\mu\chi\,(\rho - 1) < 0$. Thus, the complete markets model predicts that in a recession, when the labor market is less tight, the real interest rate increases. Intuitively, a transitory decline in income motivates households to borrow, pushing up the equilibrium real interest rate.

When Θ is sufficiently high, however, the relation between the real interest rate and market tightness is positive. Under incomplete markets, high labor market tightness dampens the precautionary savings motive and hence pushes up the equilibrium real interest rate. Figure 3 presents the relation between the two variables over the period since Paul Volcker left the Federal Reserve. Data are expressed in percentage deviations from a linear trend, estimated over the period up to the end of 2007. The two series display a striking positive correlation, with a coefficient of 0.84. Thus, support in the data for the relevance of the incomplete-markets mechanism appears overwhelming.

The data can also be used to directly parametrize Θ and get a sense of the quantitative importance of the key mechanism in the model. For simplicity, consider a model with sticky wages

Figure 4: Consumption loss upon unemployment.



($\chi = 0$). The log-linearized Euler equation implies that the ration of unconditional variances of the two variables are given by $\frac{Var(\widehat{R}_s^r)}{Var(\widehat{\theta}_s)} = (1-\alpha)^2\,\Theta^2\beta^2\overline{R}^2\rho^2$. Suppose for example that $\alpha = \frac{1}{2}$, $\rho = 0.99$ and $\beta\overline{R} = 1$.[18] Given the ratio of variances observed in the data, this implies that $\Theta = 0.0061$. To facilitate interpretation of this number, Figure 4 plots for a range of assumptions on the coefficient of risk aversion μ, the implied consumption loss upon unemployment (ϑ/w). For example, for a coefficient of risk aversion of 2, the calculation implies a consumption loss of about 15 percent, which seems reasonable in the light of empirical evidence.

8 Conclusion

We have proposed a simple and intuitive heterogeneous-agents New Keynesian (NK) model with endogenous unemployment, and highlighted that the interaction between market frictions can give rise to belief-driven fluctuations. Throughout the analysis, we have assumed that government policies are summarized by a simple interest rate rule, subject to the zero lower bound. It would be interesting to think use the framework to obtain insights into the stabilization

[18] Under incomplete markets it typically holds that $\beta\overline{R} < 1$. Even then, however, the number tends to be close to one, and lowering it has very limited effects on the results.

effects of other government policies, such as fiscal policy or labor market policies. Also, the framework could be used to consider optimal policies. We leave these issues for future research.

We have also demonstrated that under incomplete markets the NK model becomes useful to analyze the link between monetary policy and financial asset prices. While we have limited the analysis to simple analytical exercises, it would be interesting to evaluate the extent to which a full-scale heterogeneous-agents NK can explain observed asset prices. Vice versa, financial markets data may be useful to impose empirical discipline on the new generation of NK models.

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10 Appendix

10.1 Steady-state Nash bargaining solution

The steady-state expressions of the asset-poor households' surplus and value functions are:

$$\begin{aligned} V^e\left(1-\beta\left(1-\omega\left(1-\eta\right)\right)\right) &= \frac{w^{1-\mu}}{1-\mu}+\beta\omega\left(1-\eta\right)V^u, \\ V^u\left(1-\beta\left(1-\eta\right)\right) &= \frac{\vartheta^{1-\mu}}{1-\mu}+\zeta+\beta\eta V^e. \end{aligned}$$

where we have exploited that in equilibrium the asset-poor households are the same and consume their incomes. Now substitute out V^u in the second equation:

$$V^e\left(1-\beta\left(1-\omega\left(1-\eta\right)\right)\right)=\frac{w^{1-\mu}}{1-\mu}+\frac{\beta\omega\left(1-\eta\right)}{1-\beta\left(1-\eta\right)}\left(\frac{\vartheta^{1-\mu}}{1-\mu}+\zeta+\beta\eta V^e\right),$$

We can now express the two values as functions of η and w:

$$\begin{aligned} V^e\left(\eta,w\right) &= \frac{\frac{w^{1-\mu}}{1-\mu}+\frac{\beta\omega(1-\eta)}{1-\beta(1-\eta)}\left(\frac{\vartheta^{1-\mu}}{1-\mu}+\zeta\right)}{1-\beta\left(1-\omega\left(1-\eta\right)\right)-\frac{\beta\omega(1-\eta)\beta\eta}{1-\beta(1-\eta)}} \\ V^u\left(\eta,w\right) &= \frac{\frac{\vartheta^{1-\mu}}{1-\mu}+\zeta+\beta\eta V^e\left(\eta,w\right)}{1-\beta\left(1-\eta\right)} \end{aligned}$$

The first-order condition to the Nash Bargaining problem is given by

$$\left(1-\upsilon\right)S^e=\upsilon S^f,$$

or,

$$\left(1-\upsilon\right)\left(V^e\left(\eta,w\right)-V^u\left(\eta,w\right)\right)=\upsilon\kappa\eta^{\alpha/(1-\alpha)}.$$

The above is an equation in two variables, which implicitly defines the wage as a function of the job finding rate, i.e the function $w(\eta)$.

