The Macroeconomics of Model T

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Abstract

We study a model of endogenous growth where firms invest both in product and process innovations. Product innovations (that open up completely new product lines) satisfy the advanced wants of the rich. Subsequent process innovations (that decrease costs per unit of quality) transform the luxurious products of the rich into conveniences of the poor. A prototypical example for such a product cycle is the automobile. Initially an exclusive product for the very rich, the automobile became affordable to the middle class after the introduction of Ford's *Model T,* the car that "*put America on wheels*". We show that an egalitarian society creates strong incentives for process innovations (such as the Model T) whereas an unequal society creates strong incentives for product innovations (new luxuries). We show that the inequality-growth relationship depends on which type of innovative activity drives technical progress, analyzing both the characteristics of and the transition to the balanced growth path.

JEL classification: O15, O30, D30, D40

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"Consumer goods inventions that cut both cost and quality but reduce the former more than the latter, such as the Model T, have historically been an important means for transforming the luxuries of the rich into the conveniences of the poor."

Jacob Schmookler, *Invention and Economic Growth* (1966)

1 Introduction

This paper develops a model of endogenous growth based on a cycle of product and process innovations. Product innovations introduce new goods satisfying the advanced wants of rich consumers. Process innovations lead to the adoption of new production processes that reduce the cost per unit of quality, making the goods also affordable to the poorer classes. As emphasized by Schmookler (1966), such a cycle of product and process innovations has historically been important to transform the luxuries of the rich into mass consumption markets.

The automobile, one of the most important durable goods in modern industrial societies, provides a prototypical example for such an innovation cycle. In the United States, the history of the commercial automobile production started with Charles and Frank Duryea who founded the Duryea Motor Wagon Company in 1893, the first American automobile manufacturing company; in 1902 and 1903 Oldsmobile (by Ransom E. Olds Company) and Cadillac (by Henry Ford Company) followed. At the time, the automobile was a luxury good consumed only by very rich households. Things started to change in 1908, when Ford introduced the *Model T,* the car that "*put America on wheels*". The concept was the use of assembly lines to produce a low-cost, low-quality car affordable to the middle class. Model T became a huge success and initiated the takeoff in car ownership in the U.S. Between 1908 and 1927 more than 15 million units of Model T were manufactured. The introduction of Model T contributed crucially to the fast diffusion of the automobile in the U.S.[1]

Product cycles where a new invention created a luxury good for the rich and subsequent innovations turned the luxury into a mass consumption good for lower classes are not confined to the auto industry. It has been important for many other consumer durables such as the refrigerator, the radio, the TV, and the computer, showing very similar patterns of innovation cycles.

We develop a formal endogenous growth model where firms engage both in product and process innovations of indivisible consumption goods. These indivisibilities let the composition of demand by rich consumers systematically differ from that of poorer households. The rich do not only purchase a larger variety of consumption goods, but also do consume these goods in better quality. Poorer households consume only a fraction of the available varieties and prefer

[1] Encyclopaedia Britannica

lower qualities to higher ones. Income inequality thus shapes product cycles and generates substantially different incentives for product and process innovation. Put differently, inequality determines the direction of technical change. Whereas an egalitarian society creates strong incentives for process innovations (such as the Model T), an unequal society creates strong incentives for product innovations (new luxuries).

Our analysis shows how the growth process and the associated mix of product and process innovations depend on the interaction between two major forces: the particular source of technical progress; and the extent of economic inequality in a society. If technical progress is mainly driven by product innovations, inequality is beneficial for long-run growth. Rising inequalities allow innovators to charge high prices both during the early period when the product is introduced as well as during the later period when the new product has generated mass markets but is still available in high quality and at a high price that the rich but not the poor are willing to pay. In contrast, if technical progress is mainly driven by process innovations, the relationship between inequality and growth is turned upside down and inequality becomes harmful for long-run growth. When the large majority of households is extremely poor, there is little potential to open up mass consumption markets and hence investments in low-quality low-cost process innovations are weak. In the presence of complementarities between process and product innovations, the relationship between inequality and growth becomes hump-shaped. Complementarities imply that an economy which has invested relatively little in process innovation is likely to benefit more from process innovations and vice versa. In that case, both very high levels and very low levels of inequality are harmful for growth, and growth is maximized at an intermediate extent of economic inequality.

Our analysis does not only characterize the balanced growth path of such an economy but also the transitional dynamics towards this path. Transitional dynamics reveal that both demand and supply shocks may trigger periods of industrial change in which a series of process innovations increases production and access to consumption markets. A large drop in inequality (such as the one that followed the Great Depression and WWII) triggers an initial phase where innovation activity is purely directed towards process innovations that facilitate mass production, while product innovation temporarily stops. Hence our model provides an explanation for the boom in consumer durables in the U.S. (and other industrialized countries) in the post-war era. A positive productivity shock lowering the costs of process innovation triggers an industrial revolution where an initially stagnant economy of craftsmanship and highly exclusive production is transformed into a modern society with broad participation and growth. We show that inequality – while initially beneficial for growth in the exclusive society – may eventually become harmful for growth after the economy has run through the transition phase and the economy has become a mass consumption society. In particular, our analysis predicts that in early stages of development (before the introduction of mass production technologies) inequality is beneficial for growth because technical progress is mainly driven by the introduction of new products for which the rich are willing to pay high prices. In later stages of development (after the introduction of mass production technologies) growth is higher in more egalitarian societies because process innovations become important drivers for growth. To generate the incentives for adopting these technologies, large markets and a high purchasing power of the lower classes are prerequisites.[2]

Our analysis extends the existing literature in at least three dimensions. *First*, our paper is related to the literature on directed technical change (Acemoglu, 1998 and 2002, Acemoglu and Zilibotti, 2001, and others). This literature analyzes the forces that generate biases in technical change towards one particular production factor. Similar to our paper, directed technical change models emphasize the tension between price and market size effects. However, the emphasis is on the relative demand for production factors, i.e. the supply/cost side of the economy. In contrast, our model focuses on demand/income effects. This channel generates an important role for the distribution of income across households, a mechanism that is absent in directed technical change models.

Second, our paper highlights the distinct role that product and process innovations can play in the process of long-run growth. In this dimension our paper differs from the large literature on the determinants of the aggregate technical progress (Romer 1990, Aghion and Howitt, 1992, Grossman and Helpman 1991, etc.). Aggregate models of product and process innovations are often mathematically similar (Acemoglu, 2009), that is the source of technical change is not essential to answer the question of what factors influence economic growth. This is different in our framework where incentives for product inventions and process innovations are subject to systematic differences, in particular with respect to the extent of inequality in the society.

