# Wealth, Wages, and Employment

Preliminary

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# Introduction

- We want a of the joint distribution of employment, wages, and wealth.
  - Workers are risk averse, only use self-insurance.
  - The employment and wage risk is endogenous.
  - The economy aggregates into a modern economy (total wealth, labor shares, consumption/investment ratios)
  - Business cycles can be studied.
- Such a framework does not exist in the literature.
  1. Requires heterogeneous agents.
  2. No (search-matching) closed form solutions possible.
  3. Wage formation? Nash bargaining not very promising:
     - A bargaining problem where wages become a(n increasing) function of worker wealth.
     - Not time-consistent and bargaining with commitment makes no sense.
     - Not numerically well-behaved.
- We offer an alternative: competitive job search with commitment to a wage (or wage schedule) while the job lasts.

# Literature

- At its core is Aiyagari (1994) meets Moen (1997).
- Developing empirically sound versions of these ideas compels us to
  - Add extreme value shocks to transform decision rules from functions into densities to weaken the correlation between states and choices.
  - Pose quits, on the job search, and explicit role for leisure so quitting is not only to search for better jobs
  - Use new potent tools to address the study of fluctuations in complicated economies Boppart, Krusell, and Mitman (2018)
- Related to Lise (2013), Hornstein, Krusell, and Violante (2011), Krusell, Mukoyama, and Şahin (2010), Ravn and Sterk (2016, 2017), Den Haan, Rendahl, and Riegler (2015).
- Especially, Eeckhout and Sepahsalari (2015), Chaumont and Shi (2017), Griffy (2017).

# What are the uses?

- The study of Business cycles including gross flows in and out of employment, unemployment and outside the labor force
- Policy analysis where now risk, employment, wealth (including its distribution) and wages are all responsive to policy.

# Today: Discuss various model Ingredients & Fluctuations

1. Baseline: Exogenous Destruction, no Quits. Built on top of Growth Model. (GE version of Eeckhout and Sepahsalari (2015)): Not a lot of wage dispersion. Not a lot of job creation in expansions.

2. Quits: Higher wage dispersion may arise to keep workers longer. (Endogenous quits via extreme value shocks). But Wealth trumps wages and wage dispersion collapses.

   2.1 Commitment to wage schedule.

   2.2 Variable Search Intensity by firms

3. On the Job Search workers may get outside offers and take them. (Some in Chaumont and Shi (2017)). Fluctuations.

4. Multiple types Workers differ in the value of leisure, i.e. attachment to the labor market. Explicit role of Outside Labor Force. Under development.

# The Baseline Model

# Baseline: Precautionary Savings, Competitive Search

- Jobs are created by firms (plants). A plant with capital plus a worker produce one ($z$) unit of the good.
  - Firms pay flow cost $\bar{c}$ to post a vacancy in market $\{w, \theta\}$. Cannot change wage (or wage-schedule) afterwards.
  - Plants (and their capital) are destroyed at rate $\delta$. Workers will not want to quit (for now).
- Households differ in wealth and wages (if working). There are no state contingent claims, nor borrowing.
  - If employed, workers get $w$ and save.
  - If unemployed, workers produce $b$ and search in some $\{w, \theta\}$.
- General equilibrium: Workers own firms.

# Order of Events of Baseline Model

1. Households enter the period with or without a job: $\{e, u\}$.

2. Production & Consumption: Employed produce $z$ on the job. Unemployed produce $b$ at home. They choose savings.

3. Firm Destruction and Exogenous Quits : Some Firms are destroyed (rate $\delta^f$) They cannot search this period. Some workers quit their jobs for exogenous reasons $\delta^h$. Total job destruction is $\delta$.

4. Search: Potential entrants and the unemployed choose wage $w$ and market tightness $\theta$.

5. Job Matching : $M(V, U)$ : Some vacancies meet some unemployed job searchers. A match becomes operational the following period. Job finding and job filling rates $\psi^h(\theta) = \frac{M(V,U)}{U}$, $\psi^f(\theta) = \frac{M(V,U)}{V}$.

# Baseline Model: Household Problem

- Individual state: wealth and wage
  - If employed: $(a, w)$
  - If unemployed: $(a)$

- Problem of the employed: (Standard)

$$V^e(a,w) = \max_{c,a'} u(c) + \beta \ [(1-\delta)V^e(a',w) + \delta V^u(a)]$$

$$\text{s.t.} \quad c + a' = a(1+r) + w, \quad a \geq 0$$

- Problem of the unemployed: Choose which wage to look for

$$V^u(a) = \max_{c,a',w} u(c) + \beta \left\{ \psi^h[\theta(w)] \ V^e(a',w) + [1 - \psi^h[\theta(w)]] \ V^u(a') \right\}$$

$$\text{s.t.} \quad c + a' = a(1+r) + b, \quad a \geq 0$$

$\theta(w)$ is an equilibrium object

# Firms Post vacancies: Choose wages & filling probabilities

- An idle firm is worth the resale value of capital
  $\Omega = \overline{k}(1 - \delta^f - \delta^k)$,

- Value of a job with wage $w$: uses constant $\overline{k}$ capital that depreciates

$$\Omega(w) = z - \overline{k}\delta^k - w + \frac{1-\delta^f}{1+r}\left[(1-\delta^h)\,\Omega(w) + \delta^h\,\overline{k}\right]$$

- Affine in $w$: $\Omega(w) = [z + \overline{k}(\frac{1-\delta^f}{1+r}\delta^h - \delta^k) - w]\frac{1+r}{r+\delta^f+\delta^h-\delta^f\delta^h}$

  Block Recursivity Applies (firms can be ignorant of Eq)

- Value of creating a firm: $\psi^f[\theta(w)]\ \ \Omega(w) + [1 - \psi^f[\theta(w)]]\,\Omega$

- Free entry condition requires that for all offered wages

$$\bar{c} + \overline{k} = \psi^f[\theta(w)]\ \ \frac{\Omega(w)}{1+r} + [1 - \psi^f[\theta(w)]]\ \ \frac{\Omega}{1+r},$$

# Baseline Model: Stationary Equilibrium

- A stationary equilibrium is functions $\{V^e, V^u, \Omega, g'^e, g'^u, w^u, \theta\}$, an interest rate $r$, and a stationary distribution $x$ over $(a, w)$, s.t.

  1. $\{V^e, V^u, g'^e, g'^u, w^u\}$ solve households' problems, $\{\Omega\}$ solves the firm's problem.

  2. Zero profit condition holds for active markets

$$\bar{c} + \overline{k} = \psi^f[\theta(w)] \ \frac{\Omega(w)}{1+r} + [1 - \psi^f[\theta(w)]] \ \frac{\overline{k}(1-\delta-\delta_k)}{1+r}, \quad \forall w \ \text{offered}$$

  3. An interest rate $r$ clears the asset market

$$\int a \ dx = \int \Omega(w) \ dx.$$

# Characterization of a worker's decisions

- Standard Euler equation for savings

$$u_c = \beta (1 + r) \; E \{u'_c\}$$

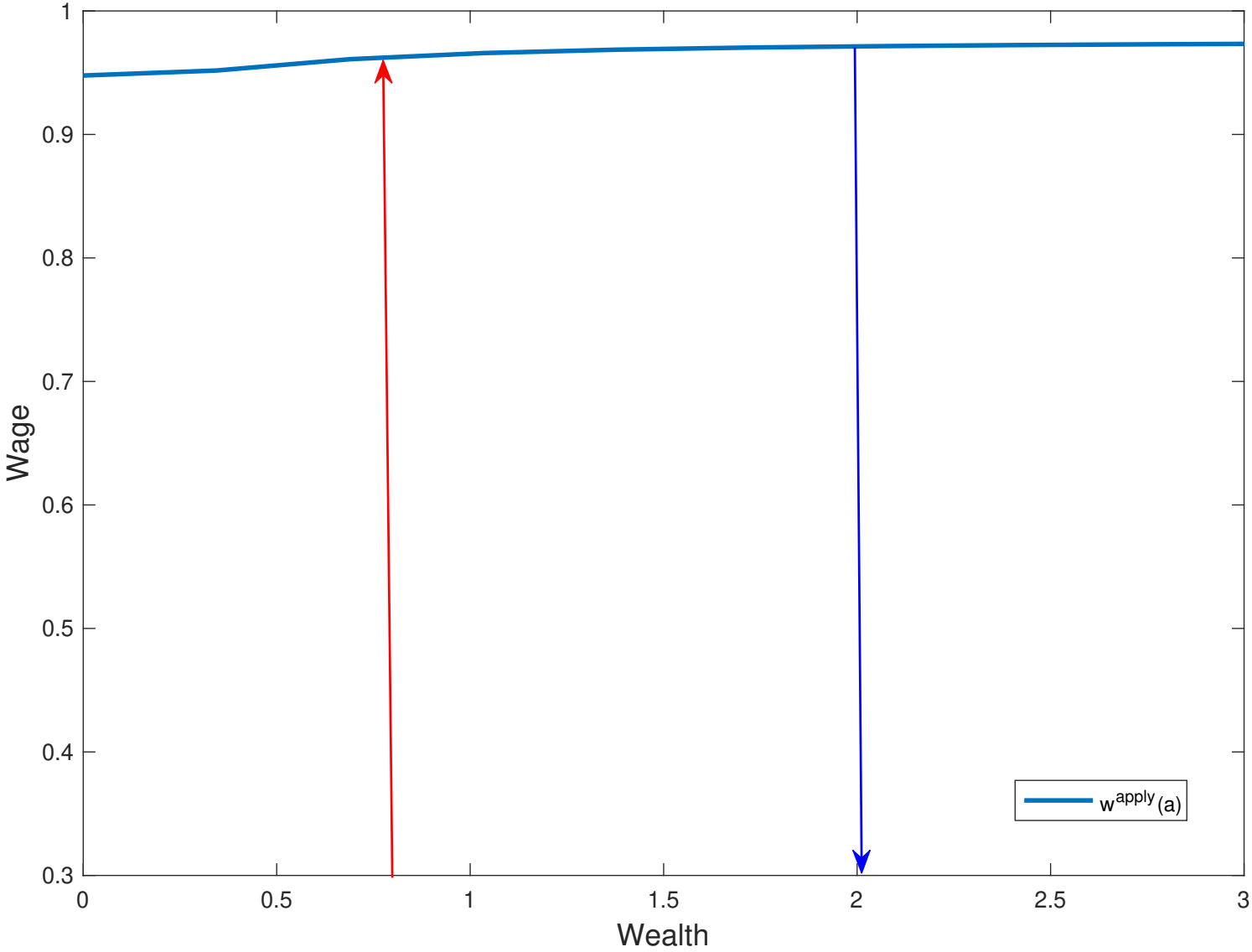
- A F.O.C for wage applicants

$$\psi^h[\theta(w)] \; V^e_w(a', w) = \psi^h_\theta[\theta(w)] \; \theta_w(w) \; [V^u(a') - V^e(a', w)]$$

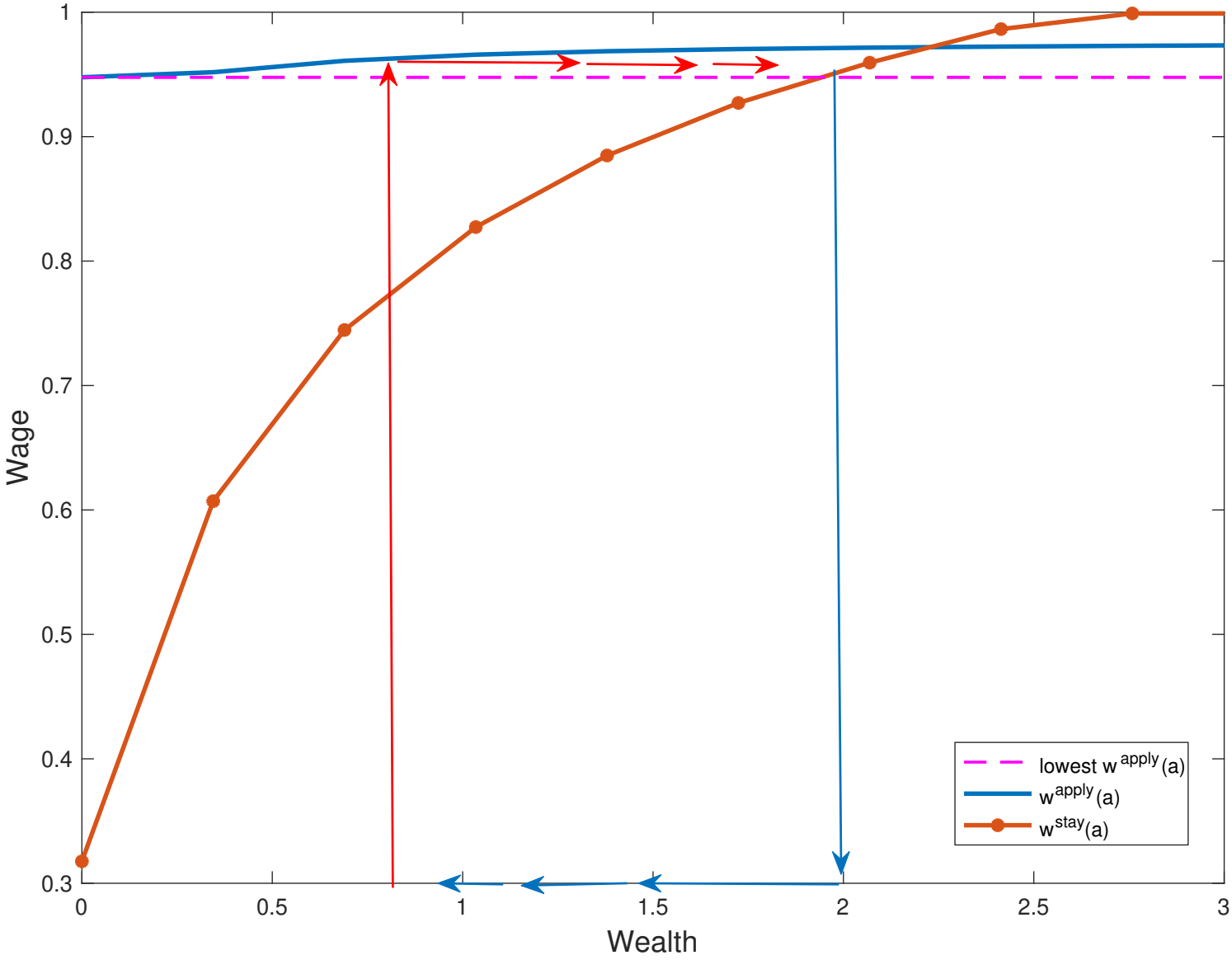
- Households with more wealth are able to insure better against unemployment risk.

- As a result they apply for higher wage jobs and we have dispersion

## Worker's wage application decision



## Worker's saving decision



# Summary: Baseline Model

1. Very Easy to Compute Steady-State with key Properties
   - i Risk-averse, only partially insured workers, endogenous unemployment
   - ii Can be solved with aggregate shocks too
   - iii Policy such as UI would both have insurance and incentive effects
   - iv Wage dispersion small—wealth doesn't matter too much
   - v $\cdots$ so almost like two-agent model (employed, unemployed) of Pissarides despite curved utility and savings

2. In the following we will examine the implications of a quitting choice

# Endogenous Quits

# Endogenous Quits: Beauty of Extreme Value Shocks

1. Temporary Shocks to the utility of working or not working: Some workers quit.

2. Adds a (smoothed) quitting motive so that higher wage workers quit less often: Firms may want to pay high wages to retain workers.

3. Conditional on wealth, high wage workers quit less often.

4. But Selection (correlation 1 between wage and wealth when hired) makes wealth trump wages and higher wages imply quit less often: Wage inequality collapses due to firms profit maximization.

5. We end up with a model with little wage dispersion but with endogenous quits that respond to the cycle.

# Quitting Model: Time-line

1. Workers enters period with or without a job: $\{e, u\}$.

2. Production occurs and consumption/saving choice ensues:

3. Exogenous job/firm destruction happens.

4. Quitting:
   - $e$ draw shocks $\{\epsilon^e, \epsilon^u\}$ and make quitting decision.
     Job losers cannot search this period.
   - $u$ draw shocks $\{\epsilon_1^u, \epsilon_2^u\}$. No decision but same expected means.

5. Search: New or Idle firms post vacancies. Choose $\{w, \theta\}$.
   Wealth is not observable. (Unlike Chaumont and Shi (2017)).
   Yet it is still Block Recursive

6. Matches occur

# Quitting Model: Workers

- Workers receive i.i.d shocks $\{\epsilon^e, \epsilon^u\}$ to the utility of working or not
- Value of the employed right before receiving those shocks:

$$\widehat{V}^e(a', w) = \int \max\{V^e(a', w) + \epsilon^e, V^u(a') + \epsilon^u\}\, dF^\epsilon$$

  $V^e$ and $V^u$ are values after quitting decision as described before.

- If shocks are Type-I Extreme Value dbtn (Gumbel), then $\widehat{V}$ has a closed form and the ex-ante quitting probability $q(a, w)$ is

$$q(a, w) = \frac{1}{1 + e^{\alpha[V^e(a,w) - V^u(a)]}}$$

  higher parameter $\alpha \to$ lower chance of quitting.

- Hence higher wages imply longer job durations. Firms could pay more to keep workers longer.

# Quitting Model: Workers Problem

- Problem of the employed: just change $\widehat{V}^e$ for $V^e$

$$V^e(a,w) = \max_{c,a'} u(c) + \beta \left[(1-\delta)\widehat{V}^e(a',w) + \delta V^u(a)\right]$$
$$\text{s.t.} \quad c + a' = a(1+r) + w, \quad a \geq 0$$

- Problem of the unemployed is like before except that there is an added term $E\{\max[\epsilon_1^u, \epsilon_2^u]\}$

So that there is no additional option value to a job.