Basic properties: Consider the special case in which $\eta=0$. From the Nash bargaining solution it follows that the wage must satisfy $V^e\left(0,w(0)\right)=V^u\left(0,w(0)\right)=\frac{\frac{\vartheta^{1-\mu}}{1-\mu}+\zeta}{1-\beta}$. It follows that $\frac{w(0)^{1-\mu}}{1-\mu}=\frac{\vartheta^{1-\mu}}{1-\mu}+\zeta$ and hence $w(0)>\vartheta$ whenever $\zeta>0$.

At the other extreme, under $\eta=1$ we get from the Nash Bargaining solution $V^e\left(1,w\right)=V^u\left(1,w\right)+\frac{\upsilon}{1-\upsilon}\kappa$. Also, the worker value functions imply that $V^e\left(1,w\right)-V^u\left(1,w\right)=\frac{w(1)^{1-\mu}}{1-\mu}-\frac{\vartheta^{1-\mu}}{1-\mu}-\zeta$. It follows that $\frac{w(1)^{1-\mu}}{1-\mu}=\frac{\vartheta^{1-\mu}}{1-\mu}+\zeta+\frac{\upsilon}{1-\upsilon}\kappa$ and hence $w(1)>w(0)$, $V^e\left(1,w(1)\right)>V^e\left(0,w(0)\right)$ and $V^u\left(1,w\right)>V^u\left(0,w\right)$.

Finally, consider a case in which the worker has no bargaining power ($\upsilon=0$). It follows from the Nash bargaining solution that in this case $V^e\left(\eta,w\right)=V^u\left(\eta,w\right)$ which implies that $\frac{w(\eta)^{1-\mu}}{1-\mu}=\frac{\vartheta^{1-\mu}}{1-\mu}+\zeta$. As a result, the real wage does not depend of η, i.e. the real wage is sticky.

10.2 Log-linearizing the model

10.2.1 Nash Bargaining block

The first-order condition to the Nash, together with the asset-poor workers' value functions are given by:

$$\begin{aligned} V_s^e - V_s^u &= \upsilon \kappa \eta_s^{\alpha/(1-\alpha)}, \\ V_s^e &= \frac{w_s^{1-\mu}}{1-\mu} + \beta \mathbb{E}_s \omega \left(1-\eta_{s+1}\right) V_{s+1}^u + \beta \mathbb{E}_s \left(1 - \omega \left(1-\eta_{s+1}\right)\right) V_{s+1}^e, \\ V_s^u &= \frac{\vartheta^{1-\mu}}{1-\mu} + \zeta + \beta \mathbb{E}_s \left(1-\eta_{s+1}\right) V_{s+1}^u + \beta \mathbb{E}_s \eta_{s+1} V_{s+1}^e, \end{aligned}$$

and note that this a system of three equations and three unknowns (V_s^e, V_s^u, w_s and η_s). After log-linearization, the above system can be written in the following form:

$$\mathbf{A} \begin{bmatrix} \widehat{V}_s^e \\ \widehat{V}_s^u \\ \widehat{w}_s \end{bmatrix} + \mathbf{B} \widehat{\eta}_s = \mathbb{E}_s \mathbf{C} \begin{bmatrix} \widehat{V}_{s+1}^e \\ \widehat{V}_{s+1}^u \\ \widehat{w}_{s+1} \end{bmatrix} + \mathbb{E}_s \mathbf{D} \widehat{\eta}_{s+1}$$

where **A** and **D** are 3×3 matrices and **B** and **D** are 3×1 vectors, all consisting of parameter values. Note that none of the variables $\widehat{V}_s^e$, $\widehat{V}_s^u$ and $\widehat{w}_s$ is a state variable. Provided that $\widehat{\eta}_s$ follows some linear law of motion, we can apply the method of undetermined coefficients to find solutions for $\widehat{V}_s^e$, $\widehat{V}_s^u$ and $\widehat{w}_s$ as linear functions of $\widehat{\eta}_s$. We denote the solution for the wage as $\widehat{w}_s = \chi \widehat{\eta}_s$, where it follows that χ is a function of the parameters that enter **A**, **B**, **C** and **D**.