Third, we speak to a small literature that has studied the impact of income inequality on technical progress. Matsuyama (2002) demonstrates the virtuous cycle between learning-by-doing and a large middle class, enabling the Flying Geese pattern discussed later in our paper. Foellmi and Zweimüller (2006) focus on product inventions and the scope of innovators' price setting power in the presence of a wealthy upper class. The present paper can be viewed as a synthesis of these classes of models. Our analysis highlights the conditions under which an unequal society suffers from lack of process innovations (and/or learning-by-doing) and from a small range of mass markets. Our analysis also makes precise the conditions under which such

[2] In Galor and Moav (2004, 2006) the inequality-growth relationship also changes across stages of development. Due to non-homothetic preferences over consumption and bequests, inequality leads to higher growth in early stages of development and to lower growth in later stages.

a society benefits from large mark-ups and high incentives to open up completely new product lines.[3]

The paper is organized as follows: Section 2 analyzes empirical and historical evidence motivating the key assumptions and mechanisms of our model. Section 3 introduces the formal framework, section 4 presents the solution of the balanced growth equilibrium, and section 5 discusses the relationship between inequality and growth. Section 6 studies transitional dynamics. We conclude with a summary and a list of potential extensions to our framework.

2 Motivating evidence

Casual observations and empirical evidence suggest that there is a strong impact of income on the number of varieties purchased by households, which is at odds with homothetic preferences.[4] Figure 1 illustrates this point by exhibiting the shares of ownership of various consumer durables of urban Chinese households (National Bureau of Statistics of China). At any given point in time, most types of consumer durables are only consumed by a fraction of the households. The figure also shows that levels of penetration rise over time. This is what Matsuyama (2002) calls the "Flying Geese pattern", in which a series of products takes off one after another, following an increase in productivity and income. This gradual increase in penetration levels was first emphasized by Katona (1964) who observed that the mass consumption society is the last stage of a process in which former luxury goods, consumed only by a few, privileged households, have been transformed into necessities for most households (i.e. mass consumption goods). Many products such as cars, radios, television sets, washing machines, refrigerators, vacuum cleaners and, more recently, computers have gone through such product cycles in the developed world, and are presently going through similar cycles in developing countries. Besides plain income effects, key elements of such product cycles are process innovations that cut the costs of production sufficiently. After a product has been invented, initial manufacturing costs are usually quite high, and sales volumes linger as the good can only be afforded by a few rich households. The takeoff and subsequent proliferation of the product is often ignited

[3] Murphy, Shleifer, and Vishny (1989) study the role of income distribution on technology adoption in a static context. Falkinger (1994) develops a model where inequality affects technical progress via aggregate output of consumer goods. The effect of inequality on technical progress in quality ladder models is explored in Li (2003) and Zweimüller and Brunner (2006).

[4] Jackson (1984) finds that the richest income class consumed twice as many different goods as the poorest class, using micro data from the Consumer Expenditure Survey of the Bureau of Labor Statistics. Falkinger and Zweimueller (1996) generate similar results using aggregate cross-country data from the International Comparison Project of the UN on per-capita expenditure levels on ninety-one different consumption categories.

and enabled by a series of process innovations that reduce manufacturing costs significantly.[5]

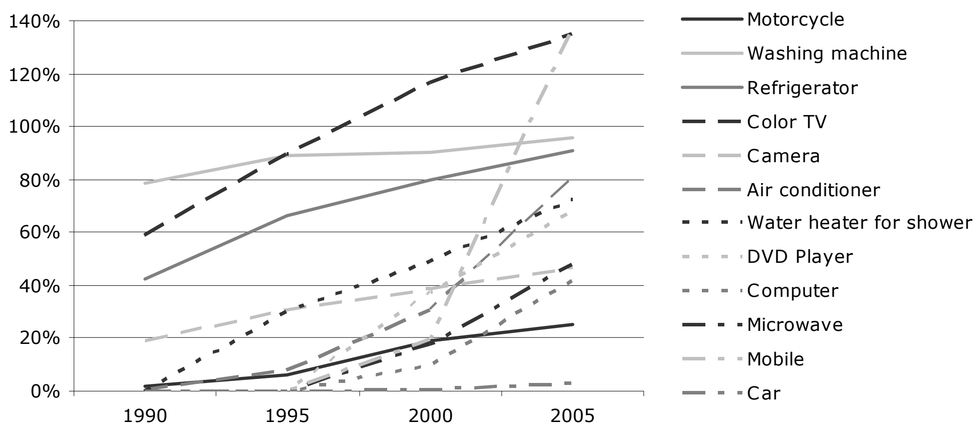

Figure 1: Ownership of consumer durables in Urban Chinese households (National Bureau of Statistics of China)

As mentioned above, one of the most famous historical examples for such an innovation pattern is the Ford Model T. It is generally regarded as the first affordable automobile, the car that "put America on wheels". One major reason behind the huge success story of Model T were Ford's innovations, including assembly line production instead of individual hand crafting, as well as the concept of paying the workers a wage proportionate to the cost of the car, so that they would provide a ready made market. Both innovations led to a huge increase in productivity. In total, Ford manufactured more than 15 million Model T's from 1908 to 1927, which contributed critically to the fast diffusion of the automobile. Figure 2 shows automobile and truck registrations in the U.S. from 1900 to 1970. The number of car registrations took off in the period of the Model T, and reached 23 million in 1927. Whereas 1% of households in the U.S. owned a car in 1908, the hour of birth of the Model T, penetration reached 50% in 1924.[6]

The product cycle that led to the Model T is not specific to the U.S. but can be observed in other parts of the world. Most of the large European economies had their own Model T which brought the car to the people. In Germany, a "people's car" – Volkswagen ("Beetle") – was initially introduced in the 1930s (and fostered by the Nazi regime). Citroën[7], Fiat and

[5] Our analysis highlights the relevance of major product and process innovations that create new product lines and subsequent mass consumption goods. Notice that in reality both mass consumption goods and luxury goods are continuously improved in quality. While this is clearly of high relevance in practice, we abstract from continuous quality improvements in our framework.

[6] See Model T Facts on media.ford.com, Encyclopaedia Britannica, and Bowden and Offer (1994) for penetration levels.

[7] Citroën director Pierre-Jules Boulanger's early design brief for the 2CV supposedly asked for "a vehicle

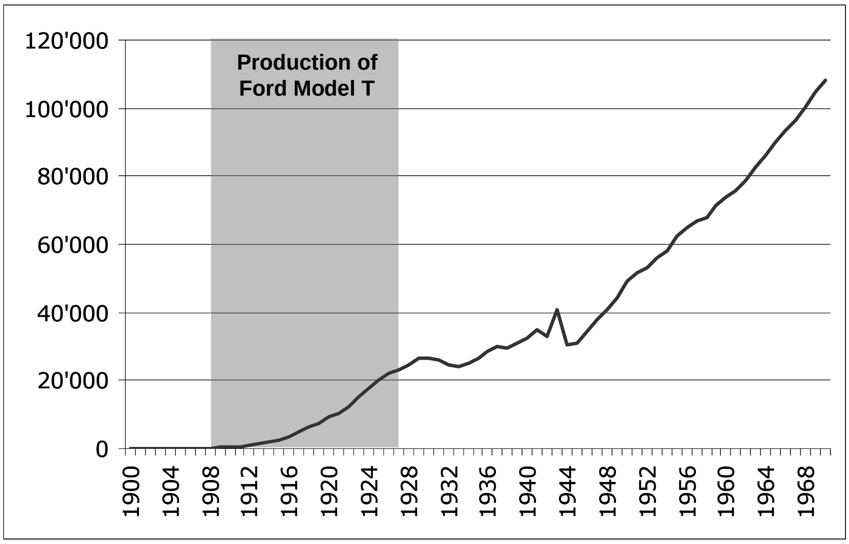

Figure 2: Automobile and truck registrations in the US in 1'000 units (US Census)