# Quitting Model: Value of the firm

- $\Omega^j(w)$: Value with with $j$-tenured worker.
  Free entry condition requires that for all offered wages

$$\bar{c} + \overline{k} = \frac{1}{1+r} \left\{ \psi^f[\theta(w)]\, \Omega^0(w) + [1 - \psi^f[\theta(w)]]\, \Omega \right\},$$

- Probability of retaining a worker with tenure $j$ at wage $w$ is $\ell^j(w)$.
  (One to one mapping between wealth and tenure)

$$\ell^j(w) = 1 - q^e[g^{e,j}(a, w), w]$$

  $g^{e,j}(a, w)$ savings rule of a $j-$ *tenured* worker that was hired with wealth $a$

- Firm's value

$$\Omega^j(w) = z - \overline{k}\delta^k - w + \frac{1-\delta^f}{1+r} \left\{ \ell^j(w)\Omega^{j+1}(w) + [1 - \ell^j(w)]\, \Omega \right\}$$

# Quitting Model: Solving forward for the Value of the firm

$$\Omega^0(w) \quad = \quad (z - w - \delta^k k)\, Q^1(w) + (1 - \delta^f - \delta_k) k\, Q^0(w),$$

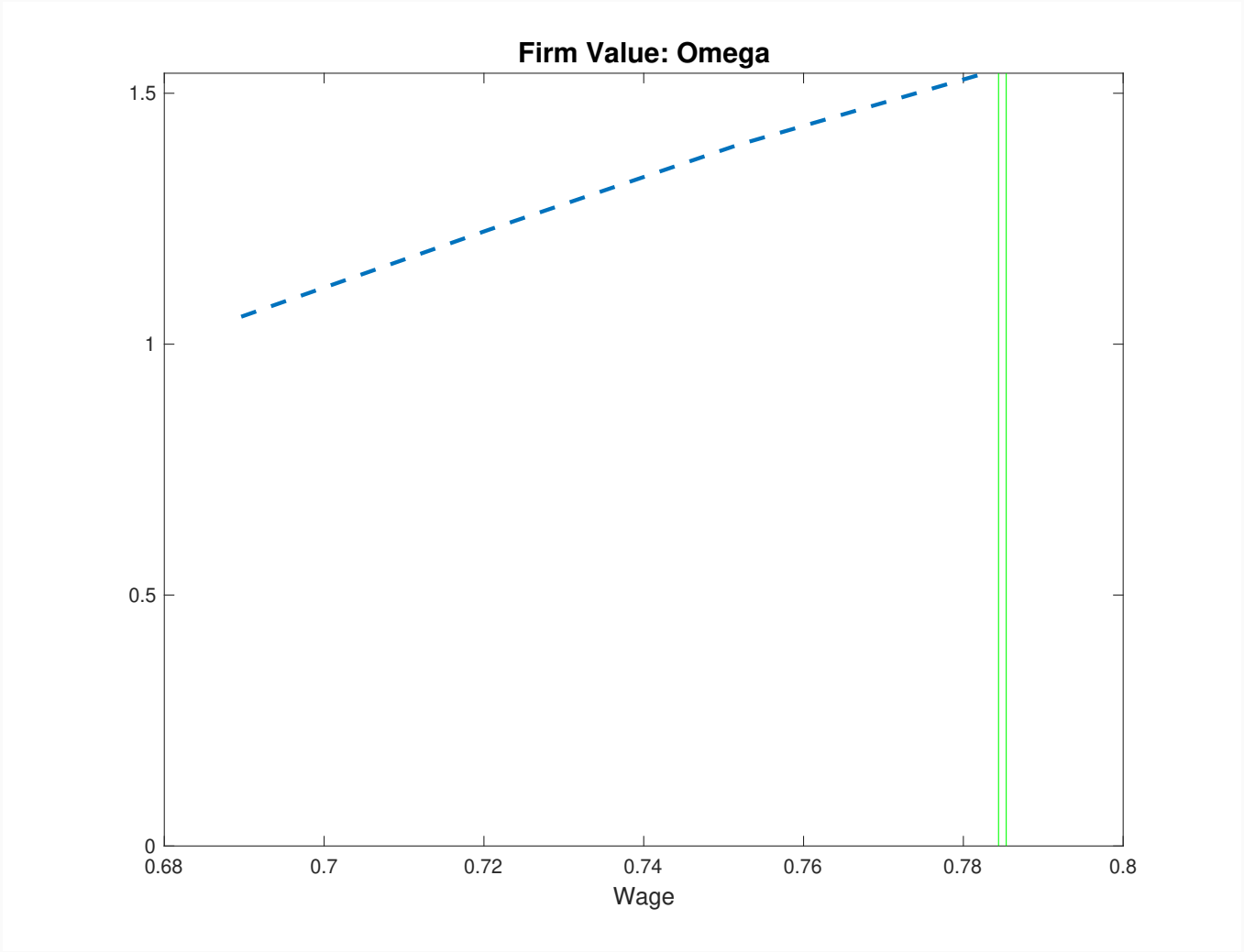
$$Q^1(w) = 1 + \sum_{\tau=0}^{\infty} \left[ \left( \frac{1-\delta^f}{1+r} \right)^{1+\tau} \prod_{i=0}^{\tau} \ell^i(w) \right],$$

$$Q^0(w) = \sum_{\tau=0}^{\infty} \left[ \left( \frac{1-\delta^f}{1+r} \right)^{1+\tau} [1 - \ell^\tau(w)] \left( \prod_{i=0}^{\tau-1} \ell^i(w) \right) \right].$$

- New equilibrium objects $\{Q^0(w), Q^1(w)\}$. Rest is unchanged.
- It is Block Recursive because wealth can be inferred from $w$ and $j$. (No need to index contracts by wealth (as in Chaumont and Shi (2017)) ).

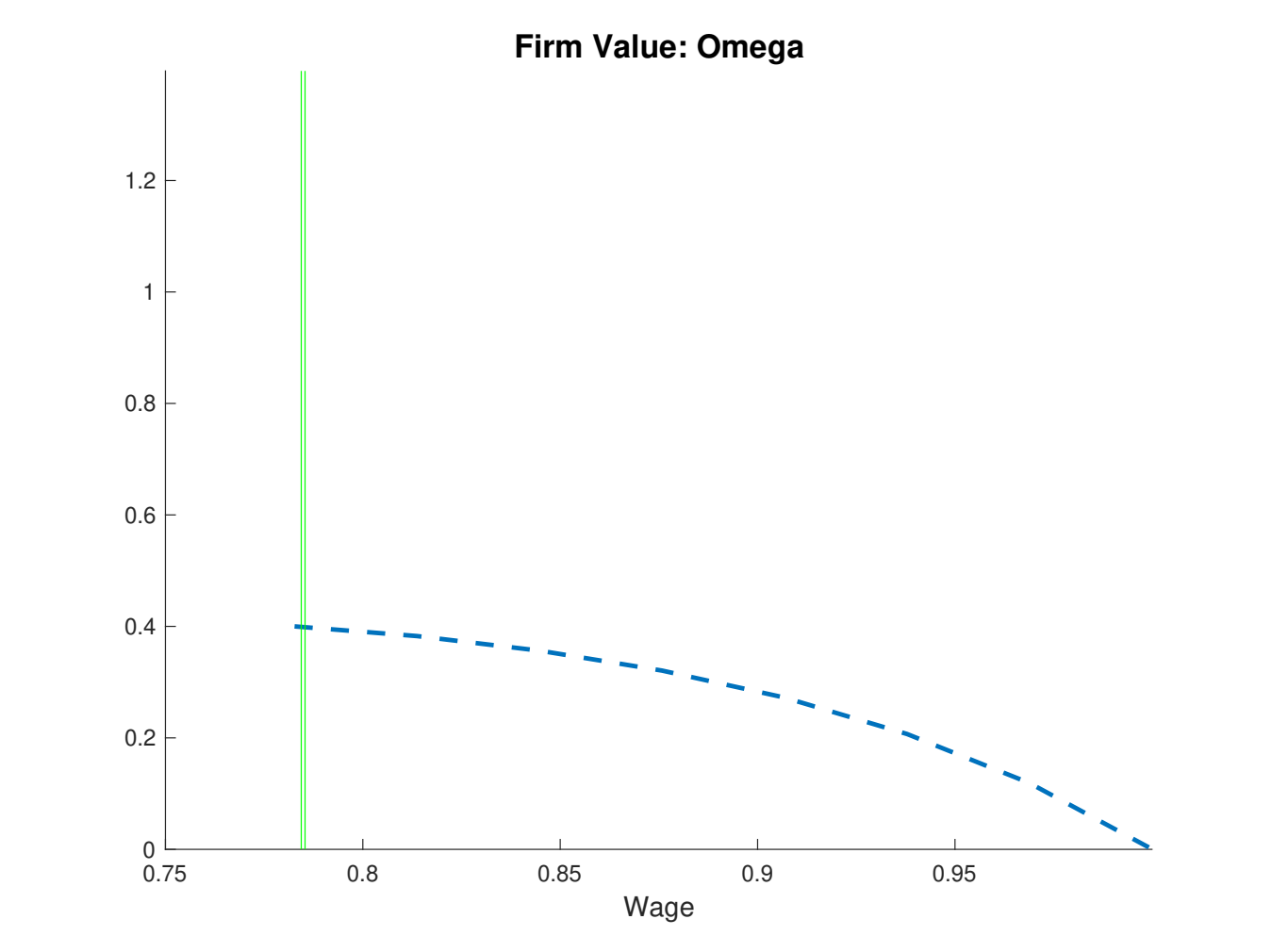
# Value of the firm as wage varies: The Poor

- For the poorest, employment duration increases when wage goes up.
- Firms value is increasing in the wage



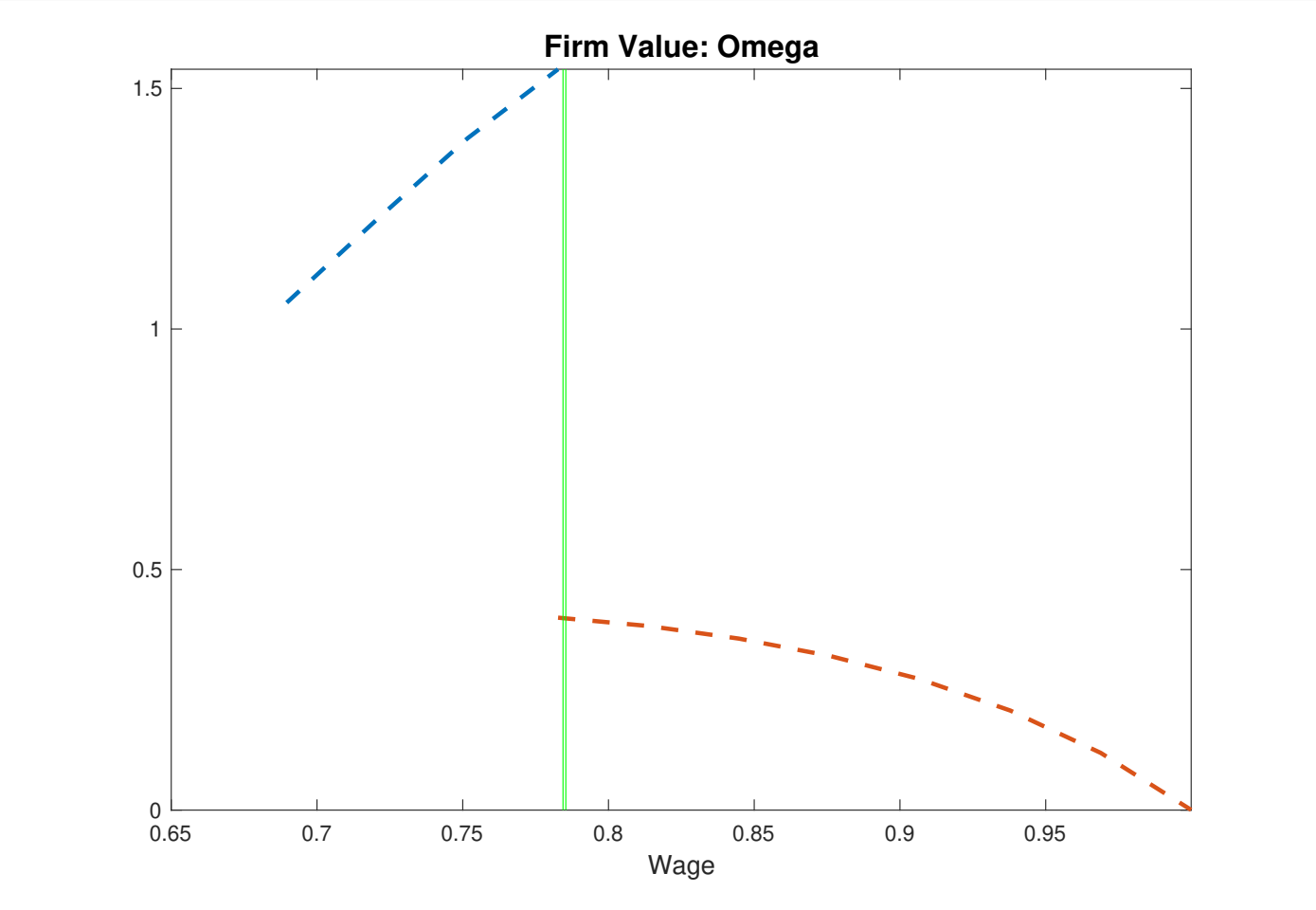
# Value of the firm as wage varies: The Rich

- For the richest, employment duration increases but not fast enough.
- Firm value is slowly decreasing in wages (less than static profits).



# Value of the firm: Accounting for Worker Selection

- Large drop from below to above equilibrium wages.
- In Equilibrium wage dispersion COLLAPSES due to selection.




- So be it.

# Effect of Quitting: The Mechanism

- Two forces shape the dispersion of wages
  - Agents quit less at higher paid jobs, which enlarge the spectrum of wages that firms are willing to pay (for a given range of vacancy filling probability).
  - However, by paying higher wages, firms attract workers with more wealth.
- Wealthy people quit more often, shrink employment duration.
- In equilibrium, the wage gap is narrow and the effect of wealth dominates.
- Need to weaken link between wages and wealth but not today.

# On the Job Search

# On the Job Search Model: Time-line

1. Workers enter period with or without a job: $V^e, V^u$.
2. Production & Consumption:
3. Exogenous Separation
4. Quitting? Searching? Neither?: Employed draw shocks $(\epsilon^e, \epsilon^u, \epsilon^s)$ and make decision to quit, search, or neither. Those who quit become $u'$, those who search join the $u$, in case of finding a job become $\{e', w'\}$ but in case of no job finding remain $e'$ with the same wage $w$ and those who neither become $e'$ with $w$. $\widehat{V}^E(a', w)$, is determined with respect to this stage.
5. Search : Potential firms decide whether to enter and if so, the market $(w)$ at which to post a vacancy; $u$ and $s$ assess the value of all wage applying options, receive match specific shocks $\{\epsilon^{w'}\}$ and choose the wage level $w'$ to apply. Those who successfully find jobs become e', otherwise become u'.
6. $\widehat{V}^u(a'), \{\Omega^j(w)\}$ are determined with respect to this stage.
7. Match

- After saving, the unemployed problem is

$$\widehat{V}^u(a') = \int \max_{w'} \left[ \psi^h(w') V^e(a', w') + (1 - \psi^h(w')) V^u(a') + \epsilon^{w'} \right] \, dF^\epsilon$$

- After saving, the employed choose whether to quit, search or neither

$$\widehat{V}^e(a', w) = \int \max\{ V^e(a', w) + \epsilon^e, V^u(a') + \epsilon^u, V^s(a', w) + \epsilon^s \} dF^\epsilon$$

- The value of searching is

$$V^s(a', w) = \int \max_{w'} \left[ \psi^h(w') V^e(a', w') + [1 - \psi^h(w')] V^e(a', w) + \epsilon^{w'} \right] \, dF^\epsilon$$

# On the Job Search: Household choices

- The probabilities of quitting and of searching

$$q(a', w) = \frac{1}{1 + \exp(\alpha[V^e(a', w) - V^u(a')]) + \exp(\alpha[V^s(a', w) - V^u(a') + \mu^s])},$$
$$s(a', w) = \frac{1}{1 + \exp(\alpha[V^u(a') - V^s(a', w)]) + \exp(\alpha[V^e(a', w) - V^s(a', w) - \mu^s])}.$$

$\mu^s < 0$ is the mode of the shock $\epsilon^s$ which reflects the search cost.

- Households solve

$$V^e(a, w) = \max_{a' \geq 0} u[a(1 + r) + w - a'] + \beta \left[\delta V^u(a') + (1 - \delta)\widehat{V}^e(a', w)\right]$$

$$V^u(a) = \max_{c, a' \geq 0} u[a(1 + r) + b - a'] + \beta \widehat{V}^u(a')$$

- The value of the firm is again given like in the Quitting Model

$$\Omega^0(w) = (z - w - \delta^k k)\, Q^1(w) + (1 - \delta - \delta_k)k\, Q^0(w),$$

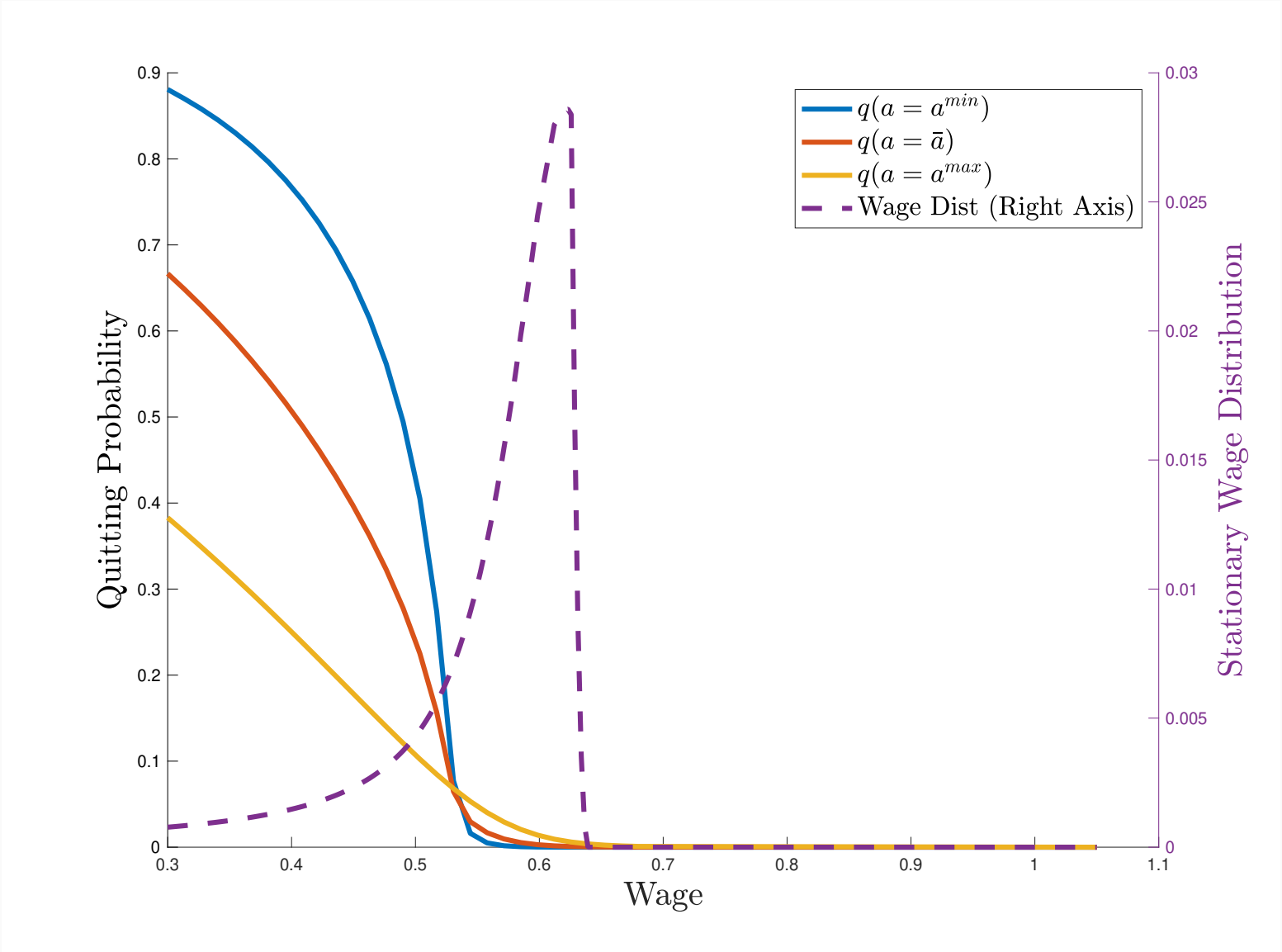
$$Q^1(w) = 1 + \sum_{\tau=0}^{\infty} \left[ \left( \frac{1-\delta}{1+r} \right)^{1+\tau} \prod_{i=0}^{\tau} \ell^i(w) \right],$$

$$Q^0(w) = \sum_{\tau=0}^{\infty} \left[ \left( \frac{1-\delta}{1+r} \right)^{1+\tau} [1 - \ell^\tau(w)] \left( \prod_{i=0}^{\tau-1} \ell^i(w) \right) \right].$$