10.2.2 Monetary Policy rule, Euler equation, Phillips Curve

The log-linerarized monetary policy rule is given by:

$$\widehat{R}_s = \delta_\pi \widehat{\Pi}_s + \delta_\theta \widehat{\theta}_s.$$

Next, consider the Euler equation of the employed households. Exploiting the fact that in Equilibrium $c_{e,s} = w_s$ and $c_{u,s} = \vartheta$, we can express the employed workers' Euler equation, Equation (20), as:

$$w_s^{-\mu} = \beta \mathbb{E}_s \frac{R_s}{\Pi_{s+1}} \left(\omega \left(1-\eta_{s+1}\right) \vartheta^{-\mu} + \left(1 - \omega \left(1-\eta_{s+1}\right)\right) w_{s+1}^{-\mu}\right)$$

and note that in the intended steady state we obtain $w^{-\mu} = \beta\overline{R}\left(\omega\left(1-\eta\right)\vartheta^{-\mu} + 1 - \omega\left(1-\eta\right)w^{-\mu}\right)$. Log-linearizing the above equation around the intended steady state gives:

$$\begin{aligned} -\mu\widehat{w}_s &= \widehat{R}_s - \mathbb{E}_s\widehat{\Pi}_{s+1} - \beta\overline{R}\omega\eta\left(\vartheta/w\right)^{-\mu}\mathbb{E}_s\widehat{\eta}_{s+1} + \beta\overline{R}\omega\eta\mathbb{E}_s\widehat{\eta}_{s+1} - \mu\beta\overline{R}\left(1-\omega\left(1-\eta\right)\right)\mathbb{E}_s\widehat{w}_{s+1} \\ &= -\mu\beta\overline{R}\mathbb{E}_s\widehat{w}_{s+1} + \widehat{R}_s - \mathbb{E}_s\widehat{\Pi}_{s+1} - \beta\overline{R}\omega\eta\left(\left(\vartheta/w\right)^{-\mu} - 1\right)\mathbb{E}_s\widehat{\eta}_{s+1} + \mu\beta\overline{R}\omega\left(1-\eta\right)\mathbb{E}_s\widehat{w}_{s+1} \\ &= -\mu\beta\overline{R}\mathbb{E}_s\widehat{w}_{s+1} + \widehat{R}_s - \mathbb{E}_s\widehat{\Pi}_{s+1} - \beta\overline{R}\Theta\mathbb{E}_s\widehat{\eta}_{s+1} \end{aligned}$$

where $\Theta = \omega\eta\left(\left(\vartheta/w\right)^{-\mu} - 1\right) - \chi\mu\omega\left(1-\eta\right)$. Exploiting that $\widehat{w}_s = \chi\widehat{\eta}_s$ gives:

$$-\mu\chi\widehat{\eta}_s = \widehat{R}_s - \mathbb{E}_s\widehat{\Pi}_{s+1} - \beta\overline{R}\left(\Theta + \mu\chi\right)\mathbb{E}_s\widehat{\eta}_{s+1}.$$

Next, consider the firms' price setting condition, which can be written as:

$$\begin{aligned} &\phi\left(\Pi_s - 1\right)\Pi_s - \phi\mathbb{E}_s\Lambda_{s,s+1}\frac{y_{s+1}}{y_s}\left(\Pi_{s+1} - 1\right)\Pi_{s+1} \\ = \;& 1 - \gamma + \gamma\left(\frac{w_s}{\exp\left(A_s\right)} + \kappa\eta_s^{\alpha/(1-\alpha)} - \left(1-\omega\right)\kappa\mathbb{E}_s\Lambda_{s,s+1}\frac{\exp(A_{s+1})}{\exp(A_s)}\eta_{s+1}^{\alpha/(1-\alpha)} + \lambda_{v,s}\right). \end{aligned}$$

and note that at the intended steady state $\lambda_{v,s} = 0$ and $\Lambda_{s,s+1} = \beta$. Log-linearizing the equation around the intended steady state with $\Pi = 1$ gives:

$$\frac{\phi}{\gamma}\widehat{\Pi}_s - \frac{\phi}{\gamma}\beta\mathbb{E}_s\widehat{\Pi}_{s+1} = w\chi\widehat{\eta}_s - wA_s + \frac{\kappa}{q}\left(\frac{\alpha}{1-\alpha}\widehat{\eta}_s - \frac{\alpha\beta\left(1-\omega\right)}{1-\alpha}\widehat{\eta}_{s+1} - \beta\left(1-\omega\right)\left(\widehat{\Lambda}_{s,s+1}\right) - \left(1-\rho\right)A_s\right)$$

where we have substituted out the wage using $\widehat{w}_s = \chi\widehat{\eta}_s$.