Austin 7 brought the car to the people of France, Italy and the UK, respectively. In rich countries, the introduction of mass-produced cars was an important step in the history of the manufacturing industry. And what has been important for rich countries in the past is starting to become relevant in poorer countries today. In Asia for example, Tata has recently announced to produce the world's cheapest car, mainly for the Indian market. The following table provides an overview of the world's major "Model T's":

Country	**Model**	**Year of introduction**
US	Ford Model T	1908
UK	Austin 7	1922
Italy	Fiat 500 Topolino & Nouva	1936
Germany	VW Käfer (Beetle)	1938
France	Citroën 2CV	1949
Japan	Subaru 360	1958
India	Tata Nano	2009

The auto industry is an example for the types of innovation and product cycles that our model aims to capture. While it provided the prototypical example, there are many other goods that experienced very similar patterns of innovation and market expansion. Two centuries after artificial refrigeration was pioneered by Dr. William Cullen, a GE home *refrigerator* cost around 700$ in 1922, compared to 450$ for a 1922 Ford Model T. Penetration barely reached

capable of transporting two peasants in boots, 100 pounds of potatoes or a barrel of wine, at a maximum speed of 40 mph, [...] Its price should be well below the one of our Traction Avant and, finally, its appearance is of little importance." (Translation, Technologie SCEREN - CNDP no. 138, 2005)

1% in the U.S. in 1925. The introduction of freon expanded the refrigerator market during the 1930s, with penetration reaching 50% by 1938. Refrigerators went into mass production after WWII, and by the year 1948 75% of all households owned a fridge.[8] The history of *television* started with first experimental transmissions made by Charles Jenkins in 1923. Television usage in the U.S. exploded after WWII. Having reached a penetration of 1% in 1948, it only took 5 years to reach 50%, and 2 more years to reach 75%. The rapid diffusion was enabled by the lifting of the manufacturing freeze, war-related technological advances, the expansion of the television networks, the drop in television prices enabled by mass production and additional disposable income.[9] A very similar evolution can be traced for *computers.* Spurred by calculation requirements for ballistics and decryption during WWII, the first electronic digital computers were developed between 1940-1945. Developments of the microprocessor led to the proliferation of the personal computer after about 1975. Mass market pre-assembled computers allowed a wider range of people to use computers, and penetration reached 1% in the U.S. around 1980. Component prices continued to fall since then, leading to continuous price declines. Penetration reached 50% around 2000. The emergence of Netbooks in 2007, a new market segment of small, energy-efficient ultra low-cost devices, is likely to advance penetration significantly, especially in developing countries.[10]

These examples demonstrate how closely process innovations and mass consumption markets are intertwined: Process innovations reducing manufacturing costs are crucial elements for tapping and proliferating mass consumption markets. Mass production, in turn, facilitates process innovation by increasing learning-by-doing and specialization benefits. We have established a close connection between inequality, product and process innovation. Product innovations introduce new goods satisfying the advanced wants of rich households who consume a wider range of goods than poorer households. Subsequent process innovations adopt manufacturing processes that reduce costs per unit of quality, making the products also affordable to poorer classes. Higher inequality raises the purchasing power of rich households, increasing demand for variety and product innovation. A more egalitarian society, on the other hand, raises the number of mass consumption markets and thus incentives for process innovation. Comparing the experience of Japan and the U.S. over the last decades provides suggestive evidence: Income concentration in Japan has remained relatively low after WWII in contrast to the U.S. (Moriguchi and Saez, 2005). During the same period of time, Japan has made itself

[8] Association of Home Appliance Manufacturers, "The Story of the Refrigerator;" Bowden and Offer (1994)

[9] Steven Schoenherr, "History of Television," History Server of University of San Diego; Bowden and Offer (1994)

[10] Jeffrey Shallit, "A Very Brief History of Computer Science," University of Waterloo; W. Warner, "Great Moments in Microprocessor History," Technical Library IBM; "Computer Use and Ownership," U.S. Census, and authors' estimates

a name as country of lean production and just-in-time management, i.e. process innovation. A recent study by Nagaoka and Walsh (2009), using data from the RIETI-Georgia Tech inventor survey, indeed shows that R&D in Japan is more biased to process innovation, in contrast to the U.S. where it is more directed to product innovation.

3 The model

3.1 The distribution of endowments

We assume there are L households that inelastically supply L units of labor. βL households are poor (indexed by P) and $(1-\beta)L$ are rich (indexed by R). Income differences arise from two sources. First, households are unequally endowed with units of labor. A poor household is endowed with $\ell_P = \theta_\ell < 1$ labor units, and the labor endowment of a rich household is $\ell_R = (1-\beta\theta_\ell)/(1-\beta) > 1$.[11] The parameters β and θ_ℓ fully characterize the distribution of labor endowments. The corresponding Lorenz-curve is piecewise linear with slope θ_ℓ for population shares between 0 and β; and slope $(1-\beta\theta_\ell)/(1-\beta)$ for population shares between β and 1. Notice that common measures of inequality (such as the Gini coefficient and the coefficient of variation) indicate an increase in inequality when θ_ℓ falls and/or β rises. It is assumed that the distribution of labor endowments is constant over time.

The second source of income differences is due to inequality in wealth, based on ownership in monopolistic firms. We denote by $v(t)$ the per-capita value of these firms at date t and assume that a poor household owns wealth $v_P(t) = \theta_v(t)v(t)$ and a rich household owns wealth $v_R(t) = [(1-\beta\theta_v(t))/(1-\beta)]\,v(t)$ where $\theta_v(t) < 1$ and $(1-\beta\theta_v(t))/(1-\beta) > 1$. In analogy to the labor endowment distribution, the distribution of wealth is determined by β and $\theta_v(t)$. Unlike the labor endowment distribution, however, the wealth distribution can change over time since $v_P(t)$ and $v_R(t)$ are endogenously determined by households' savings decisions. In sections 4 and 5 below we will study balanced growth paths. Along such paths, all households have the same savings rates and the wealth distribution is stationary, $\theta_v(t) = \theta_v$ for all t. When we analyze balanced growth paths below we will assume $\theta_\ell = \theta_v = \theta$. While this is clearly a rather special case, it keeps the analysis simple and transparent. Allowing labor endowment and wealth distributions to differ does not change the results in any economically relevant way. For instance, in comparing steady states, it does not make a difference whether the resulting incomes differences arise due to an unequal labor endowment distribution, due to an unequal wealth distribution, or both. What matters is inequality in total lifetime incomes. However, when we study transitional dynamics in section 6, we have to account for the fact

[11] Since the average labor endowment per household is unity we must have $\beta\ell_P + (1-\beta)\ell_R = 1$. Setting $\ell_P = \theta_\ell$ we get $\ell_R = (1-\beta\theta_\ell)/(1-\beta)$.

that households' savings rates need no longer be equal in the transition to a new steady state. As the wealth distribution changes over time we have to abandon the assumption $\theta_\ell = \theta_v = \theta$ and make the time-dependence of $\theta_v(t)$ explicit.