- Except that now the probability of keeping a worker after $j$ periods is

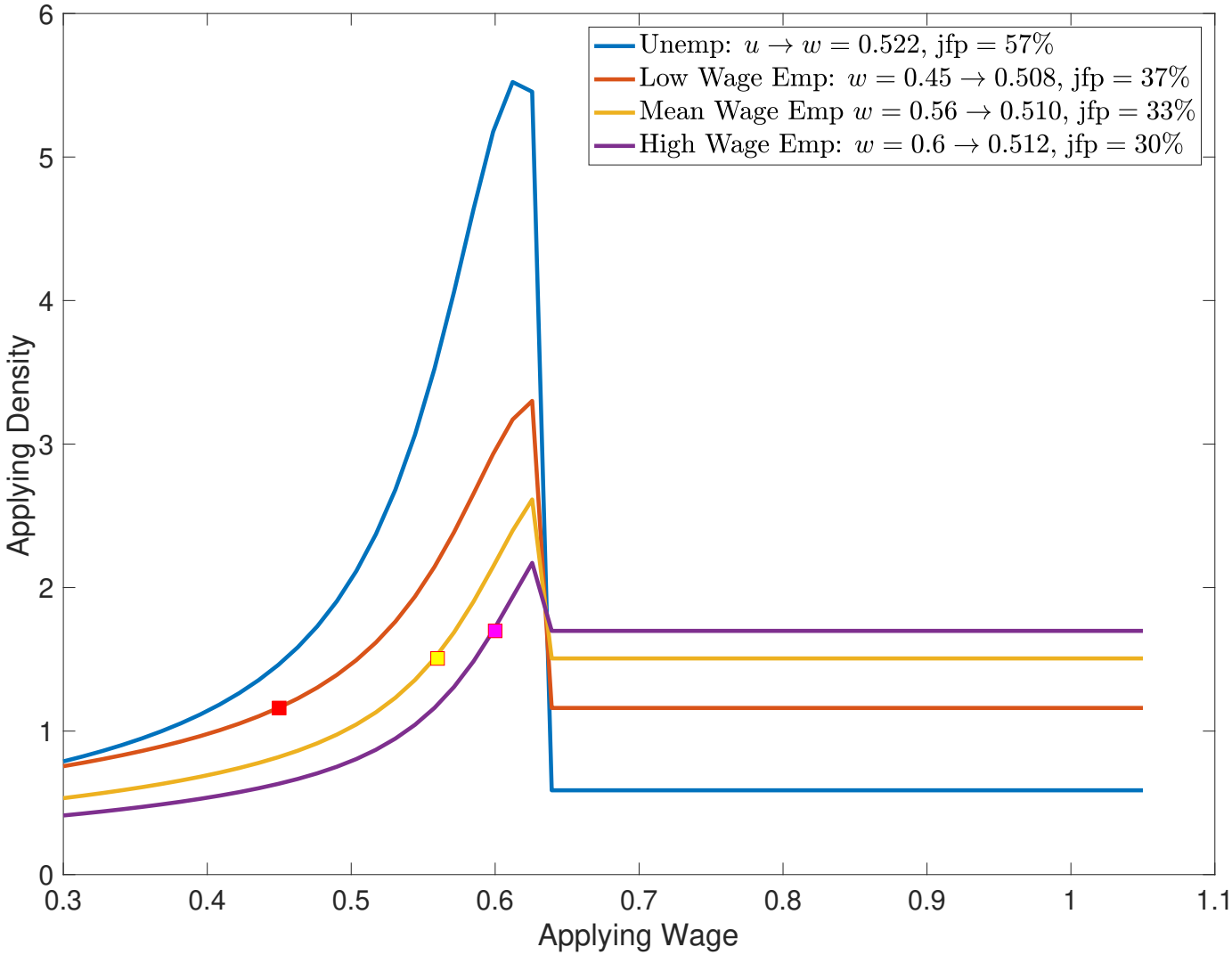
$$\ell^j(w) = 1 - \int h(w;a)\, q[g^{e,j}(a,w), w]\, dx^u(a) -$$
$$\int h(w;a)\, s[w; g^{e,j}(a,w)] \left[ \int \hat{h}[\widetilde{w}; g^{e,j}(a,w), w] \xi \phi^h(\widetilde{w})\, d(\widetilde{w}) \right] dx^u(a)$$

# OJS Quitting Probabilities, Various wealths & Wage Density

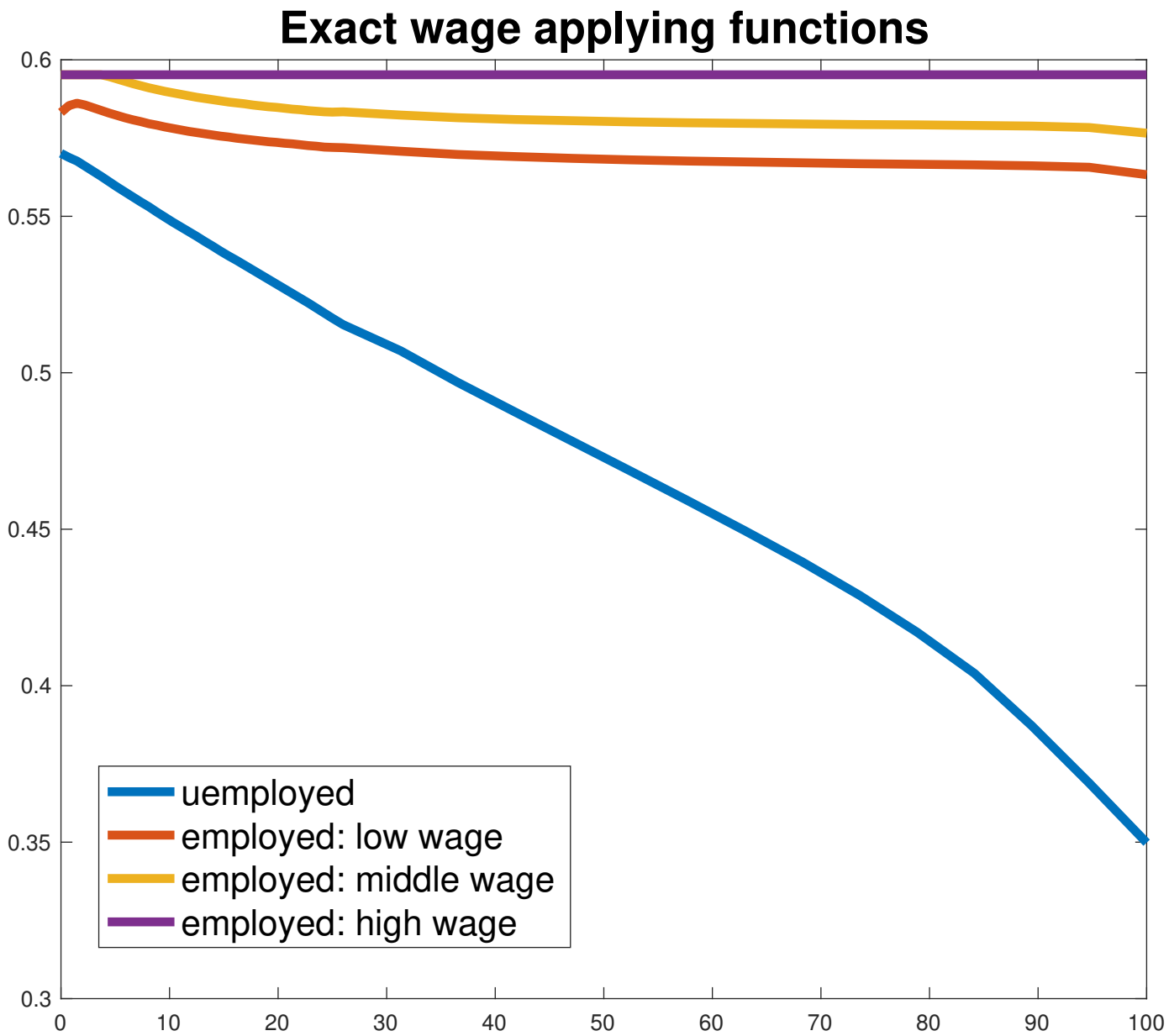



- The rich pursue often other activities (leisure?)

# OJS Which Jobs to Move to?



# OJS Exact Wages to Apply



# OJS Summary of Findings: Mediocre

- The rich pursue often other activities (leisure?)
- Unemployed get jobs faster than searchers
- But $\cdots$ to higher wages
- Higher wage guys move more and to higher wages than lower wage
- But to lower wages than their own
- Excessive quitting in expansions: Easy to come back. Quit to take advantage of a *vacation* a temporary non working opportunity.
- We are redefining the role of extreme value shocks so that searching for almost impossible to find jobs is not rewarding (t)
- Extend to types differ in value of leisure: Outside labor force.

# On the Job Search Model: Equilibrium Properties

- Some good Properties
  - Low wage workers move more often than high wage workers
  - Low wage workers move to lower wages than high wage workers
- Still some unattractive properties
  - Unemployed apply to higher wages than employed.
    - We think that it is an artifact of the way aiming shocks enter: too much wait in the application process and not in the outcome. We are now changing the process of how to implement these shocks.
  - There is excessive quitting in expansions because it is easy to come back. All quitting is to take advantage of a *vacation* a temporary non working opportunit.
    - We propose an extension where some quitting is due to a more permanent switch into a low attachment stage (retirement, schooling, parenting). Business cycles are less tempting to quite: A model of multiple types that differ in leisure valuation. Gives an explicit role to outside the labor force that is not purely temporary.

# Minor Extensions outside Steady State

# Wages move some with the Aggregate State of the Economy

- Wages are indexed to the Aggregate state $z$
- The firm is hard wired to pay not $w$ but

$$w[1 + \gamma(z - 1)]$$

- It will reduce (depending on $\gamma$ the incentive to quit and look for another job in an expansion)
- Very easy to implement
- Same steady state

# Firms choose Search Intensity

- The number of vacancies posted is chosen by firms
- Easy to implement
- Slightly Different steady state

# Free entry with variable recruiting intensity

- Let $\upsilon(\overline{c})$ be a technology to post vacancies where $\overline{c}$ is the cost paid.

- Then the free entry condition requires that for all offered wages

$$0 = \max_{\overline{c}} \left\{ \upsilon(\overline{c})\ \psi^f[\theta(w)] \ \ \frac{\Omega(w)}{1+r} + \left[1 - \upsilon(\overline{c})\ \psi^f[\theta(w)]\right] \ \ \frac{\overline{k}(1-\delta_k)}{1+r} - \overline{c} - \overline{k} \right\},$$

- With FOC given by

$$v_{\overline{c}}(\overline{c}) \ \left\{ \psi^f[\theta(w)] \ \ \left[ \frac{\Omega(w)}{1+r} - \ \ \frac{\overline{k}(1-\delta_k)}{1+r} \right] \right\} = 1,$$

# How to make it consistent with the current steady state

- If $v(\overline{c}) = \frac{\upsilon_1 \overline{c}^2}{2} + \upsilon_2 \overline{c}$, we have

$$(\upsilon_1 \overline{c} + \upsilon_2) \left\{ \psi^f[\theta(w)] \left[ \frac{\Omega(w)}{1+r} - \frac{\overline{k}(1-\delta_k)}{1+r} \right] \right\} = 1,$$

- By Choosing $\upsilon$ so that for the numbers that have now

$$\left\{ \left[ \frac{\upsilon_1 \overline{c}^2}{2} + \upsilon_2 \overline{c} \right] \psi^f[\theta(w)] \frac{\Omega(w)}{1+r} + \left[ 1 - \frac{\upsilon_1 \overline{c}^2}{2} - \upsilon_2 \overline{c} \right] \psi^f[\theta(w)] \frac{\overline{k}(1-\delta_k)}{1+r} \right\} = \overline{c} + \overline{k}$$

- Solving for $\{\upsilon_1, \upsilon_2\}$ that satisfy both equations given our choice of $\overline{c}$ we are done

# Various Economies

- Limited Comparable Results
- Right now we have three Economies
  1. Baseline Not Quitting
  2. Aiming and Not Quitting
  3. An Aiming and Quitting (Closed Economy) General Equilibrium
  4. An Aiming-Quitting & On the Job Search
  5. Same to 4 but with higher $\beta$
- Potential output is Normalized to 1.

# Half-Quarterly Calibration

## In half quarter units

- $K = 3, Y = 1/8, r = 0.37\%$
- firm destruction rate $\delta = 0.36\%$
- job destruction rate $\delta^l = 1.07\%$
- capital maintenance rate $\delta^k = 0.8\%$ from $I/Y = 25\%$.
- $\eta = 0.62$
- $\chi = 0.15$ to match $u = 10\%$.
- $\beta = 0.99928$ (need fine-tune)

# New Calibration to Half-Quarter, in Annual Terms

| | Baseline | Endog Quits | ANQ | AQ | AQOJS |
|---|---|---|---|---|---|
| beta | 0.994 | 0.994 | 0.994 | 0.994 | 0.994 |
| interest rate | 0.030 | 0.030 | 0.030 | 0.030 | 0.030 |
| avg consumption | 0.685 | 0.737 | 0.579 | 0.664 | 0.623 |
| avg wage | 0.705 | 0.730 | 0.578 | 0.674 | 0.637 |
| avg wealth | 2.974 | 5.371 | 1.888 | 3.002 | 1.251 |
| stock market value | 3.026 | 2.739 | 4.683 | 3.353 | 4.040 |
| avg labor income | 0.656 | 0.685 | 0.560 | 0.635 | 0.612 |
| consumption to wealth ratio | 0.230 | 0.137 | 0.306 | 0.221 | 0.498 |
| labor income to wealth ratio | 0.028 | 0.016 | 0.037 | 0.026 | 0.061 |
| quit ratio | 0.086 | 0.041 | 0.086 | 0.044 | 0.052 |
| OJS search ratio | - | - | - | - | 2.000 |
| unemployment rate | 0.120 | 0.104 | 0.062 | 0.104 | 0.076 |
| wage of newly hired unemployed | 0.705 | 0.730 | 0.578 | 0.598 | 0.544 |
| std consumption | 0.014 | 0.018 | 0.010 | 0.015 | 0.010 |
| std wage | 0.001 | 0.000 | 0.010 | 0.003 | 0.010 |
| std wealth | 3.031 | 5.785 | 2.346 | 3.183 | 0.957 |
| mean-min consumption | 2.282 | 2.456 | 1.928 | 2.213 | 2.078 |
| mean-min wage | 1.012 | 1.000 | 1.926 | 2.248 | 2.124 |
| mean-min wealth | Inf | Inf | Inf | Inf | Inf |
| UE transition | 0.118 | 0.080 | 0.126 | 0.083 | 0.093 |
| EE transition | - | - | - | - | 0.280 |
| total vacancy | 0.576 | 0.057 | 2.582 | 0.622 | 2.874 |
| avg unemp duration | 1.012 | 2.302 | 0.446 | 0.932 | 0.781 |
| avg emp duration | 7.469 | 11.159 | 7.469 | 10.844 | 9.920 |
| OJS move rate | - | - | - | - | 0.303 |

# Aggregate Fluctuations

# What is needed?

- Two steps
  1. Compute the TRUE impulse response to an MIT Shock
  2. Use this path as a dynamic linear approximation to generate fluctuations (Boppart, Krusell, and Mitman (2018))

- The transition is a large but doable problem:
  - Firms need to know functions $\{Q_t^0(w), Q_t^1(w), \psi^f(w)\}$ at each stage (no block recursivity)
  - Households need to know $\phi_t^h(w)$ job finding probabilities every period.
  - Also need to know sequence of interest rates (not today)

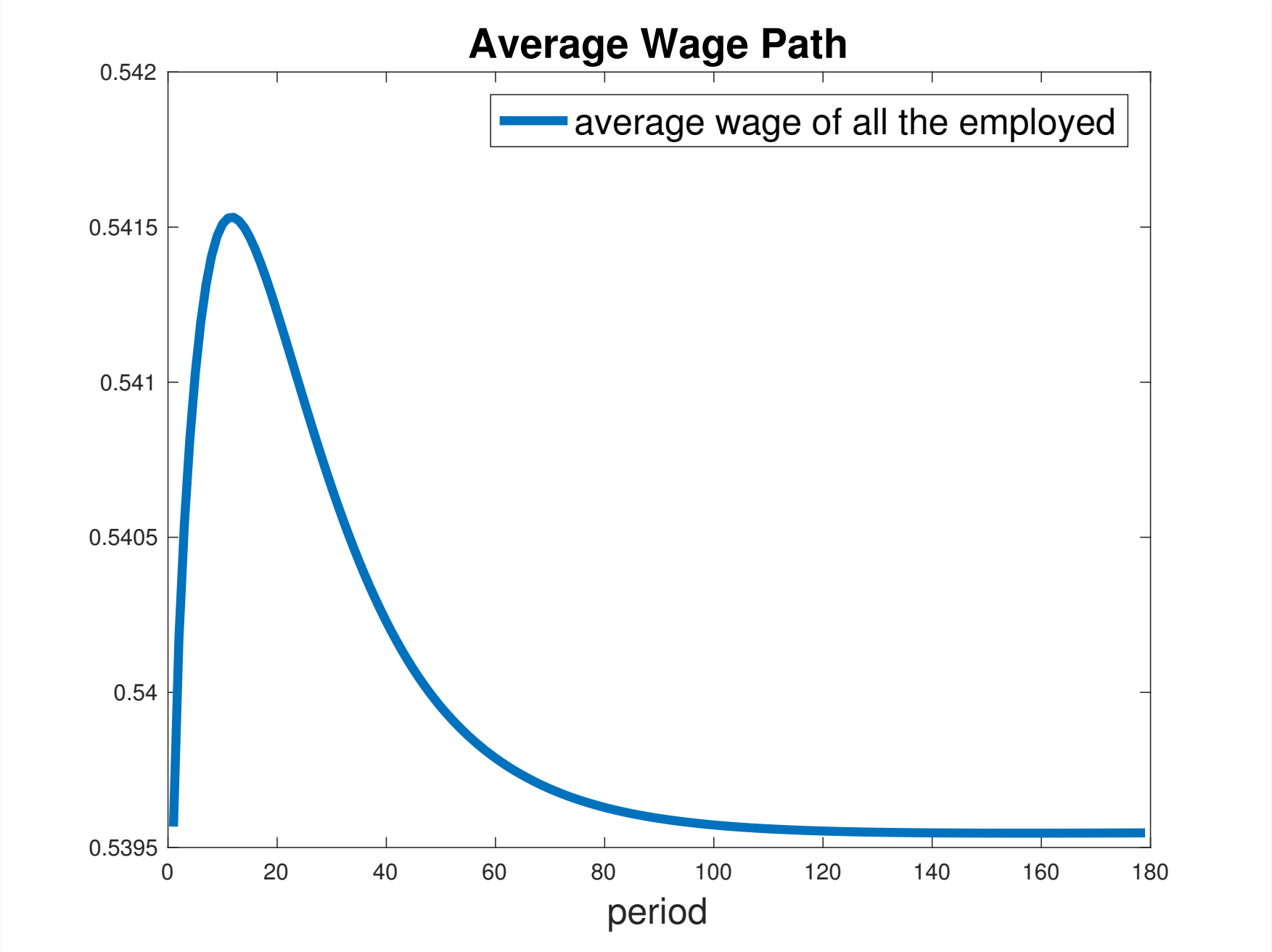
- So it is a second order difference functional equation.