10.3 Reducing the model

Under the the two assumptions ($\delta_\pi = \frac{1}{\beta}$ and risk-neutrality of the equity investors) and in the absence of productivity shocks, the log-linearized Euler equation and pricing condition become:

$$\begin{aligned} -\mu\chi\beta\widehat{\eta}_s + \mu\beta^2\overline{R}\chi\mathbb{E}_s\widehat{\eta}_{s+1} &= \widehat{\Pi}_s - \beta\mathbb{E}_s\widehat{\Pi}_{s+1} + \frac{\beta\delta_\theta}{1-\alpha}\widehat{\eta}_s - \beta^2\overline{R}\Theta\mathbb{E}_s\widehat{\eta}_{s+1} \\ w\chi\widehat{\eta}_s + \frac{\kappa}{q}\left(\frac{\alpha}{1-\alpha}\widehat{\eta}_s - \frac{\alpha\beta\left(1-\omega\right)}{1-\alpha}\mathbb{E}_s\widehat{\eta}_{s+1}\right) &= \frac{\phi}{\gamma}\left(\widehat{\Pi}_s - \beta\mathbb{E}_s\widehat{\Pi}_{s+1}\right) \end{aligned}$$

where in the first equation we have substituted out the interest rate using $\widehat{R}_s = \delta_\pi\widehat{\Pi}_s + \delta_\theta\widehat{\theta}_s$, and tightness using $\widehat{\theta}_s = \frac{\widehat{\eta}_s}{1-\alpha}$. Using the first equation to substitute out $\widehat{\Pi}_s - \beta\mathbb{E}_s\widehat{\Pi}_{s+1}$ in the second equation gives:

$$w\chi\widehat{\eta}_s + \frac{\kappa}{q}\left(\frac{\alpha}{1-\alpha}\widehat{\eta}_s - \frac{\alpha\beta\left(1-\omega\right)}{1-\alpha}\mathbb{E}_s\widehat{\eta}_{s+1}\right) = \frac{\phi}{\gamma}\left(-\mu\chi\beta\widehat{\eta}_s + \mu\beta^2\overline{R}\chi\mathbb{E}_s\widehat{\eta}_{s+1} - \frac{\beta\delta_\theta}{1-\alpha}\widehat{\eta}_s + \beta^2\overline{R}\Theta\mathbb{E}_s\widehat{\eta}_{s+1}\right).$$

Collecting terms gives:

$$\mathbb{E}_s\widehat{\eta}_{s+1} = \Psi\widehat{\eta}_s,$$

where

$$\Psi = \frac{\frac{\phi}{\gamma}\mu\chi\beta + \frac{\phi}{\gamma}\frac{\beta\delta_\theta}{1-\alpha} + w\chi + \frac{\kappa}{q}\frac{\alpha}{1-\alpha}}{\frac{\kappa}{q}\frac{\alpha\beta(1-\omega)}{1-\alpha} + \frac{\phi}{\gamma}\mu\beta^2\overline{R}\chi + \frac{\phi}{\gamma}\beta^2\overline{R}\Theta}.$$

10.4 Adding productivity shocks

With productivity shocks the model becomes:

$$\begin{aligned} & w\chi\widehat{\eta}_s - wA_s + \frac{\kappa}{q}\left(\frac{\alpha}{1-\alpha}\widehat{\eta}_s - \frac{\alpha\beta(1-\omega)}{1-\alpha}\mathbb{E}_s\widehat{\eta}_{s+1} - \beta(1-\omega)\rho A_s\right) \\ = & \frac{\phi}{\gamma}\left(-\mu\chi\beta\widehat{\eta}_s + \mu\beta^2\overline{R}\chi\mathbb{E}_s\widehat{\eta}_{s+1} - \frac{\beta\delta_\theta}{1-\alpha}\widehat{\eta}_s + \beta^2\overline{R}\Theta\mathbb{E}_s\widehat{\eta}_{s+1}\right) \\ A_s = & \rho A_{s-1} + \sigma_A\varepsilon_s^A \end{aligned}$$