3.2 Technology and technical progress

Labor is the only production factor, the labor market is competitive and the market clearing wage is denoted by $w(t)$. Production activities are undertaken in monopolistic firms that supply differentiated products and operate with an increasing returns-to-scale technology. The creation of a firm requires a *product innovation*, i.e. an investment of $\tilde{F}(t)$ units of labor that yields the blueprint for a completely new product (e.g. the automobile). Once such a product innovation has been made, the innovating firm obtains a patent of infinite length granting the exclusive right to market this product. We think of a product innovation as a luxury good that initially satisfies the wants of rich households and that is costly in production. We assume a new product has quality q_h and requires a (high) labor input $\tilde{a}_h(t)$ per unit of output. After a successful product innovation, the firm has the option to undertake a *process innovation* that cuts both the quality of the product and its production cost. More precisely, we assume that after a further investment of $\tilde{G}(t)$ labor units, the product can also be supplied in lower quality $q_l < q_h$ and produced with a lower labor input $\tilde{a}_l(t) < \tilde{a}_h(t)$, the quality-cost ratio is higher, however, $q_l/\tilde{a}_l(t) > q_h/\tilde{a}_h(t)$. This captures Schmookler's idea that mass consumer good inventions cut both costs and quality but the former more than the latter. (Think of the high quality as the Cadillac and of the low quality as the Model T.)

In what follows we will refer to firms that have incurred both the product and the process innovation as "*mass producers*". Firms that have made only the product but not the process innovation will be called "*exclusive producers*". (The term "exclusive" is suggestive in the sense that it refers to both a high "exclusive" quality and to a situation where firms "exclude" the poor from consumption by setting prices that only rich but not poor households can afford.)[12]

Product and process innovations are the driving forces behind technical progress and long-run growth. We assume that the (non-excludable, non-rival) aggregate stock of knowledge $A(t)$ is determined by past product innovations and past process innovations. Assuming that labor requirements in the various activities are inversely related to the aggregate stock of knowledge $A(t)$ we have $\tilde{F}(t) = F/A(t)$, $\tilde{a}_h(t) = a_h/A(t)$, $\tilde{G}(t) = G/A(t)$, and $\tilde{a}_l(t) = a_l/A(t)$ where F,

[12] Note that the way we use the terms "exclusive producers" and "mass producers" refers to access to technology rather than to quantity of production. It may be that a mass producer makes a higher profit by selling only to the rich and a luxury producer may be better off by selling to the rich and the poor. We will see that such "strange" outcomes never happen along a balanced growth path but may be temporarily relevant during transitions towards a new steady state (see section 6 below).

G, a_h, and a_l are exogenous, positive constants. We assume that $A(t)$ is linked to past product and process innovations via the CES-function

$$A(t) = \left[\psi N(t)^{\gamma} + (1-\psi) M(t)^{\gamma}\right]^{1/\gamma}, \tag{1}$$

with $\gamma \in (0,1)$ and $\psi \in [0,1]$. $N(t)$ denotes the range of product varieties and $M(t)$ the range of varieties that underwent process innovations. The linear homogeneity of knowledge accumulation (1) satisfies the knife-edge condition for endogenous growth.[13] Note also that both R&D sectors benefit equally from spillovers, corresponding to the basic case of no state dependence in models of directed technical change (cf. Acemoglu, 2002).[14]

3.3 Preferences and consumer choices

Households have an infinite horizon and choose consumption both within and across periods to maximize lifetime utility. At a given point in time, a household chooses consumption from the continuum of $N(t)$ goods. Among the $N(t)$ firms that exist at date t there are those that made a product innovation but have not yet made a process innovation (exclusive producers); and other firms that have made both the product and the process innovation (mass producers). This means $M(t)$ goods are supplied both in high and low quality and $N(t) - M(t)$ goods are supplied in high quality only. In general, the prices may vary both across goods and across qualities and may change over time. We denote the price of good j and quality q at date t by $p(j, q, t)$.

The crucial assumption adopted here is that goods are indivisible. More precisely, the household has to decide whether or not to consume good j, and if yes, whether to consume it in high or low quality. There are three outcomes: either a household consumes (i) one unit in high quality, (ii) one unit in low quality, or (iii) does not consume at all. It turns out that such a discrete specification of preferences is a simple and tractable way to introduce non-homotheticities and to allow for a situation where rich households do not only consume a broader menu of goods but also consume the purchased goods in higher quality. Denote by $x_i(j,t)$ an indicator function that takes value 1 if household i consumes good j at date t, and takes value 0 if not. Similarly, denote by $q_i(j,t)$ the chosen quality level which can take only one of the two values $\{q_h, q_l\}$. The household's objective function is given by

$$U_i(\tau) = \int_{\tau}^{\infty} \log\left[\int_0^{N(t)} x_i(j,t) q_i(j,t) dj\right] e^{-\rho(t-\tau)}\, dt$$

[13] The knowledge-driven specification is more simple and transparent in a setting with final good varieties in different qualities whereas a lab-equipment model yields formally equivalent results and does not add economic substance.

[14] An extension of the model could study the role of state dependence, e.g. $\tilde{F}(t) = F/N(t)$, $\tilde{a}_h(t) = a_h/N(t)$, $\tilde{G}(t) = G/M(t)$, and $\tilde{a}_l(t) = a_l/M(t)$.

where ρ is the rate of time preference. The term in brackets can be interpreted as an instantaneous consumption aggregator which, for later use, we denote by $c_i(t) \equiv \int_0^{N(t)} x_i(j,t)q_i(j,t)dj$. The consumer chooses the time paths of $x_i(j,t)$ and $q_i(j,t)$ so as to maximize the above lifetime utility subject to the lifetime budget constraint

$$\int_\tau^\infty \left[\int_0^{N(t)} p(j,q_i,t)x_i(j,t)dj\right] e^{-R(t,\tau)}dt \leq \int_\tau^\infty \ell_i w(t)e^{-R(t,\tau)}dt + v_i(\tau),$$

where $R(t,\tau) = \int_\tau^t r(s)ds$ is the cumulative discount factor between dates τ and t, $r(t)$ is the interest rate, ℓ_i is the (time-invariant) labor endowment of household i, and $v_i(\tau)$ is the initial wealth level owned by the household.

The first-order conditions for the discrete consumption choice of good j are given by

$$\{x_i(j,t), q_i(j,t)\} = \begin{cases} \{1, q_h\} & \text{if} \quad q_h\mu_i(t) - p(j,q_h,t) \geq \max\left[0, q_l\mu_i(t) - p(j,q_l,t)\right], \\ \{1, q_l\} & \text{if} \quad q_l\mu_i(t) - p(j,q_l,t) \geq \max\left[0, q_h\mu_i(t) - p(j,q_h,t)\right], \\ \{0, \cdot\} & \text{otherwise,} \end{cases} \tag{2}$$

where

$$\mu_i(t) = \left[c_i(t)\lambda_i(t)\right]^{-1}$$

is household i's willingness to pay per unit of quality and $\lambda_i(t)$ the marginal utility of wealth at date t. These first order conditions are very intuitive. The condition in the first line of (2) says that good j will be consumed in high quality if the consumer's willingness to pay for the high quality $q_h\mu_i(t)$ is sufficiently larger than its price $p(j,q_h,t)$ so that both alternatives (purchasing not at all and purchasing the low quality) lead to a worse outcome. In other words, there needs to be a utility gain and it needs to be larger than the utility gain from purchasing the low quality. Similarly, the consumer will purchase the low quality if there is a utility gain that is larger than when purchasing the high quality. Otherwise, the household does not consume good j at all.