# Aiming and Quitting Model.

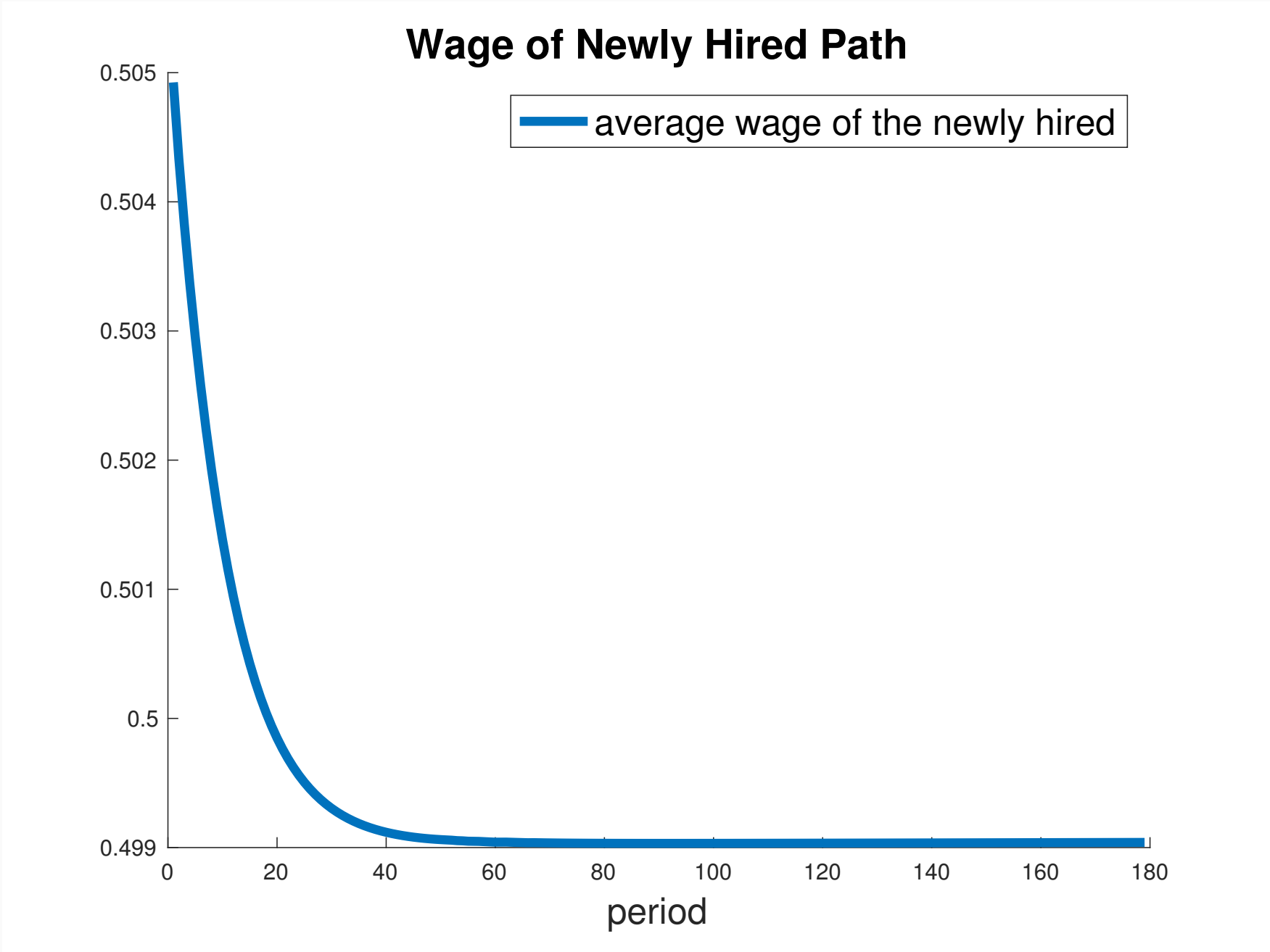
## 5% Productivity Shock ($\rho = .9$)

- Average wages don't move much but wages of new workers do!
- Newly hired Wage Distribution Shifts upward
- Quits are pro-cyclical but excessive
- Employment moves more (not so much of Shimer puzzle)

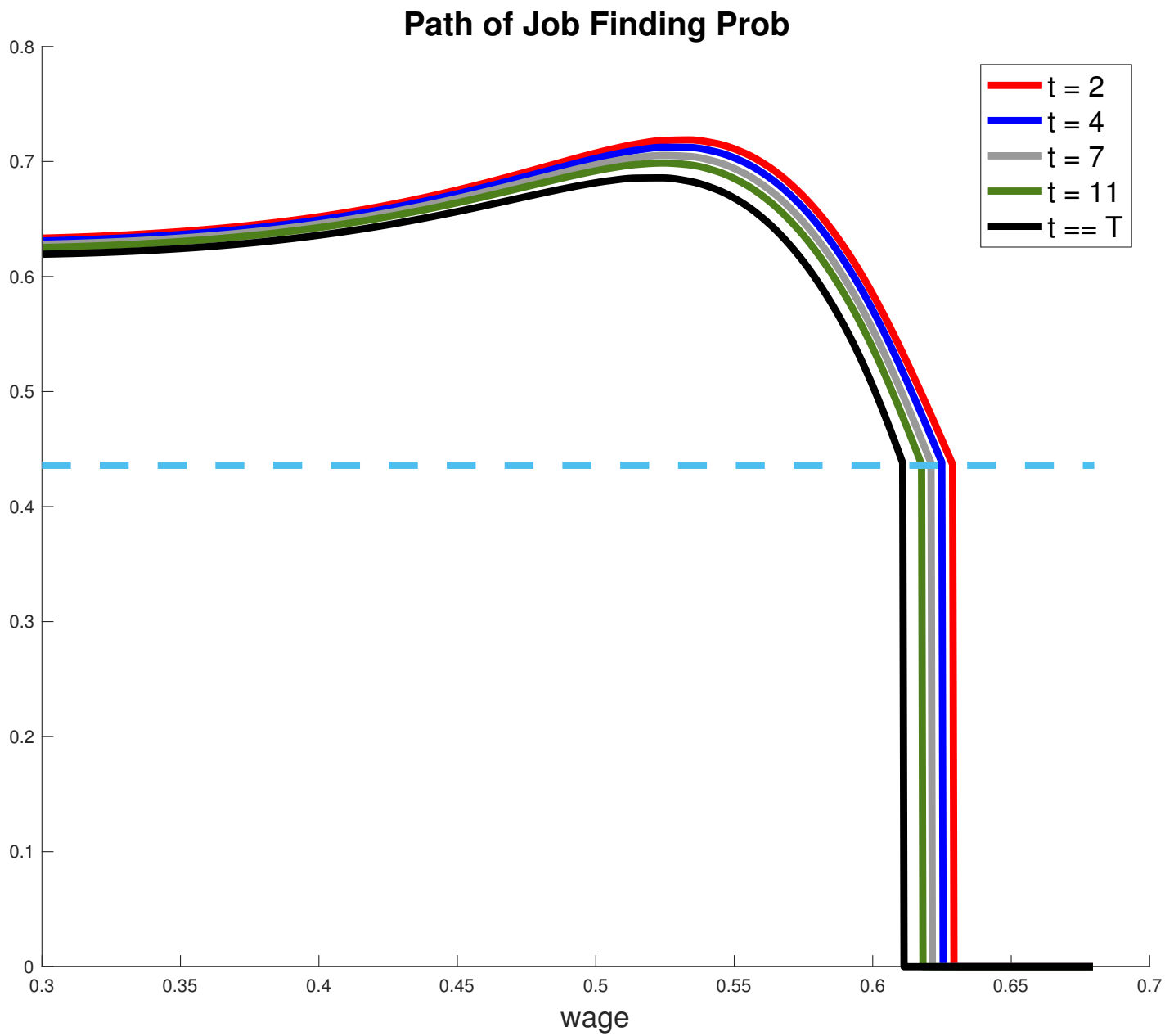
# Aiming and Quitting Model. 5% Productivity Shock ($\rho = .9$)



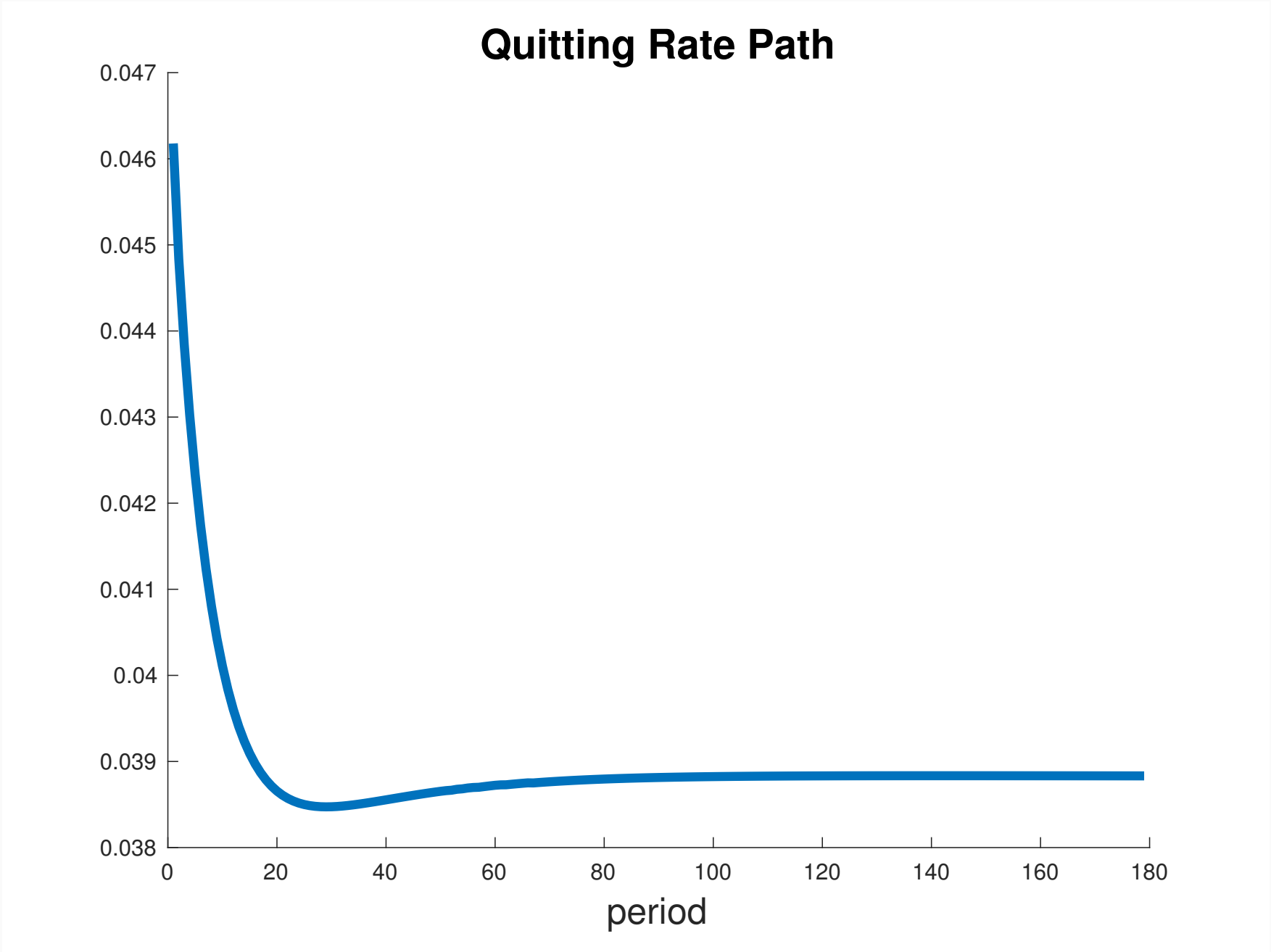
# Aiming and Quitting Model. 5% Productivity Shock ($\rho = .9$)



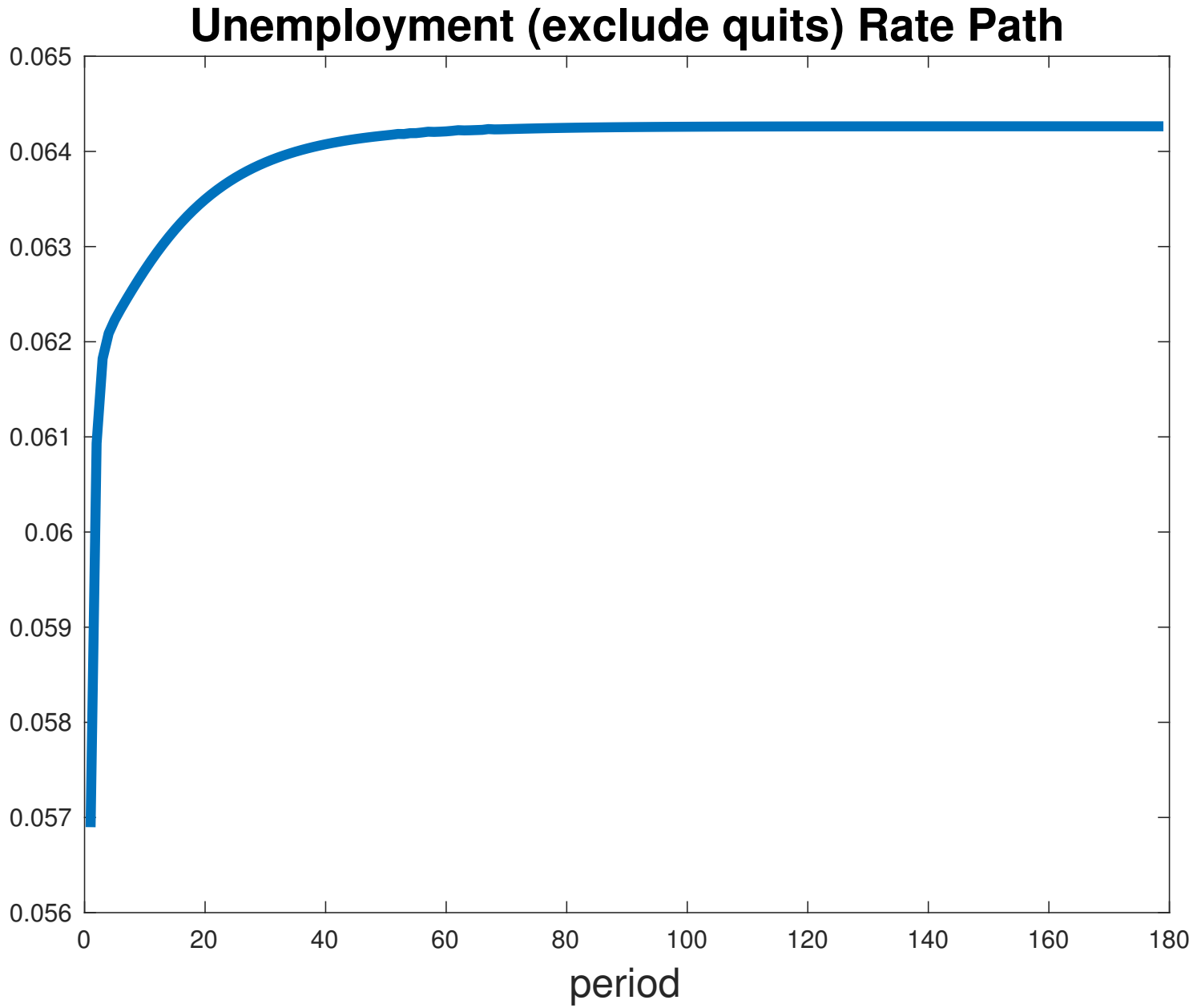
# Aiming and Quitting Model. 5% Productivity Shock ($\rho = .9$)