which we can rewrite as

$$\begin{aligned} \mathbb{E}_s\widehat{\eta}_{s+1} &= \Psi\widehat{\eta}_s - \Omega A_s \\ A_s &= \rho A_{s-1} + \sigma_A\varepsilon_s^A \end{aligned}$$

where

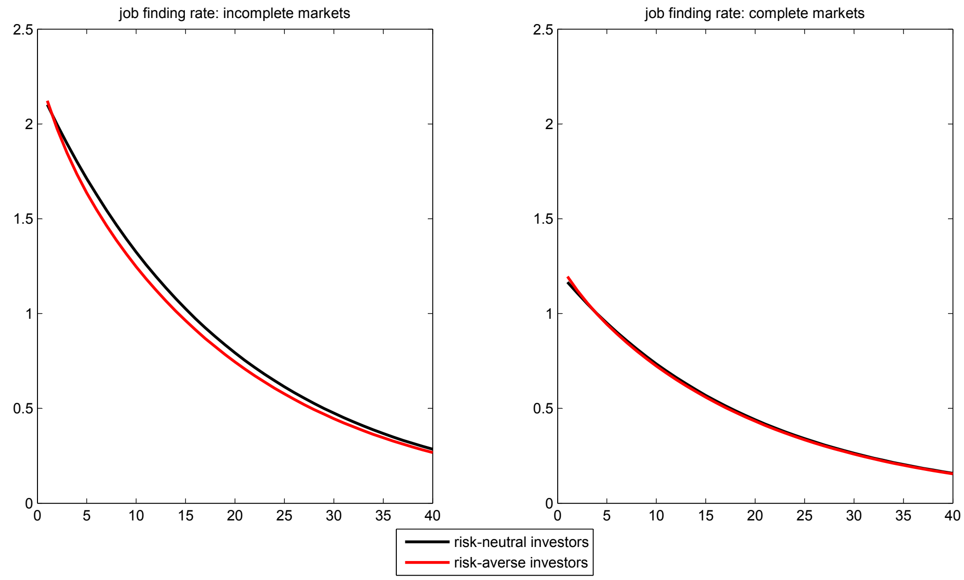
$$\Omega = \frac{w + \frac{\kappa}{q}(1-\rho)}{\frac{\kappa}{q}\frac{\alpha\beta(1-\omega)}{1-\alpha} + \frac{\phi}{\gamma}\mu\beta^2\overline{R}\chi + \frac{\phi}{\gamma}\beta^2\overline{R}\Theta}$$

10.5 Risk-averse investors

When we log-linearized the model, we have assume for simplicity that the asset-rich firm owners are risk neutral. The reason is that, technically, the unemployment rate becomes a state variable for inflation and the job finding rate, once we assume risk averse investors. With an additional state variable, the analytical solution of the model becomes more cumbersome, detracting from the key intuitions of the model.

Below, we use numerical simulations to compare versions with risk-neutral and risk-averse investors, showing only very small differences. We parametrize the model as follows. We choose the subjective discount factor β target a steady-state interest rate of 3 percent per annum. The coefficient of risk aversion, μ, is set to 2, whereas the elasticity of substitution between goods, γ, is set to 6. To calibrate the price-stickiness parameter ϕ, we exploit the observational equivalence between the Calvo and Rotemberg versions of the log-linearized New Keynesian

Figure 5: Responses to a positive technology shock.



model, and target an average price duration of 5 months. The home production parameter, ϑ, is set to imply a 15 percent consumption drop upon unemployment.

The vacancy cost is parametrized to target a stead-state hiring cost of about 4 percent of the quarterly wage, following Silva and Toledo (2009). We further target a monthly job finding rate of 0.3 and set the job loss rate, ω, to 2 percent. The matching function elasticity parameter, α, is set to 0.5. Regarding the monetary policy rule, we set $\delta_\pi = 1.5$ and $\delta_\theta = 0$. The persistence parameter of the technology shock is set to $\rho = 0.95$. For simplicity we assume sticky wages ($\chi = 0$).

The left panel of the figure below ("incomplete markets") plots the response of the job finding rate to a positive technology shock. On impact, the response is larger with risk-averse investors. In subsequent periods, the pattern reverses and the response is smaller with risk-averse investors. Quantitatively, however, the differences are very small.

Next, we consider a version of the model in which we set the home production parameter ϑ such that there is no consumption loss upon job loss. Effectively, this removes the incomplete-markets wedge from the model. The right panel of the figure below ("complete markets") again compares the versions with risk-averse and risk-neutral investors. The differences are similar to the complete markets case. Most importantly, differens are again very small.