3.4 Price setting and profits

Firms make their pricing decisions on the basis of market demand functions that derive from households' optimal consumption choices given by the conditions in (2). Figure 3 shows the market demand curves graphically, both for the high quality (panel a) and for the low quality (panel b). Notice that the willingness to pay for quality $k \in \{l, h\}$ is always larger for a rich household than for a poor household, $q_k\mu_R > q_k\mu_P$. (For simplicity, we omit time indices in this section).

FIGURE 3

An *exclusive producer* can supply the product only in high but not in low quality. For such a firm only panel a) of Figure 3 is relevant. When the firm charges a price below (or equal to)

$q_h\mu_P$ both rich and poor households will purchase the good and market demand is L. When the price is above $q_h\mu_P$ but below (or equal to) $q_h\mu_R$ only rich households purchase the good and market demand is $(1-\beta)L$. When the price is larger than $q_h\mu_R$ not even the rich are willing to purchase and market demand is zero. The exclusive producer has essentially two options: (i) set price $q_h\mu_R$ and sell to rich households only; or (ii) set price $q_h\mu_P$ and sell to the whole customer base.

A *mass producer* can supply the good both in high and low quality. Such a firm faces demand curves as drawn in panels a) and b) of Figure 3. Stricly speaking, the demand curve for the high quality takes the form drawn in panel a) only if the low quality is not supplied. Similarly, panel b) is only relevant if the high quality is not supplied. (This is because each household consumes at most one unit.) To determine the optimal prices of a mass producer we have to consider panels a) and b) simultaneously. The mass producer has in principle the following options: (i) supply the low quality at price $q_l\mu_P$ and do not sell the high quality at all; (ii) supply the low quality at price $q_l\mu_R$ and do not sell the high quality at all; (iii) supply the high quality at price $q_h\mu_R$ and do not sell the low quality at all; or (iv) supply the high quality at price $q_h\mu_P$ and do not sell the low quality at all.

Actually, the mass producer has a fifth option and this option is the most interesting one in the present context: (v) set price $q_l\mu_P$ for the low quality and sell it to poor households and set price $q_l\mu_P + (q_h - q_l)\,\mu_R$ for the high quality and sell it to rich households. (This means rich consumers still purchase the Cadillac-version of a new product even when a Model T-version becomes available.) Notice that under this fifth option the firm cannot fully exploit the willingness to pay of rich consumers since they can switch to the low quality. To attract the rich households as customers for the high quality, the firm needs to set a price that is not larger than the price that makes a rich household indifferent between consuming the low quality and consuming the high quality. From (2) it is straightforward to verify that, when the low quality has price $q_l\mu_P$, the highest price that induces the rich to purchase the high rather than the low quality is $q_l\mu_P + (q_h - q_l)\,\mu_R$. To ensure that in equilibrium a situation emerges, where a mass producer sells the high quality to the rich and the low quality to the poor, we make the following assumption:

Assumption 1 *Inequality is sufficiently high such that the following three conditions are satisfied: (i)* $(q_h - q_l)\,\mu_R > (\tilde{a}_h - \tilde{a}_l)\,w$*, (ii)* $(1-\beta)\,(q_h\mu_R - \tilde{a}_h w) \geq (q_h\mu_P - \tilde{a}_h w)$*, and (iii)* $q_l\mu_P - (1-\beta)\,q_l\mu_R - \beta\tilde{a}_l w \geq 0$.

Obviously the willingnesses to pay of rich and poor households, μ_R and μ_P, will be determined endogenously in general equilibrium (see next section). Condition (i) says that the willingness to pay of rich households for the quality gap $q_h - q_l$ is sufficiently high relative to the cost gap $(\tilde{a}_h - \tilde{a}_l)\, w$ so that a mass producer strictly prefers selling the high quality to the rich and the low quality to the poor at prices $q_l \mu_P$ and $q_l \mu_P + (q_h - q_l)\mu_R$, respectively, to selling the low quality at price $q_l \mu_P$ to all consumers. Condition (ii) says that an exclusive firm weakly prefers selling only to rich households at price $q_h \mu_R$ rather than selling to all households at price $q_h \mu_P$. Finally, condition (iii) says that a producer with access to the mass production technology is weakly better off separating the market (selling the low quality to the poor and the high quality to the rich) rather than selling the high quality only to the rich at a higher price $q_h \mu_R$. Our assumption $q_l/\tilde{a}_l > q_h/\tilde{a}_h$ guarantees that (ii) and (iii) are compatible.

In the next section we study the balanced growth path where all exclusive producers sell their high quality only to the rich, and all mass producers sell the low quality to the poor and the high quality to the rich. Along this path all inequalities in Assumption 1 hold strictly. This does not need to be the case during a transition towards the balanced growth path. The case where condition (ii) holds with equality and condition (iii) holds with strict inequality corresponds to a situation where the economy has few mass producers, so that the poor purchase all mass consumption goods in low quality but also purchase some luxuries. The case where condition (iii) holds with equality and condition (ii) holds with strict inequality corresponds to a situation where there are so many mass producers that the poor cannot afford to purchase all mass consumption goods but only a subset of them.[15]

Proposition 1 *a) Suppose conditions (ii) and (iii) in Assumption 1 hold with strict inequality. Then every exclusive producer sells only to the rich, charges price $p_e = q_h \mu_R$ and earns profit $\pi_e = (1-\beta)\, L\, (p_e - \tilde{a}_h w)$. Every mass producer sells the low quality to the poor at price $p_l = q_l \mu_P$ and the high quality to the rich at price $p_h = q_l \mu_P + (q_h - q_l)\, \mu_R$ and earns profit $\pi_m = (1-\beta)\, L\, (p_h - \tilde{a}_h w) + \beta L\, (p_l - \tilde{a}_l w)$. b) When condition (ii) holds with equality, exclusive firms are indifferent between selling only to rich and to all households. c) When condition (iii) holds with equality, mass firms are indifferent between selling the high quality only to the rich at price $q_h \mu_R$ and separating the market. In that case we have $\pi_e = \pi_m$.*

Proof. *See Appendix A.* ■

It is also instructive to see what happens if some of the conditions of Assumption 1 are violated. In that case, mass producers supply only one quality. They may sell only the low quality to the whole customer base. This case is similar to the one we will study below and will emerge when inequality is not too large. Alternatively, mass producers may not have an

[15] The assumption $q_l/\tilde{a}_l > q_h/\tilde{a}_h$ precludes that both (ii) and (iii) hold with equality. Also notice that the rich purchase all goods in every case. Both exclusive and mass producers which do not sell to the rich have strictly lower profits and hence will undercut prices to get the rich as customers. Similarly, firms that sell to some poor households sell to all poor households.

incentive to supply the low quality. This case is obviously not interesting because there does not exist an incentive to undertake a process innovation and the model essentially reduces to one of expanding product varieties.[16]