# Aiming and Quitting Model. 5% Productivity Shock ($\rho = .9$)

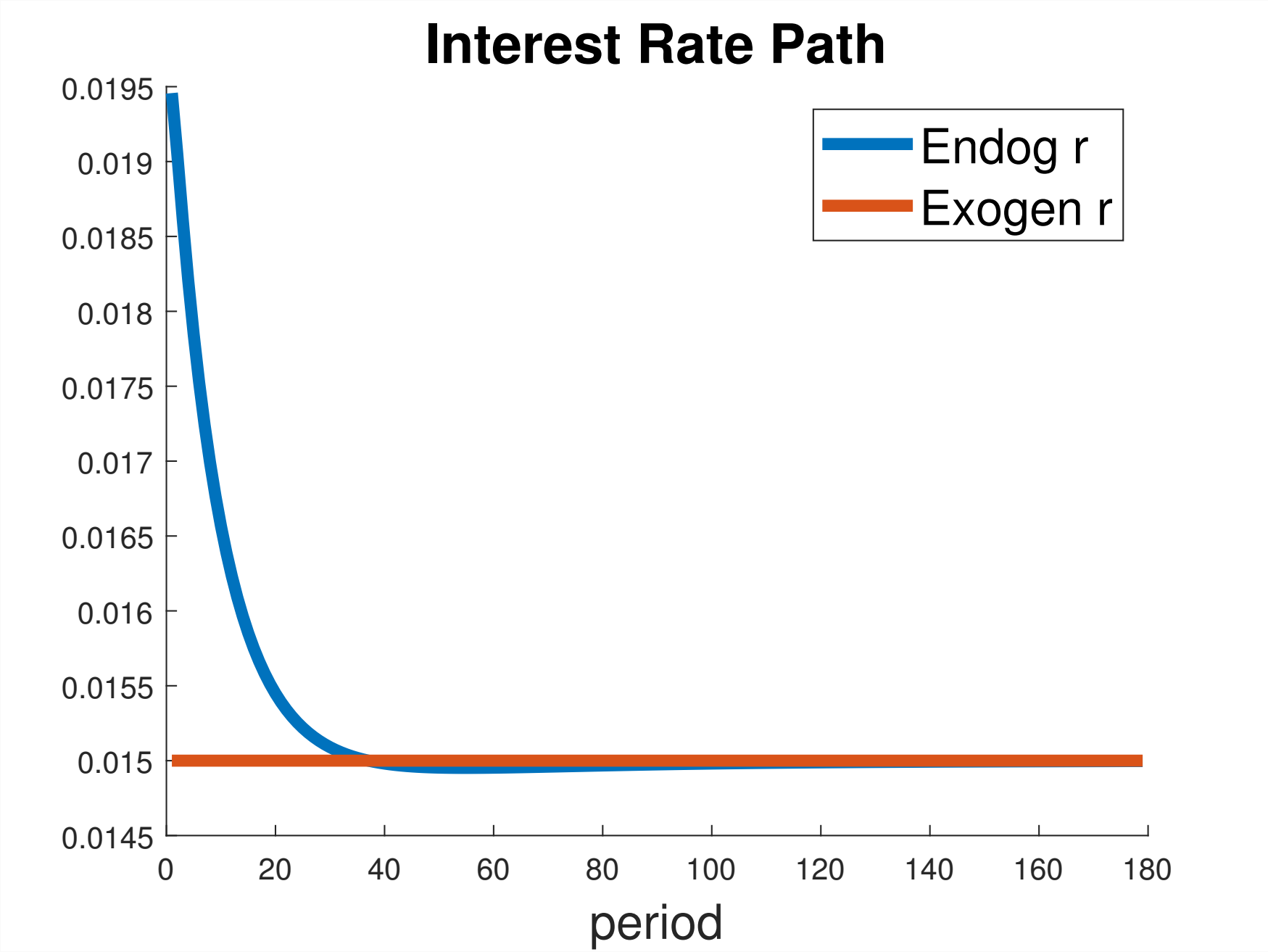


# Aiming and Quitting Model. 5% Productivity Shock ($\rho = .99$)

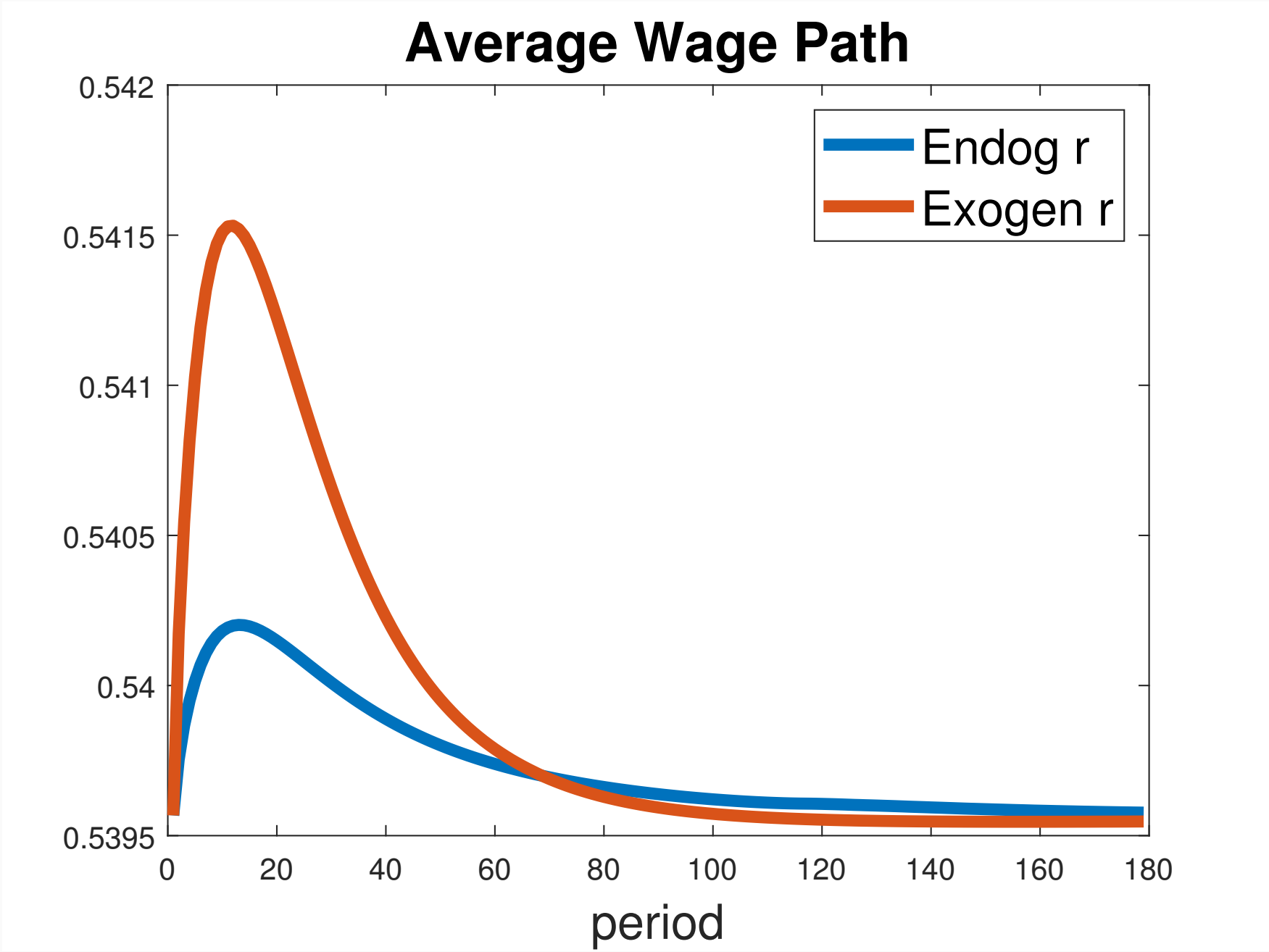


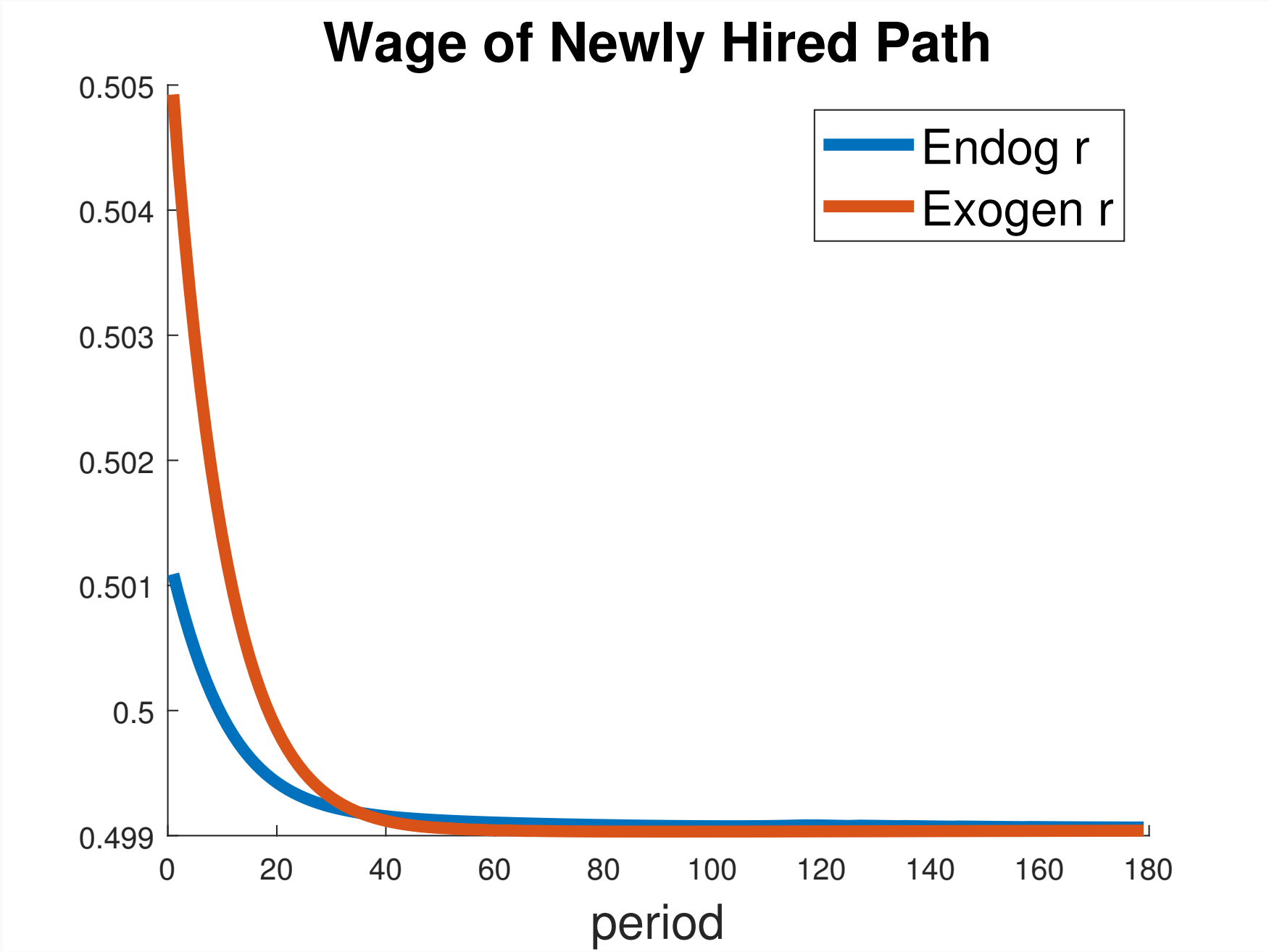
# Aiming and Quitting Model (Endogenous $r$). 5% Productivity Shock ($\rho = .9$)

- Interest rate $r$ goes up endogenously as a response of positive technology shocks
- As a consequence wages and employment move less
- Quits are still pro-cyclical but much less in magnitude
- Massive movements in mutual fund value but little in wages and employment

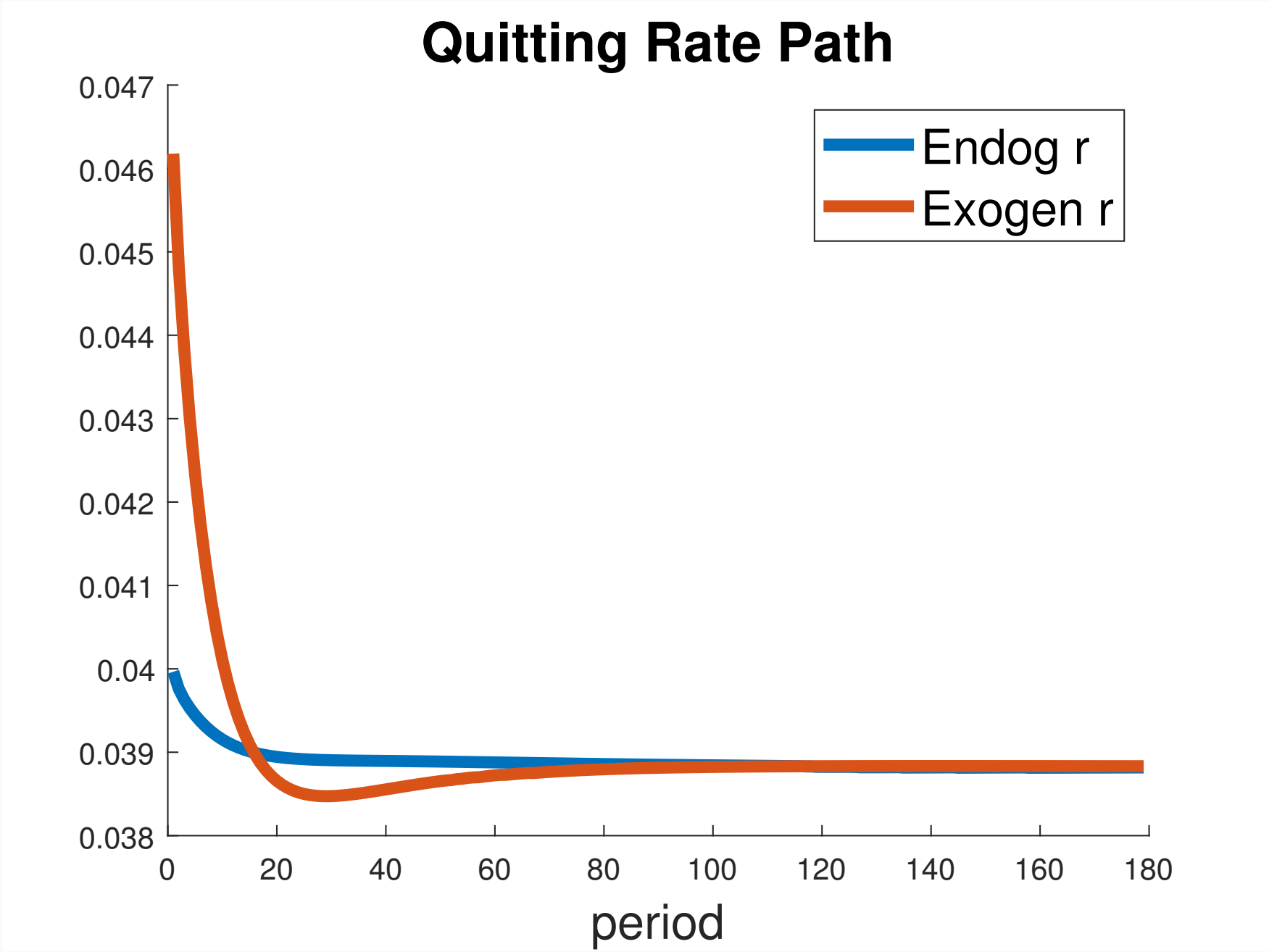

Interest Rate Path
Endog r
Exogen r
0.0195
0.019
0.0185
0.018
0.0175
0.017
0.0165
0.016
0.0155
0.015
0.0145
0
20
40
60
80
100
120
140
160
180
period

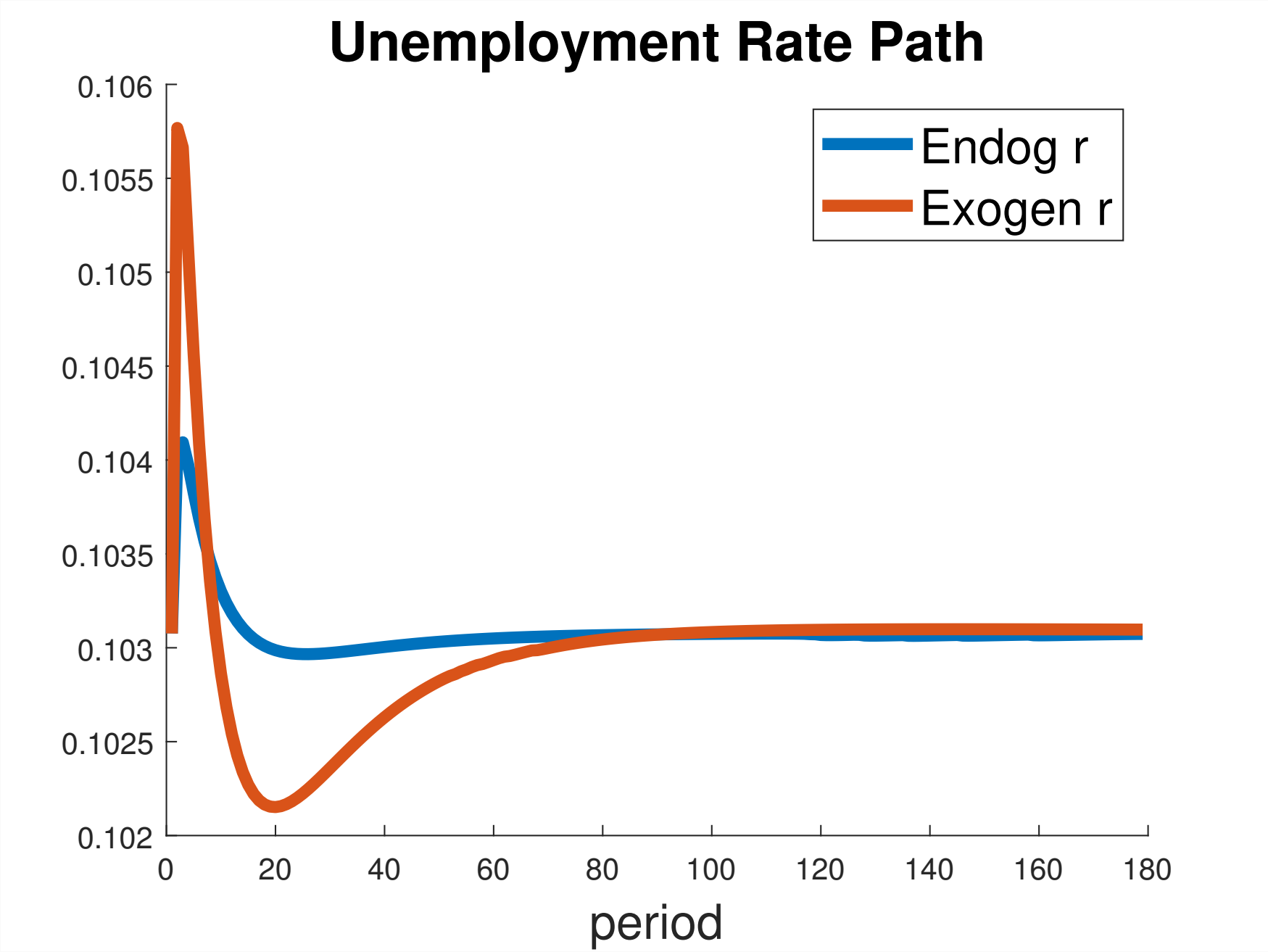
# Aiming and Quitting Model. 5% Productivity Shock ($\rho = .9$)



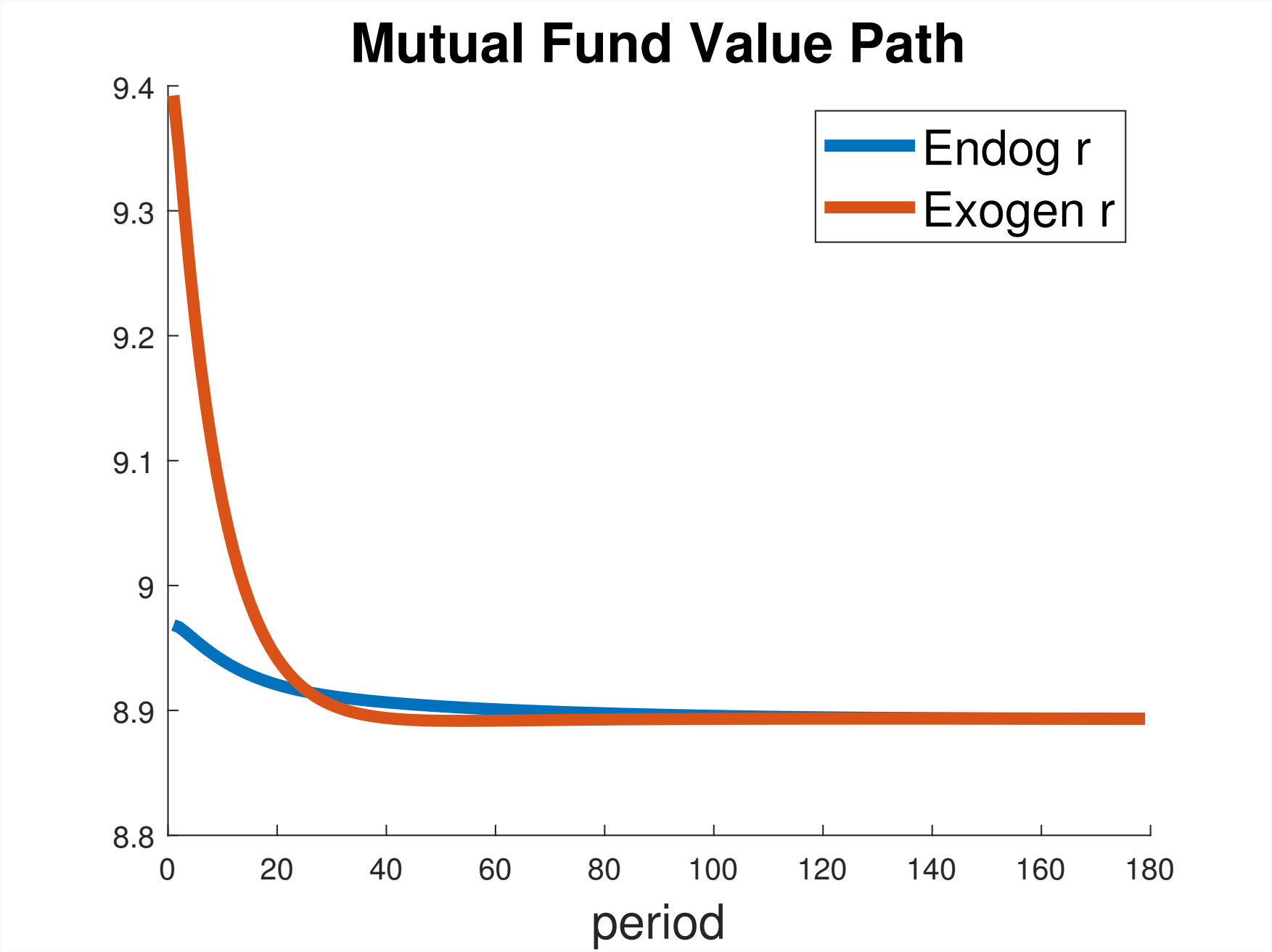

Wage of Newly Hired Path
0.505
0.504
0.503
0.502
0.501
0.5
0.499
0
20
40
60
80
100
120
140
160
180
period
Endog r
Exogen r