3.5 R&D and resources

Inventing a new good and setting up a new exclusive firm is attractive as long as the value of this product innovation (the present value of future cash flows) does not fall short of the initial R&D cost. Initial R&D costs are $w(t)\tilde{F}(t)$ and, taking labor as the numeraire so that $w(t) = A(t)$, we have $w(t)\tilde{F}(t) = F$. The present value of a new innovation depends on whether and, if so, when the firm implements the mass production technology. The process innovation costs are $w(t)\tilde{G}(t) = G$. Denote by Δ the duration between the product innovation and the process innovation, i.e. the firm "age" at which to implement the mass production technology; and by $\pi_e(j,t)$ and $\pi_m(j,t)$ the profits before and after implementing mass production, respectively. Then the value of a firm that introduces a new product at date τ is given by

$$V(j,\tau) = \max_{\Delta} \left[\int_{\tau}^{\tau+\Delta} \pi_e(j,t) e^{-R(t,\tau)} dt + \int_{\tau+\Delta}^{\infty} \pi_m(j,t) e^{-R(t,\tau)} dt - G e^{-R(\tau+\Delta,\tau)} \right].$$

With free entry into the R&D sector, the general equilibrium leaves no profit opportunities unexploited. Hence the value of a product innovation cannot exceed the initial R&D cost $V(j,t) \leq F$.

Finally, the economy-wide resource constraint has to be satisfied at all times. Aggregate labor supply is fixed to L. Aggregate labor demand comes from the R&D sector and the production sector which produces (high- and low-quality) output. In the R&D sector, $\dot{N}(t)\tilde{F}(t)$ units of labor are engaged in designing entirely new products, and $\dot{M}(t)\tilde{G}(t)$ units of labor are used to implement new mass production technologies. In the production sector $Y_h(t)a_h(t)$ and $Y_l(t)a_l(t)$ units of labor are employed to produce high-quality and low-quality output denoted by $Y_h(t)$ and $Y_l(t)$, respectively. The resource constraint of the economy can be written as

$$Y_h(t)\tilde{a}_h(t) + Y_l(t)\tilde{a}_l(t) + \dot{N}(t)\tilde{F}(t) + \dot{M}(t)\tilde{G}(t) \leq L.$$

4 General equilibrium and balanced growth

We are now ready to consider the dynamic general equilibrium of the economy described above. In this and the next section we analyze the balanced growth path and leave the analysis of

[16] In the dynamic context this means there is no incentive to undertake a process innovation because the return to this investment is too low. An alternative polar case would be one where firms have an extremely high incentive to undertake the process innovation because process innovations are very cheap. In that case all firms would invest in both product and process innovation right from the beginning, again reducing the framework to a situation of expanding product varieties in which the high quality is never produced.

transitional dynamics to section 6 below. In the balanced growth equilibrium, there is both continuous introduction of entirely new products and continuous adoption of new processes that allow mass production of former exclusive goods. In the main text we focus on the most interesting equilibrium situation where mass producers sell the high quality to the rich and the low quality to the poor, i.e. where Assumption 1 holds. Situations where Assumption 1 does not hold are analyzed in Appendix B.

Definition 1 *A* balanced growth equilibrium *in our economy consists of a path where the interest rate $r(t)$ is constant; the stock of knowledge $A(t)$, the wage rate $w(t)$, the total number of firms $N(t)$, and the number of mass producers $M(t)$ grow at the constant rate g. Hence the fraction of mass producers $m = M(t)/N(t)$ is constant and labor requirements $\tilde{a}_h(t)$, $\tilde{a}_l(t)$, $\tilde{F}(t)$, and $\tilde{G}(t)$ shrink at rate g. Profit maximizing prices $p_e(j,t)$, $p_h(j,t)$ and $p_l(j,t)$, and instantaneous profits $\pi_e(j,t)$ and $\pi_m(j,t)$ are the same for all firms and constant over time. Given Assumption 1, rich households consume all $N(t)$ goods in high quality and poor households consume all $M(t)$ mass consumption goods in low quality. Hence the level of consumption of rich $c_R(t) = q_h N(t)$ and poor $c_P(t) = q_l M(t)$ also grows at rate g. Both types of households have the same savings rate, so the distribution of wealth is stationary.*

4.1 Product and process innovations

In a balanced growth equilibrium, the profits of exclusive and mass producers are constant over time and given by π_e and π_m defined in Proposition 1 and the interest rate r is constant. The optimal timing of the process innovation simplifies to

$$\max_{\Delta} \int_{\tau}^{\tau+\Delta} \pi_e e^{-r(t-\tau)} dt + \int_{\tau+\Delta}^{\infty} \pi_m e^{-r(t-\tau)} dt - G e^{-r\Delta}.$$

Using the Leibniz rule we obtain

$$\Delta = \begin{cases} 0 & \text{if} \quad (\pi_m - \pi_e)/r > G, \\ [0,\infty) & \text{if} \quad (\pi_m - \pi_e)/r = G, \\ \infty & \text{if} \quad (\pi_m - \pi_e)/r < G. \end{cases}$$

The above condition says that the present value of the increased profit flow is compared to innovation costs. We are interested in an equilibrium outcome where exclusive producers and mass producers co-exist so the first and third case of the above condition can be ruled out. This means the optimal timing of a process innovation Δ is undetermined. In other words firms are indifferent whether and when to invest in process innovation. However, the aggregate fraction of firms which have invested in process innovation, i.e. the fraction of mass producers, is determined in equilibrium.

The indeterminacy of the individual product cycle is due to the symmetry in preferences and technology. The symmetry assumption is not critical for our results. In fact, introducing asymmetries in our basic framework generates deterministic product cycles (featuring the empirically observed mentioned in Section 2). However, asymmetries complicate the analysis without leading to any substantial changes in our results. In Appendix C we sketch two such extensions. In the *first* extension we relax the symmetry in preferences by introducing hierarchic preferences, a fixed ranking of all varieties in the product space (by attaching unequal utility weights to the various goods). It is straightforward to see that innovations follow the consumption hierarchy (i.e. high priority goods are invented first) and that new products are initially offered only to rich households. The optimal date for process innovation is when the poor have become richer and are willing to pay a sufficiently high price so that the mass production technology breaks even. A *second* extension models asymmetry into the technology of firms by introducing learning-by-doing at the level of the individual firm. When individual manufacturing experiences facilitate production, firms initially serve the smaller, exclusive market since manufacturing costs are still relatively high. As soon as sufficient production experience has been gained, it becomes optimal for the firm to invest in the mass production technology serving the entire market.

Returning to the basic model, the following no-arbitrage conditions must hold:

$$\begin{aligned} V_N = \tfrac{\pi_e}{r} &= \tfrac{(1-\beta)L(q_h\mu_R - a_h)}{r} = F, \\ V_M = \tfrac{(\pi_m - \pi_e)}{r} &= \tfrac{L[q_l\mu_P - (1-\beta)q_l\mu_R - \beta a_l]}{r} = G. \end{aligned} \tag{3}$$

Note that, along the balanced growth path, all involved variables are constant over time. The present value of the profit flow enabled by product innovation V_N must be equal to initial product R&D costs. And the present value of the incremental profit flow enabled by subsequent process innovation V_M must be equal to process innovation costs. Note that V_N increases in the purchasing power of the rich, while V_M increases in the purchasing power of the poor. Higher inequality raises incentives for product innovation relative to process innovation, while a more egalitarian society increases incentives for process innovation.

4.2 Growth and mass production

In a balanced growth equilibrium, expenditures grow at rate g and prices are constant. Hence, consumption growth of poor and rich households follows the standard Euler equation:

$$r = g + \rho, \tag{4}$$

With a constant interest rate r and a constant growth rate g, the present value of household i's lifetime income is equal to $w(t)\ell_i/\rho + v_i(t)$. Because poor households are endowed with θ units of labor and $\theta v(t)$ units of firm shares and rich households are endowed with $(1-\beta\theta)/(1-\beta)$ units of labor and $[(1-\beta\theta)/(1-\beta)]\, v(t)$ units of firm shares, a rich household receives an income stream that is $[(1-\beta\theta)/(1-\beta)]\,/\theta$ times as large as the one of a poor household.[17] The log-specification of intertemporal preferences implies that the flow of expenditures of a rich household compared to a poor on the balanced growth path needs to be $[(1-\beta\theta)/(1-\beta)]\,/\theta$ times as large, too. Recalling that the $mN(t)$ mass producers charge price p_h for the high quality and p_l for the low quality and the $(1-m)\,N(t)$ exclusive producers charge price p_e, the expenditure flow of a poor household is $p_l mN(t)$ and the expenditure flow of a rich household is $[p_h m + p_e(1-m)]\,N(t)$. Hence the ratio of the expenditure flow of a rich relative to a poor household is

$$\frac{mp_h + (1-m)p_e}{mp_l} = \frac{1-\beta\theta}{(1-\beta)\theta}, \tag{5}$$

where $p_e = q_h\mu_R$, $p_l = q_l\mu_P$, and $p_h = q_l\mu_P + (q_h - q_l)\,\mu_R$ (see Proposition 1).

We can now characterize and analyze the balanced growth equilibrium using two equations, a *no-arbitrage curve* and a *resource curve*. Using the no-arbitrage conditions (3), we can express the price of the lower quality as $p_l = q_l\mu_P = (1-\beta)\,[q_l\mu_R + (q_h\mu_R - a_h)\,G/F] + \beta a_l$. Combining this with the above expression for relative expenditures (5) lets us write the price of the exclusive good as

$$p_e = q_h\mu_R = q_h\frac{a_l\beta/(1-\beta) - a_h G/F}{\theta\,(q_h/m - q_l)\,/\,(1-\theta) - q_l - q_h G/F}, \tag{6}$$

from which we can infer $p_h = p_l + (q_h - q_l)p_e/q_h$. Plugging (6) into the no-arbitrage condition of the exclusive producer and using the Euler equation (4) yields the no-arbitrage curve (NA)

$$g = \frac{L}{F}\left[q_h\frac{\beta a_l - (1-\beta)a_h G/F}{\theta\,(q_h/m - q_l)\,/\,(1-\theta) - q_l - q_h G/F} - (1-\beta)a_h\right] - \rho, \tag{7}$$

which expresses the growth rate g in terms of the fraction of mass goods m. The NA-curve is upward sloping in m if $F\beta a_l > G(1-\beta)a_h$ and downward sloping otherwise. Keeping g constant, the fraction of mass producers m rises in θ, and falls in β to keep (7) in equilibrium.[18] This is because lower inequality raises the purchasing power of the poor. Hence there will be more mass production m and less exclusion $1-m$.

A second equation in m and g is derived from the aggregate resource constraint in the economy. Recall that along the balanced growth path the rich consume all $N(t)$ goods in high

[17] Here we stick to the simplifying assumption that the income composition of rich and poor households is identical. As mentioned above, this is a special case that makes the analysis simple and transparent. The more general (and more realistic) case when income composition differs between rich and poor housholds does not add economic substance to the analysis. However, in the next section, when we study transitional dynamics we need to give up this assumption since the wealth distribution is no longer stationary.

[18] An increase in θ is offsetting an increase in m as the denominator in the NA-curve is strictly increasing in θ given its derivative with respect to θ of $(q_h/m - q_l)/(1-\theta)^2 > 0$. Similar computations reveal that a decrease in β is offsetting an increase in m.

quality and the poor consume all $M(t)$ mass consumption goods in low quality. Hence we can write $L = (1-\beta)LN(t)a_h/A(t) + \beta LM(t)a_l/A(t) + \dot{N}(t)F/A(t) + \dot{M}(t)G/A(t)$. Using the equation of motion for the aggregate stock of knowledge (1) and the definitions $m = M(t)/N(t)$ and $g = \dot{N}(t)/N(t) = \dot{M}(t)/M(t)$ we can express the resource curve (RC) as

$$g = \frac{L\left[(\psi + (1-\psi)m^{\gamma})^{1/\gamma} - (1-\beta)a_h - \beta a_l m\right]}{F + Gm}. \tag{8}$$

Notice that the RC-curve may be upward or downward sloping. On the one hand, there is a demand effect. An increase in m is associated with higher consumption of the poor. Hence more employment is needed to satisfy this additional demand leaving fewer resources for research. On the other hand, there is a productivity effect. An increase in m means that final output is produced in a more efficient way which saves resources that become available for innovation and growth. Under our specification for the evolution of the knowledge stock (1), the productivity effect depends on the importance of process innovation in pushing ahead the knowledge frontier. This is captured by the parameter ψ. The lower is ψ, the more important are process innovations as drivers of technical knowledge and the stronger is the productivity effect. Note also that the distribution parameter θ does not enter the resource curve. The resource curve shifts up when the population share of the poor β rises.

Proposition 2 *A balanced growth equilibrium determined by the intersection of the two curves (7) and (8) exists if Assumption 1 holds with strict inequalities.*

Proof. *See Appendix B.* ■

The idea of the proof is the following: to determine whether the outcome where mass producers separate households and exclusive producers sell only to rich households is indeed an equilibrium, one needs to compute μ_R and μ_P using the above equations for a given set of parameters, and test whether Assumption 1 holds with strict inequalities. If this is the case, no firm has an incentive to deviate (see Proposition 1). Assumption 1 holds if the quality gap $q_h - q_l$ is sufficiently high (but not too high) relative to the cost gap $a_h - a_l$ and process innovation costs G; and if inequality is sufficiently high, i.e. the group of poor β is sufficiently large as well as the distribution parameter θ is not too high. Conversely, a low quality gap would induce all firms to become mass producers and supply only the low quality. Similarly, if the quality gap were too high, there would be no incentive to invest in process innovations. These outcomes are less interesting as the model essentially reduces to one of expanding product varieties. When inequality is too low, a further outcome arises in which mass producers sell the low quality to all households. We will characterize these other outcomes in Appendix B in more detail. The existence of a positive growth equilibrium is determined by comparing the horizontal m-axis intercepts of the NA- and RC-curve (denoted by m_{NA} and m_{RC}). Assumption 1 guarantees that the RC-curve (8) holds for $g > 0$ if $m = 1$. The equilibrium is unique if $m_{RC} < m_{NA}$ and the NA-curve is upward sloping since the NA-curve is convex and the RC-curve is concave when upward sloping, consider Figure 4.[19]

FIGURE 4

5 Income inequality and technical change

We will first analyze the equilibrium for the two polar cases of $\psi = 1$ when technological spillovers are generated only by product innovations, and $\psi = 0$ so that technical progress is driven only by process innovations.