# Aiming and Quitting Model. 5% Productivity Shock ($\rho = .9$)




Unemployment Rate Path
Endog r
Exogen r
0.106
0.1055
0.105
0.1045
0.104
0.1035
0.103
0.1025
0.102
0
20
40
60
80
100
120
140
160
180
period

# Aiming and Quitting Model. 5% Productivity Shock ($\rho = .9$)



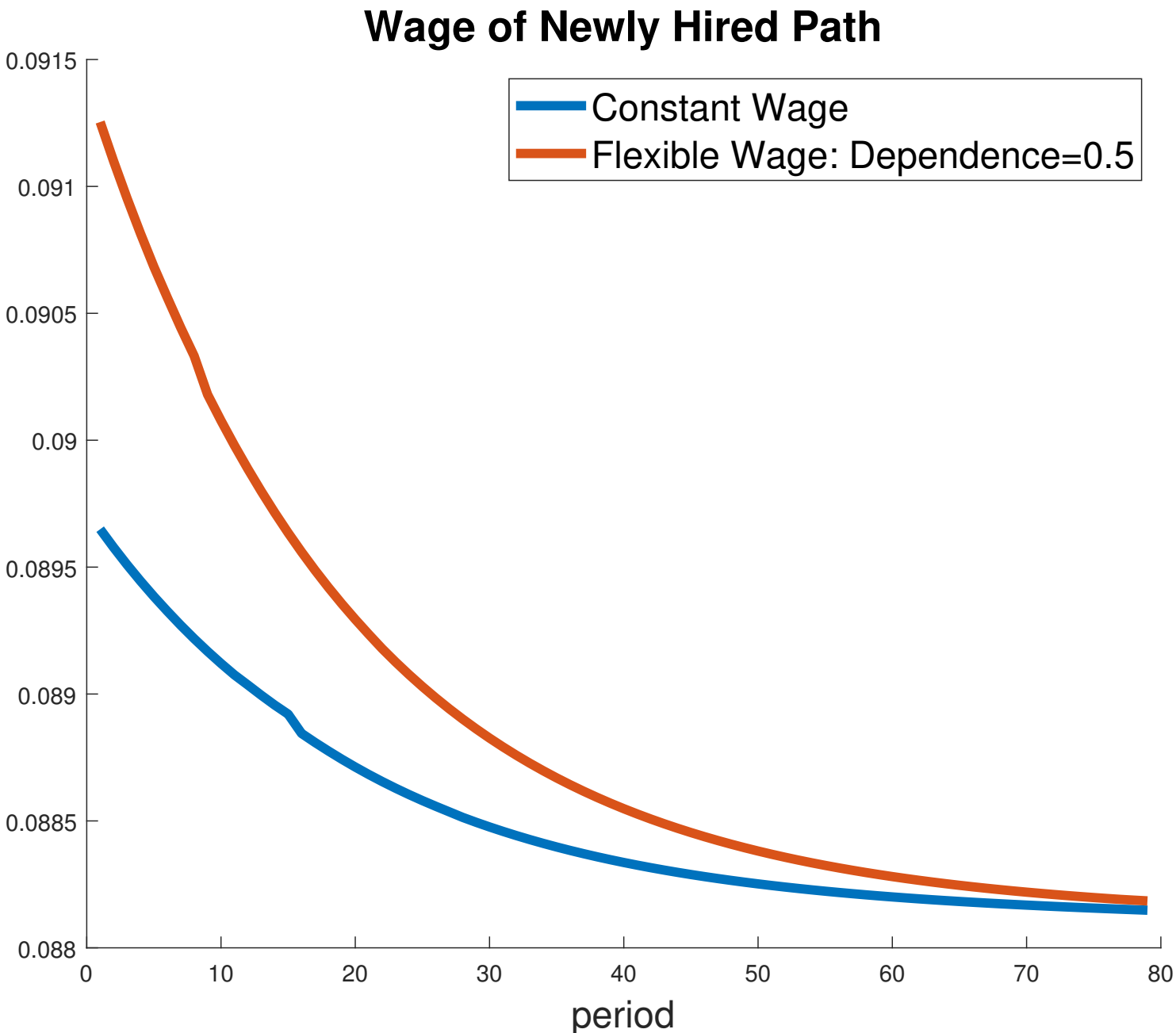
# Constant Wage v.s. Flexible Wage

- For all above we assume the wage is constant once a job match is formed.
- The alternative is to allow wages of existing jobs to fluctuate with business cycles.

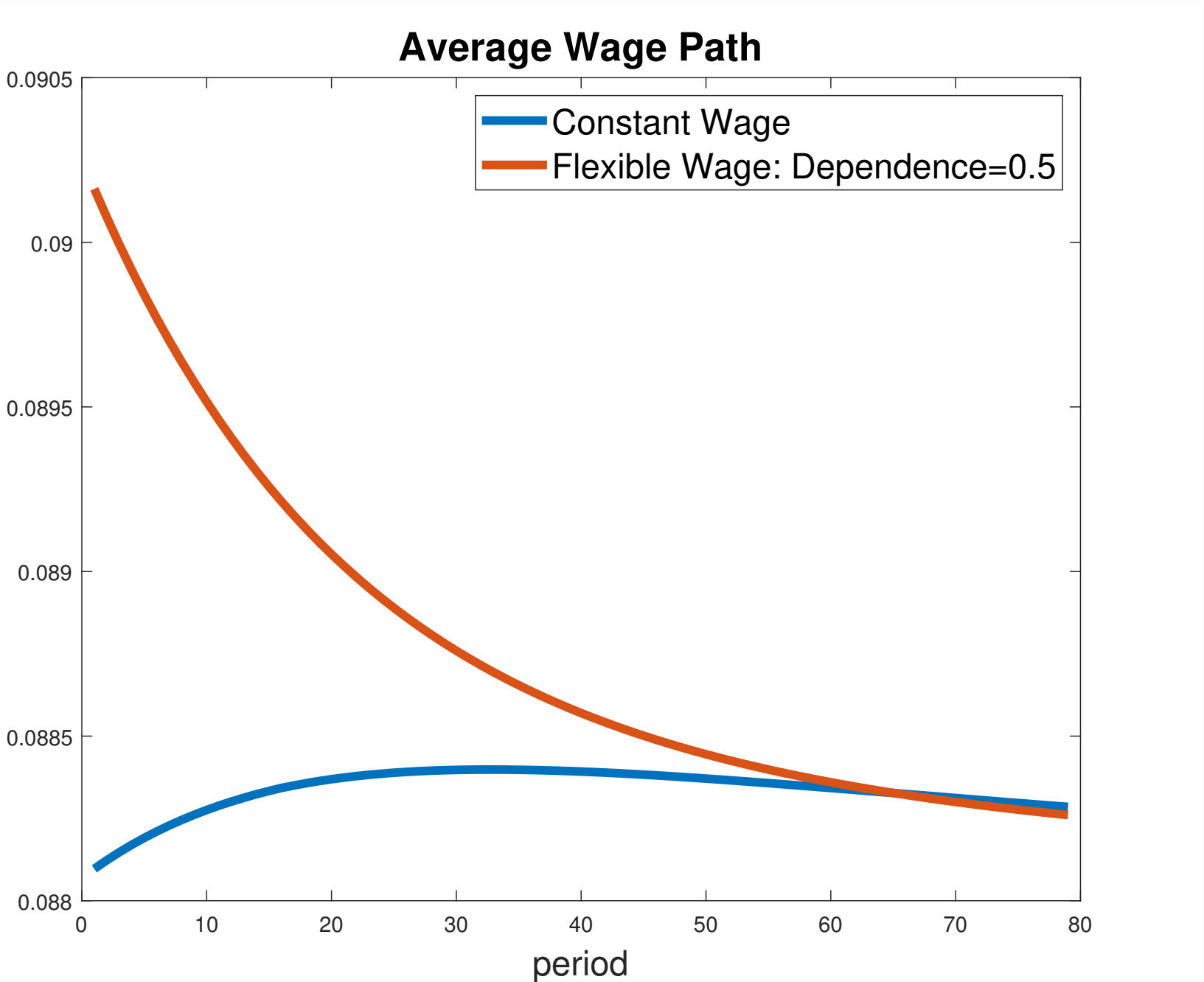
$$w = (1 + \sigma^w z)\bar{w}$$

- For NANQ economy, this does not change employment much.
- For economy with quits, flexible wages rein in the cyclicality of quitting, reducing job loss and generating pro-cyclical employment.

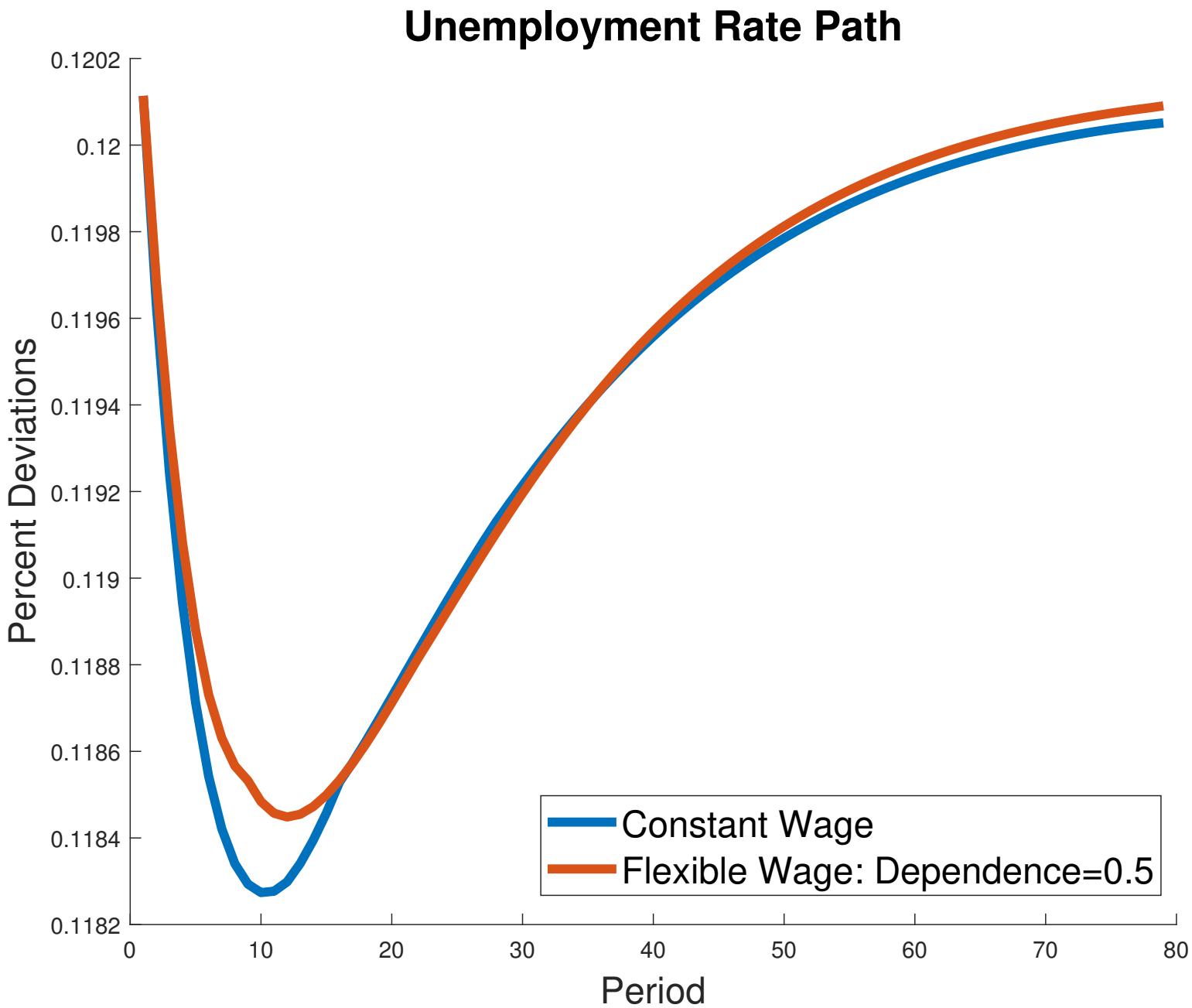
# No Aiming No Quitting. 5% Productivity Shock ($\rho = .95$)



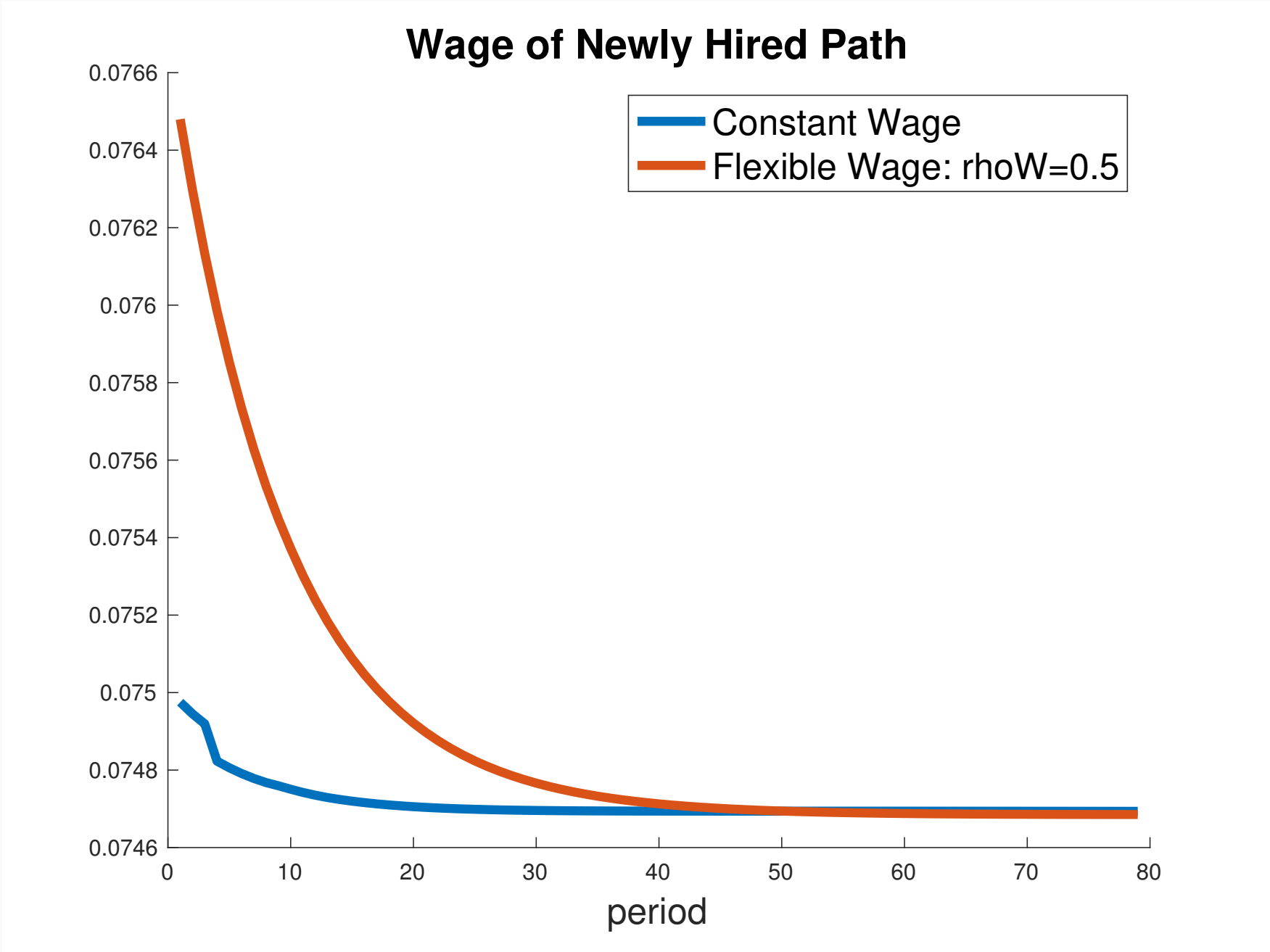
# No Aiming No Quitting. 5% Productivity Shock ($\rho = .95$)



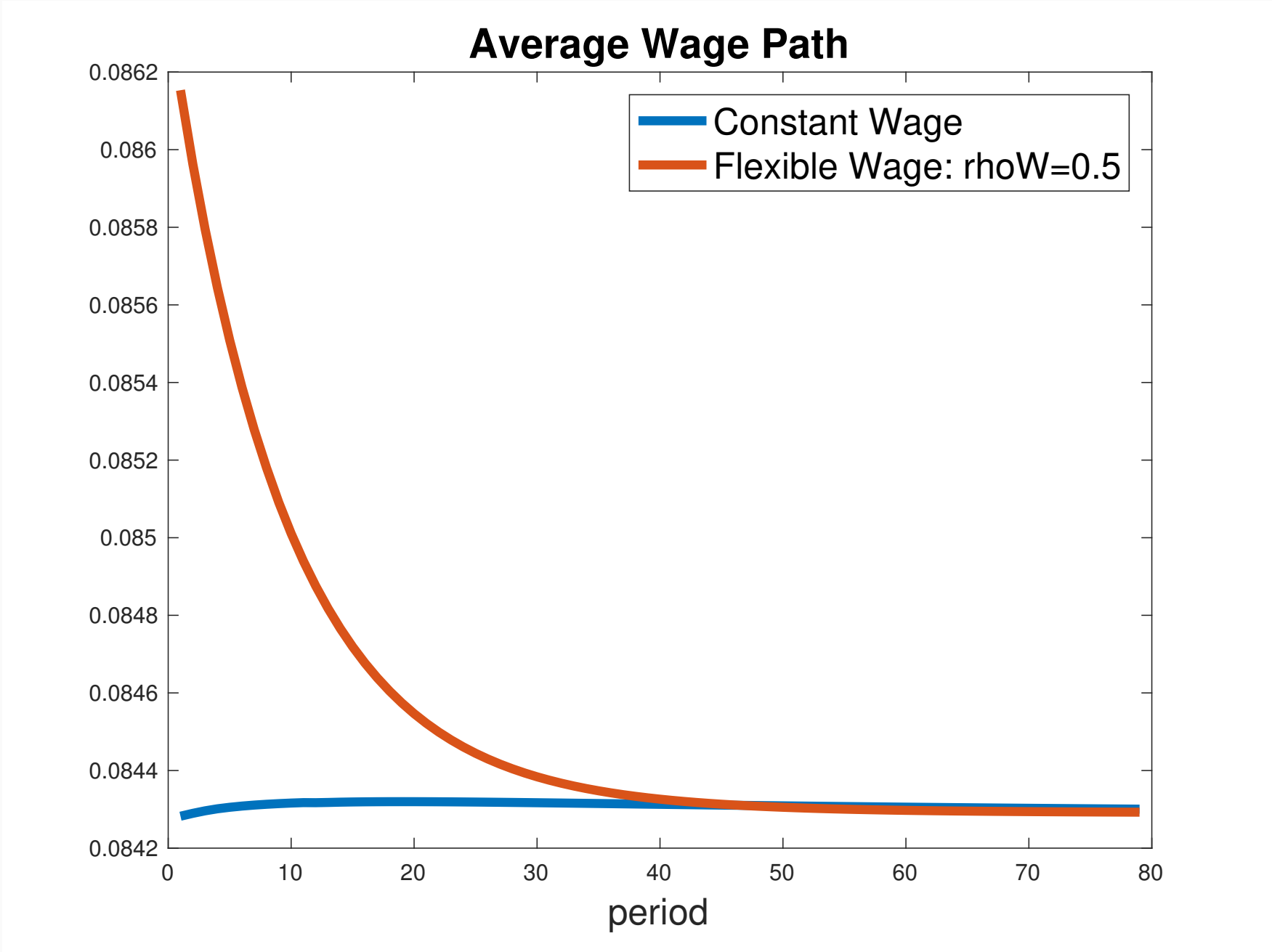
# No Aiming No Quitting. 5% Productivity Shock ($\rho = .95$)



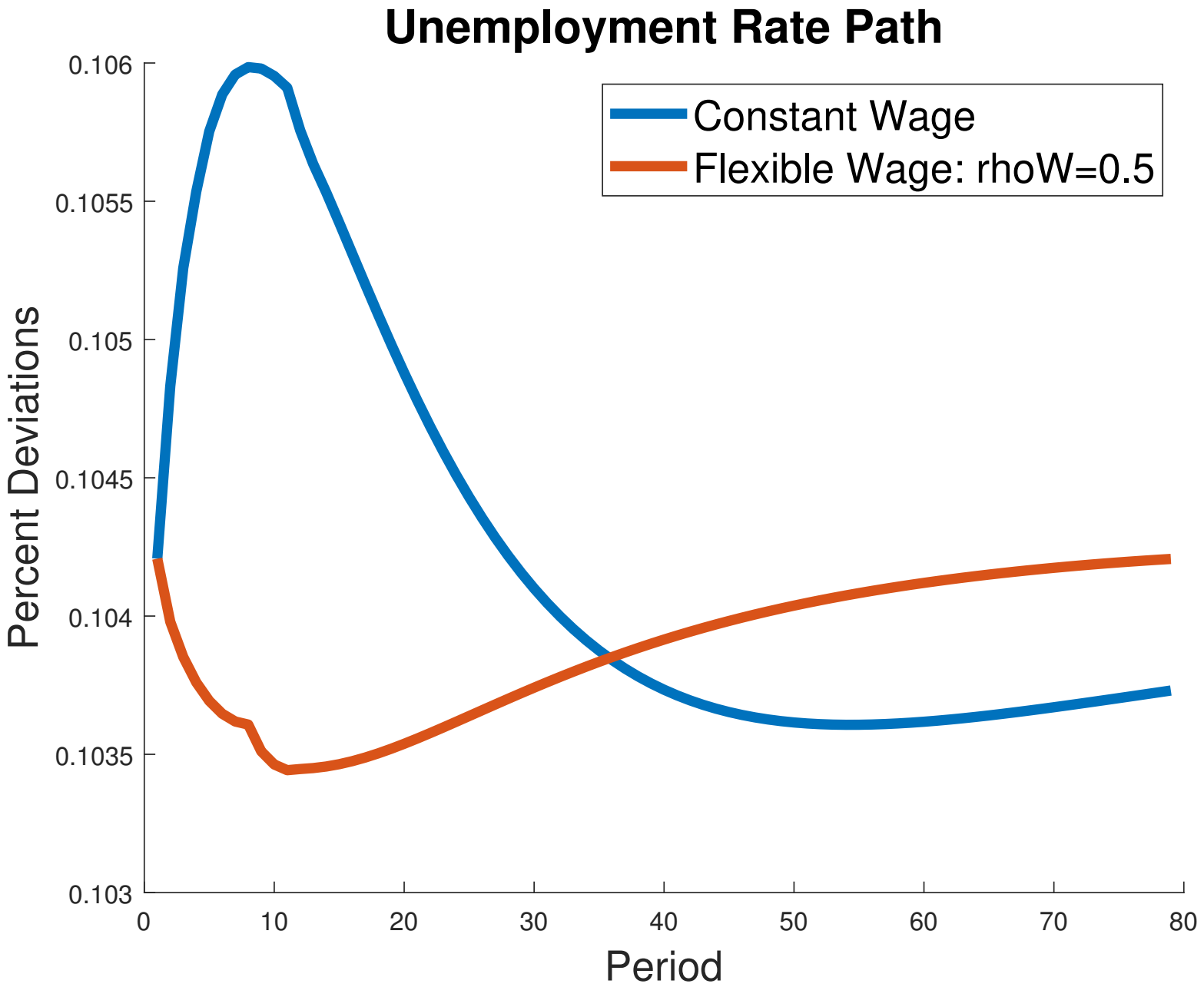
# Aiming and Quitting Model. 5% Productivity Shock ($\rho = .9$)



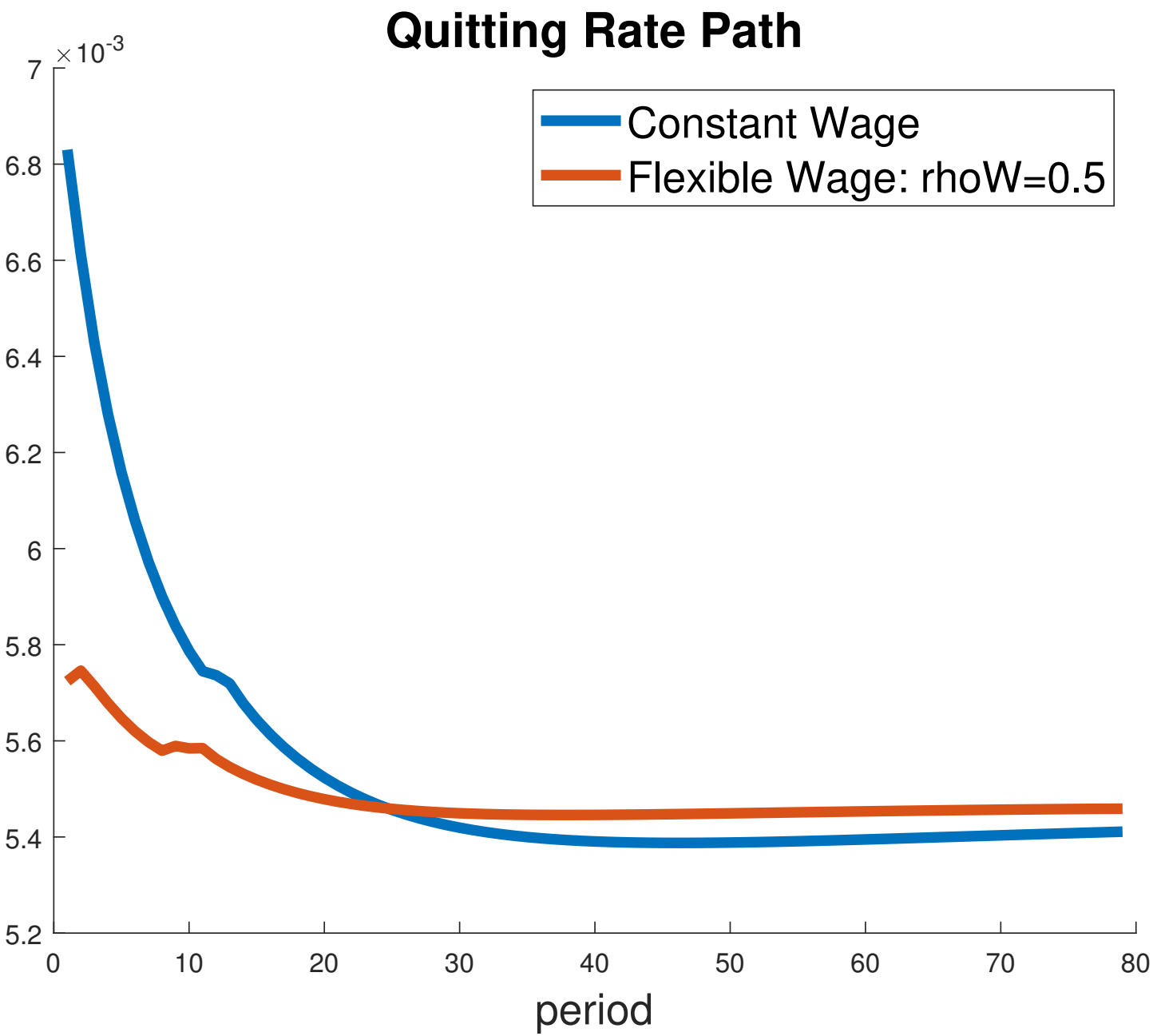
# Aiming and Quitting Model. 5% Productivity Shock ($\rho = .9$)



# Aiming and Quitting Model. 5% Productivity Shock ($\rho = .9$)



# Aiming and Quitting Model. 5% Productivity Shock ($\rho = .9$)

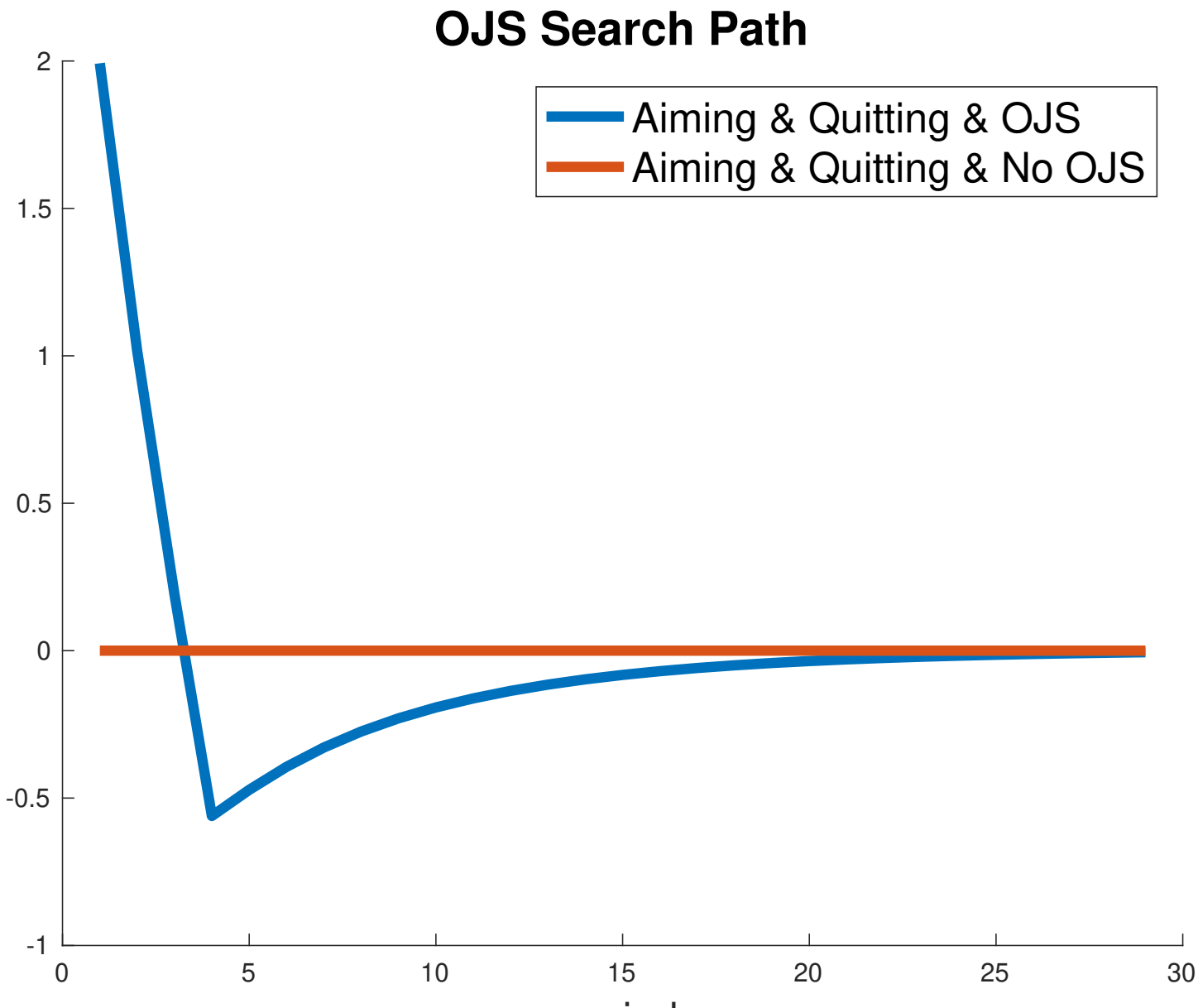


# Business Cycle Behavior of On the Job Search

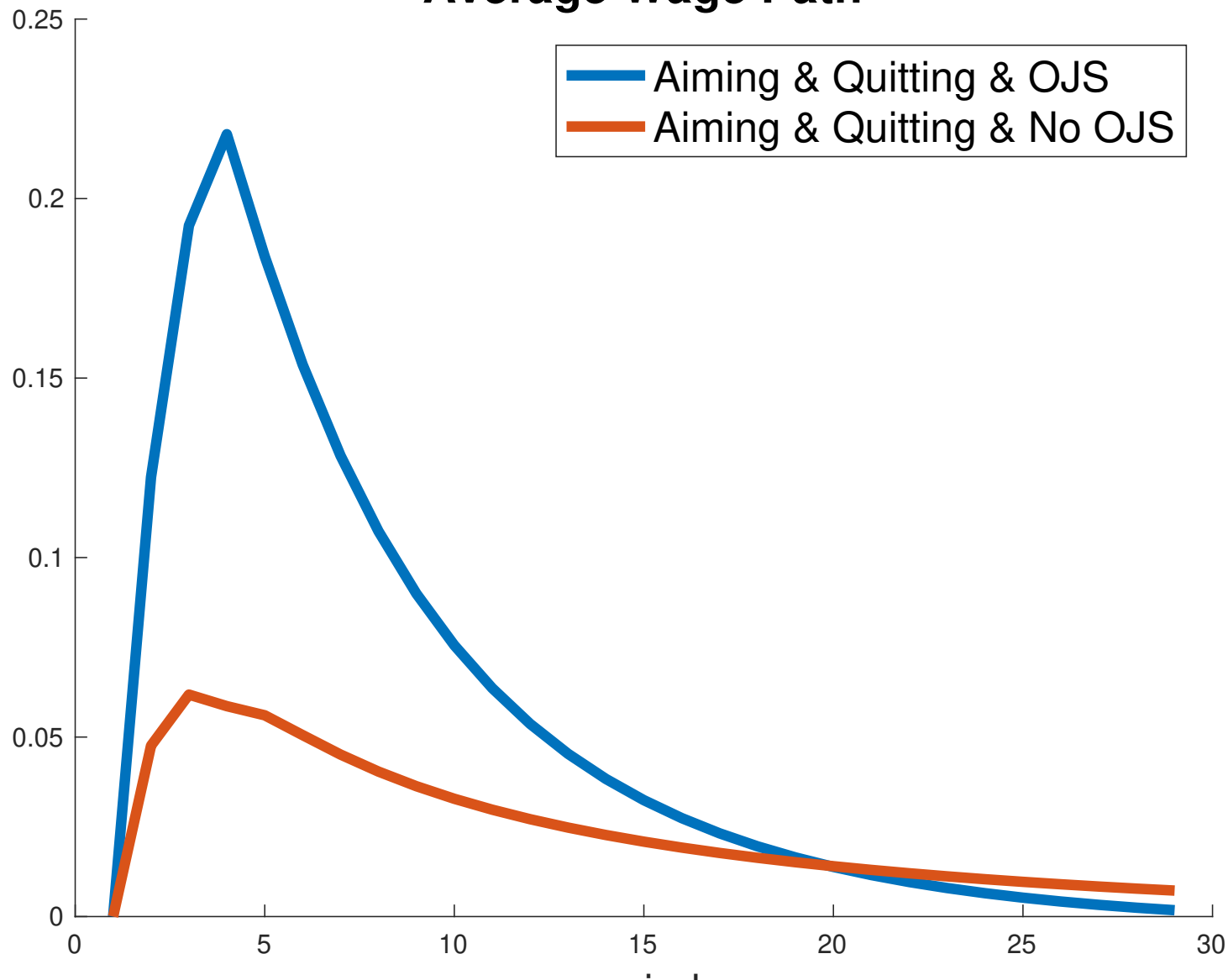
# 5% Produ. Shock ($\rho = .9$) for 5 periods

- Shocks are truncated at $t = 5$
  - Eliminating future shocks reins in the massive initial quits
  - Converge faster and less computational burden
- OJS Switches are Pro-cyclical
- OJS search amplifies the responses of wages and employment

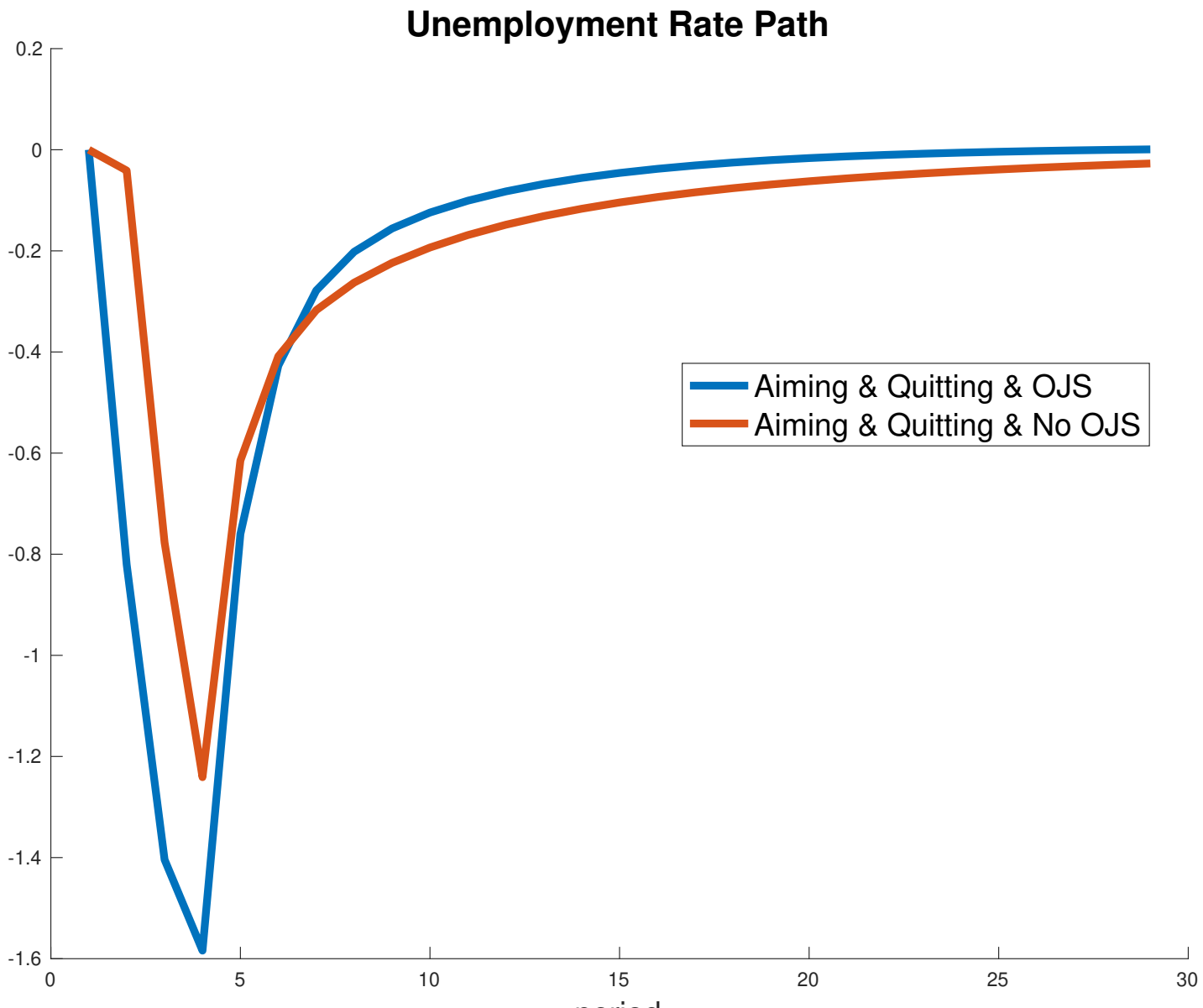
# OJS 5% Productivity Shock ($\rho = .9$, truncated at t=5) OJS Search Rate, Percent Deviations



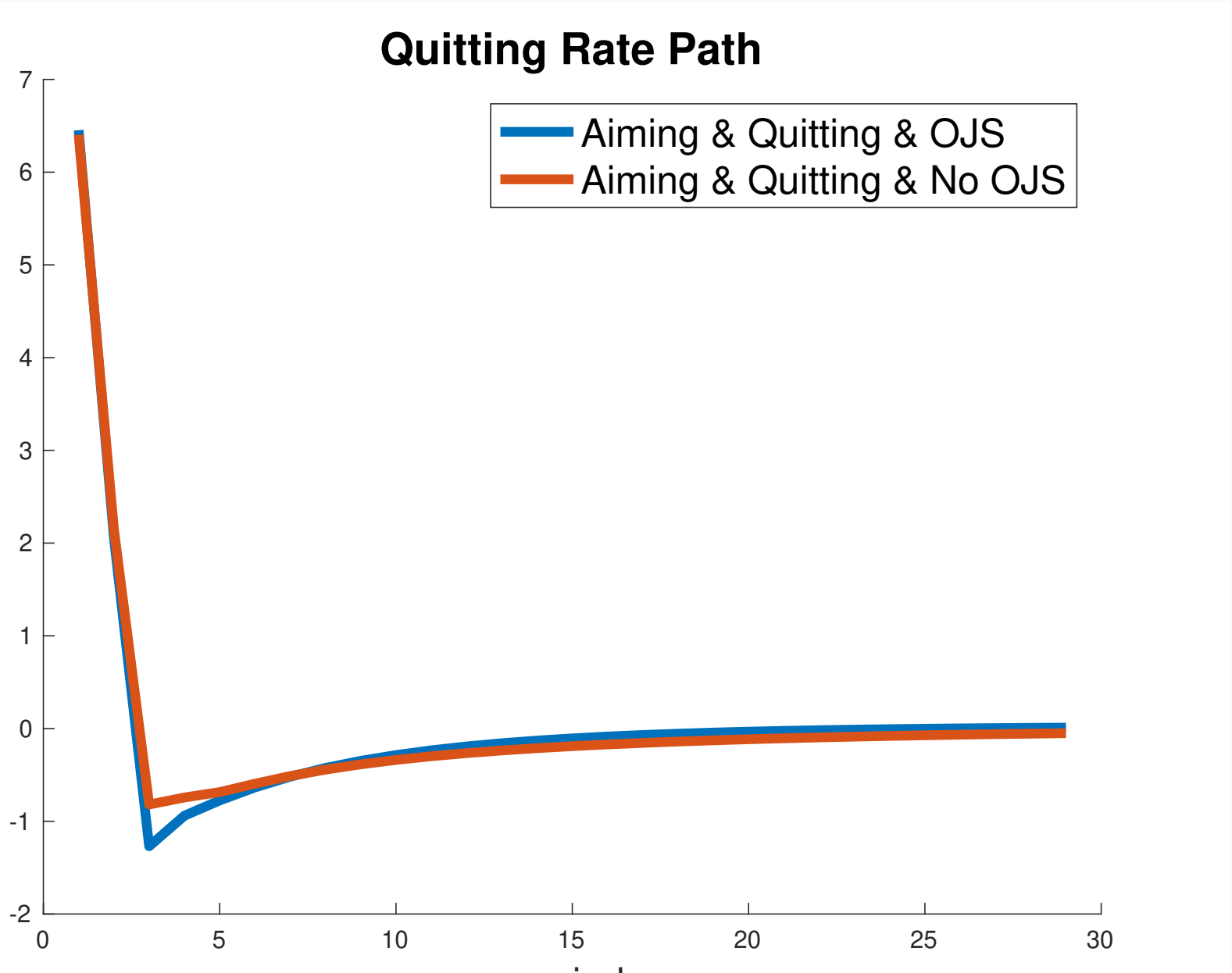
# OJS 5% Productivity Shock ($\rho = .9$, truncated at t=5) Avg Wage, Percent Deviations



# OJS 5% Productivity Shock ($\rho = .9$, truncated at t=5) Unemployment, Percent Deviations



# OJS 5% Productivity Shock ($\rho = .9$, truncated at t=5) Quits, Percent Deviations



# Conclusions

- Develop tools to get a joint theory of wages, employment and wealth that marry the two main branches of modern macro:
  1. Aiyagari models (output, consumption, investment, interest rates)
  2. Labor search models with job creation, turnover, wage determination, flows between employment, unemployment and outside the labor force.
  3. Add tools from Empirical Micro to soften wage-wealth correlations.
- Useful for business cycle analysis: We are getting procyclical
  - Quits
  - Employment after a lag
  - Investment and Consumption
  - But Perhaps Expansions and Recessions Should Arrive Slower
- Exciting set of continuation projects:
  - Efficiency Wages, Endogenous Productivity (firms use different technologies with different costs of idleness)
  - Move towards more sophisticated life cycle movements

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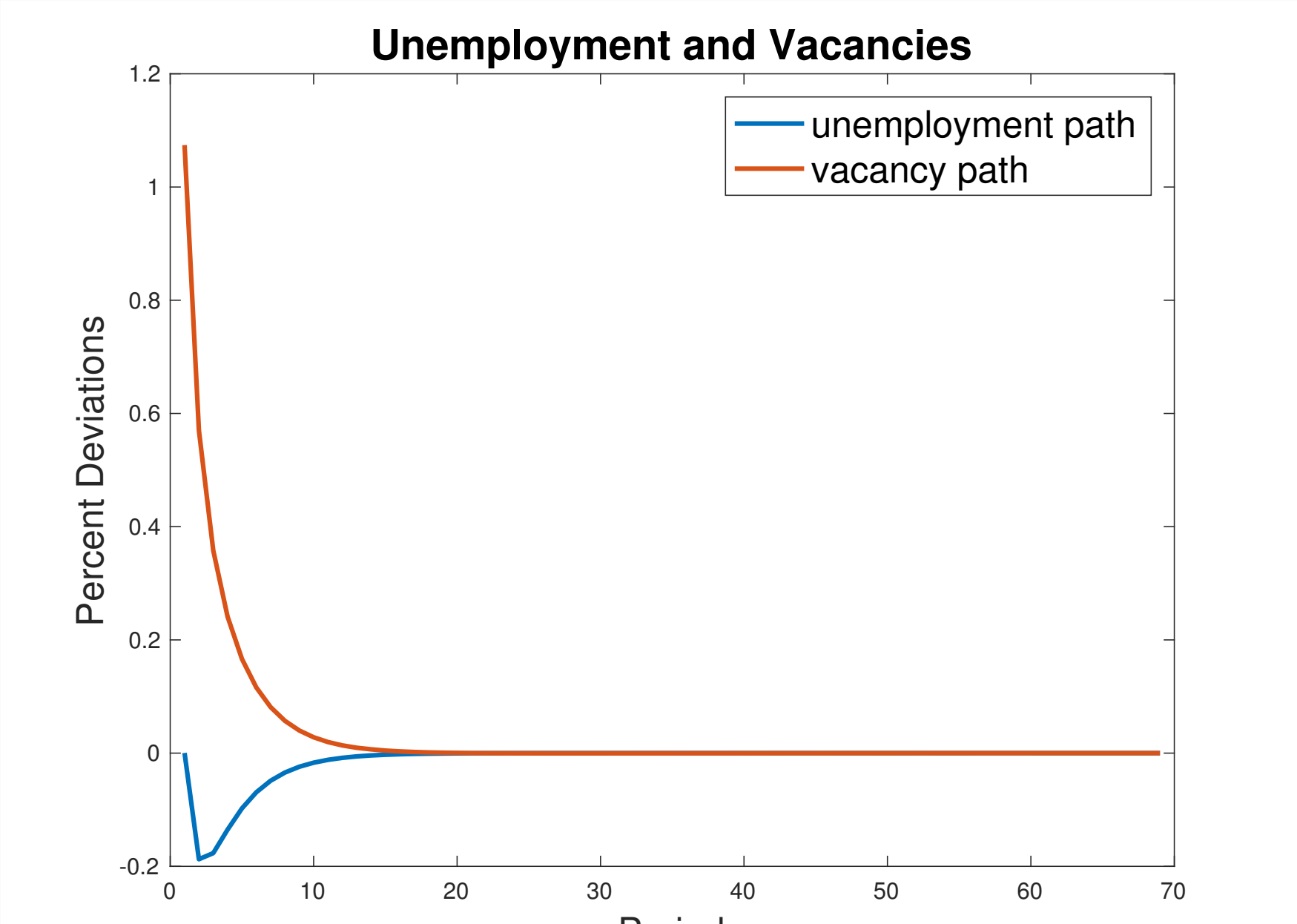
# Appendix

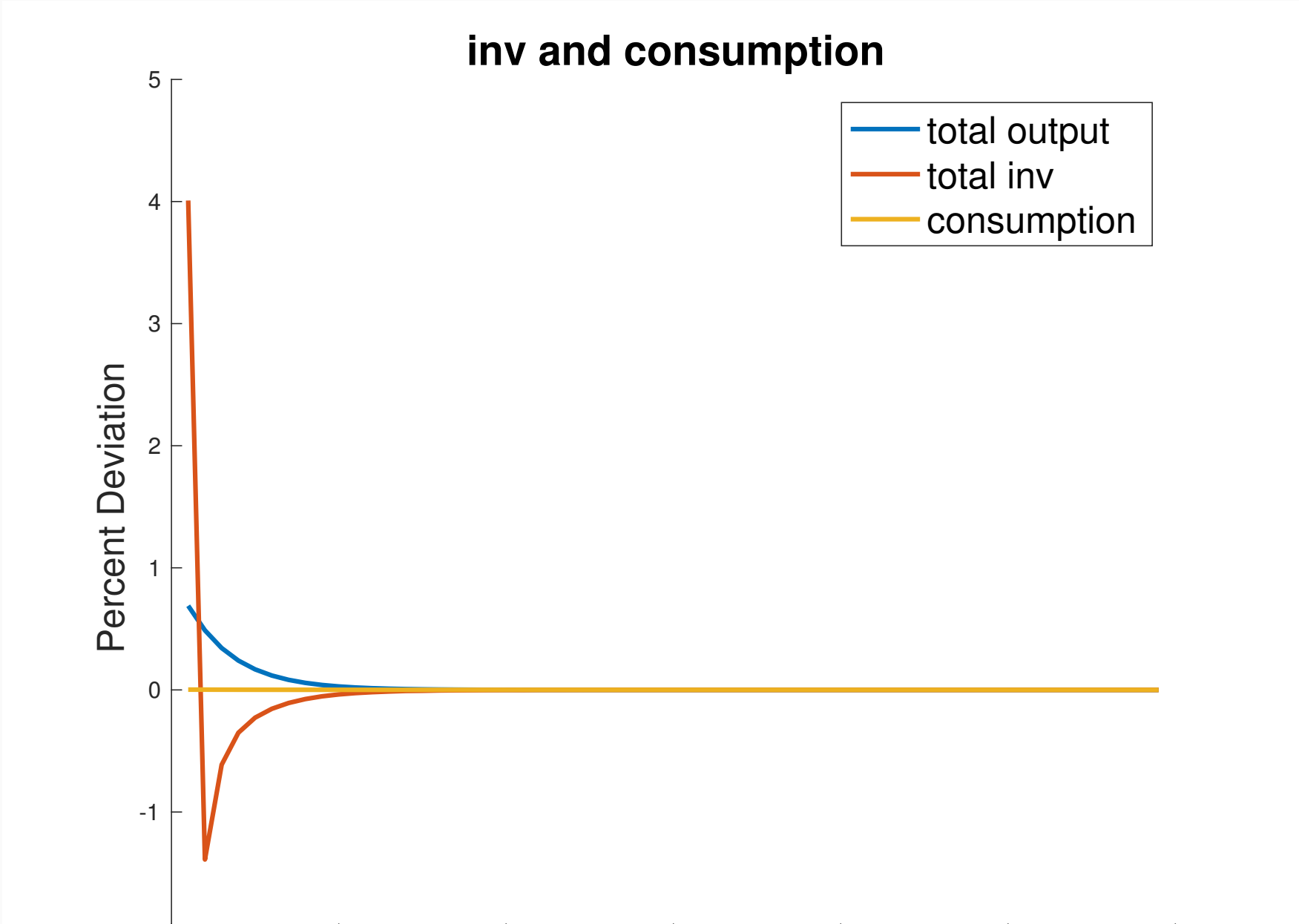
# Appendix A: Insufficient Employment Volatility

- The model features strong response of investment but insufficient response of employment.
  - We examine the mechanics of this.
- Consider for simplicity the model with aiming shocks but no quitting shocks (ANQ model). For a 1% productivity shock (with persistence 0.7), it generates
  - 1% increase of vacancies
  - 0.2% decrease of unemployment, which translates to only 0.01% increase of employment
  - and 4% increase of investment

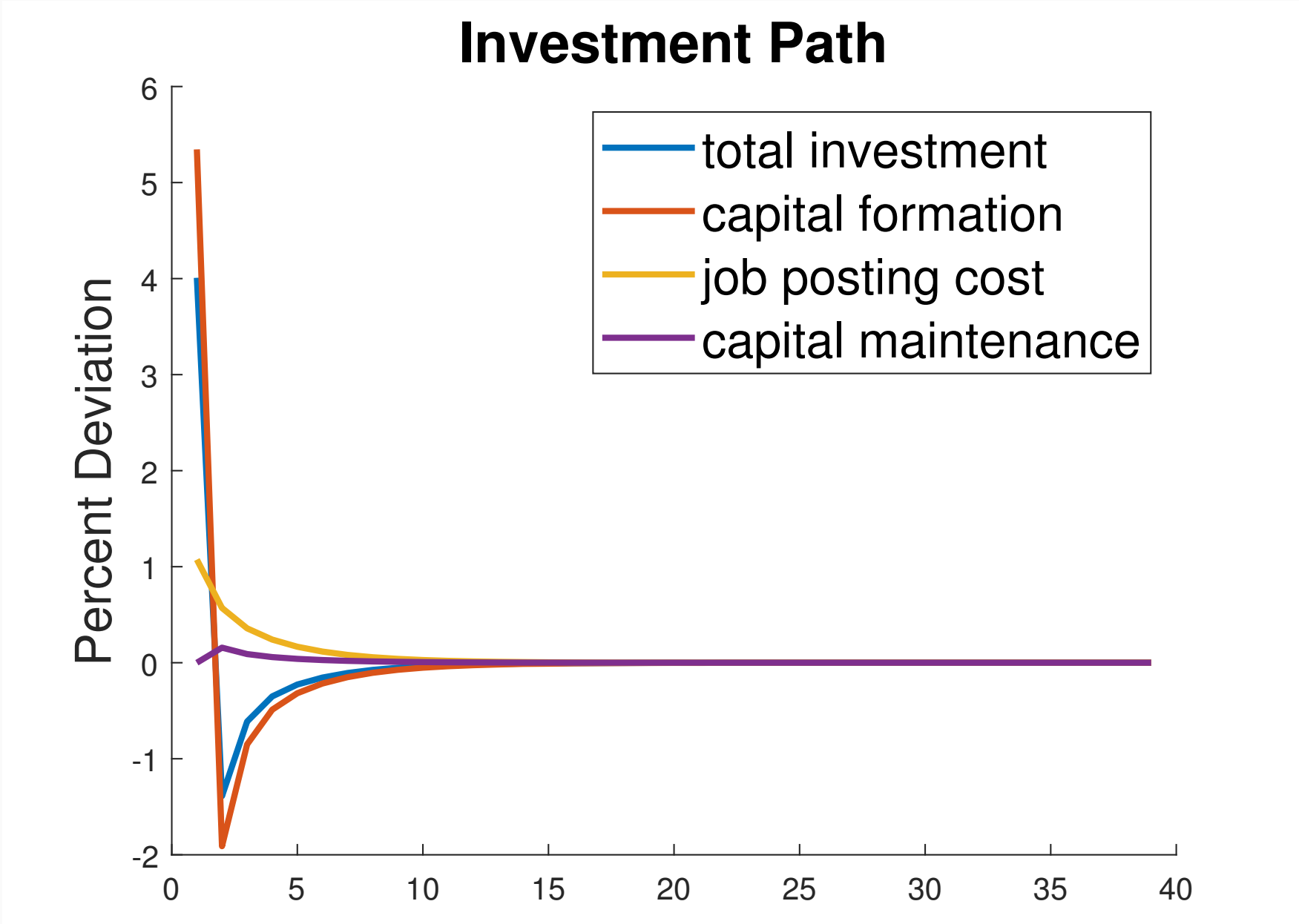
# ANQ: 1% Productivity Shock ($\rho = .7$) unemployment and vacancies



# ANQ: 1% Productivity Shock ($\rho = .7$) Output, investment and consumption



# ANQ: 1% Productivity Shock ($\rho = .7$) Decomposition of the investment



# Appendix A: Insufficient Employment Volatility

- Why does 1% increase of vacancies $v$ generate 4% increase of investment?
  - At the steady state, about 80% of the vacancies are posted by old idle firms and 20% by newly created firms.
  - Investment = wage posting cost + capital maintenance cost + new capital formation
  - As the shock hits the economy, firstly it only increases the creation of new firms, generating massive movements of investment in the form of capital formation ($ek$).
- Why does 1% increase of vacancies $v$ generate only 0.01% increase of employment?
  - As an approximation, $\hat{m} = (1-\eta)\hat{v} + \eta\hat{u}$.
  - Upon facing the shock, at first $u$ does not move. So the response of matches depend on the response of $v$ and the parameter $\eta$.
  - $\hat{m} \approx (1 - 0.72) \times 1\% = 0.28\%$, and $\frac{\Delta m}{1-u} = \frac{0.28\% \times 0.03}{0.95} \approx 0.01\%$
  - Lower $\eta$ relieves the problem (see the next page).

# Lower $\eta$ and Truncated 5% shock: AQ Economy