5.1 Product innovation as driver of productivity growth

When product innovation is the only driver of productivity growth, we have $\psi = 1$ and equation (1) becomes $A(t) = N(t)$. While the no-arbitrage curve (7) remains unchanged, the resource constraint simplifies to

$$g = \frac{L\left[1 - (1-\beta)a_h - \beta a_l m\right]}{F + Gm}. \tag{9}$$

The resource curve is downward sloping in m, since a larger share of mass producers requires more labor for manufacturing and process innovation, leaving less labor for product R&D, the driver of growth. Panel a) of Figure 4 displays the two curves and the equilibrium in this case.

In the case of $A(t) = N(t)$, inequality is beneficial for growth. A redistribution of income from the poor to the rich (reducing θ) leaves the resource curve unchanged, but shifts the no-arbitrage curve to the left, as depicted in the left-hand panel of Figure 5. A richer upper class has a higher willingness to pay for products, and this price effect increases profits. Product inventions become more attractive, spurring technical progress and growth. From a resource point of view, redistributing wealth from the poor to the rich raises exclusion in the economy, setting free resources from the manufacturing and the process R&D sectors, which become available for product R&D, the driver of growth.

FIGURE 5

Increasing the size of the group of poor households β, while holding θ constant, raises inequality (see section 3.1. above). As can be seen from the right-hand panel of Figure 5, the resource curve shifts up and the no-arbitrage curve shifts to the left. The reason is that a

[19] The condition $m_{RC} < m_{NA}$ trivially holds if the RC-curve has a vertical axis intercept in the positive (m, g)-quadrant, which is true whenever $\psi^{1/\gamma} > (1-\beta)a_h$. When $m_{RC} \geq m_{NA}$ or the NA-curve is downward sloping, there may (but need not) be multiple balanced growth equilibria. Apart from the locally stable steady state, there exists an intermediate unstable steady state (and a stagnation equilibrium) in that case. See Appendix B.

higher β is associated with higher inequality. (With θ given, relative incomes of rich households, $(1-\beta\theta)\,/\,[(1-\beta)\theta]$, increase). While there are less rich households reducing the market for the exclusive goods, the (remaining) rich have a higher willingness to pay. It turns out that the latter (price) effect dominates the former (market size) effect so profits for exclusive producers increase for a given m and g. In the new equilibrium we have fewer mass producers m which releases (manufacturing and process R&D) resources which are channeled into product R&D, and hence growth g is higher.

In sum, higher inequality (either due to a lower θ or due to a higher β, or both) is beneficial for growth, provided that growth is driven purely by product innovations.

5.2 Process innovations as productivity drivers

The result that inequality is beneficial for growth hinges upon the assumption that only product innovations affect productivity growth whereas process innovation activities do not at all impact technical progress. We now consider the other extreme, when $\psi = 0$, so that technical knowledge is entirely determined by past process R&D activities, $A(t) = M(t)$. The resource curve becomes

$$g = \frac{L\,[1-(1-\beta)a_h/m - \beta a_l]}{F/m + G}, \tag{10}$$

and is now upward sloping. As process innovation is the key to become a mass producer, a higher share of mass production m is beneficial for growth. A higher prevalence of mass production raises aggregate productivity. In contast to before a higher m implies less (low-productive) exclusive sectors which saves resources for process R&D. Panel b) of Figure 4 illustrates the two curves and the equilibrium graphically.

A higher extent of inequality due to lower incomes of poor households θ shifts the no-arbitrage curve to the left, as depicted graphically in the left-hand panel of Figure 6. The result is less mass production m and also a lower incentive to undertake process innovations. Hence the growth rate g falls.

FIGURE 6

Increasing the group size of poor households β shifts the no-arbitrage curve to the left and shifts the resource curve up, as shown in the right-hand panel of Figure 6. The effect on growth is now ambiguous. When more income is concentrated in the hands of fewer rich, there will be less mass consumption m. This has two effects. On the one hand, the shift from mass consumption to exclusive markets decreases average productivity in manufacturing. On the other hand, less mass production also implies that fewer resources are needed for production which can be used for R&D and growth. Computations show that either effect may dominate.

5.3 The general case

Having analyzed the two polar cases, we have demonstrated that inequality may be either beneficial or harmful for growth, depending on the source of technical progress and productivity growth in the economy. Inequality has an effect on prices and on the size of markets. On the one hand, a higher willingness to pay of the rich households raises prices and profit margins, spurring entry and thus product innovation. On the other hand, a high level of exclusion reduces mass consumption markets, and thus incentives for process innovation.

The general case lies in between the two polar cases. Let us write down the resource curve here as a function of m,

$$g(m) = \frac{L\left[(\psi + (1-\psi)m^{\gamma})^{1/\gamma} - (1-\beta)a_h - \beta a_l m\right]}{F + Gm},$$

The inequality-growth relationship depends on the slope of this function:

Proposition 3 *Given Assumption 1, an increase in inequality due to a lower relative income of poor consumers (lower θ) leads to a higher prevalence of mass producers m. If the resource curve is (locally) decreasing in m, $g'(m) < 0$, inequality raises growth. If it is increasing, inequality hurts growth.*

We have shown above that for a given g, the fraction of mass producers increases in θ. Hence m declines in inequality (given that θ does not enter the resource curve directly), and the impact on growth depends on the slope of the resource curve. In the cases of $\psi = 1$ and $\psi = 0$, we have shown that the resource curve is (globally) downward and upward sloping, respectively. For intermediate cases of ψ, where productivity growth is driven by both product and process innovation, the sign of the inequality-growth relationship depends on the dominating source of technical change and on the extent of inequality. Under the assumption that the aggregate stock of knowledge evolves according to (1), the marginal contribution of process innovations, $\partial A(t)/\partial M(t)$ is infinite at $m = 0$, $\lim_{m->0} g'(m) = +\infty$. Hence, as long as $\psi < 1$, the RC curve slopes upwards for low m. For larger values of m the resource curve eventually becomes downward sloping. Intuitively, there are complementarities between product and process innovation. When an economy has invested relatively little in process innovation, it is likely to benefit more from process innovations and vice versa.

Taken together, for $0 < \psi < 1$, the resource curve becomes hump-shaped as depicted in Figure 7. Higher inequality fosters growth if inequality is initially low (and the fraction of mass producers is high), whereas higher inequality slows down growth if the extent of inequality is already high initially. Therefore, in a very unequal society that is dominated by exclusive markets lowering inequality is likely to increase growth. The expansion of mass consumption markets spurs process innovation and increases growth. However, in a very egalitarian society, the relationship may be reversed, when innovation incentives are based on a better funded upper class, so that the introduction of new goods becomes more attractive. As a result, both very high levels and very low levels of inequality are harmful for growth. High long-run growth rates are reached by intermediate degrees of inequality.

FIGURE 7

6 Transitional dynamics

In our framework, both demand and supply shocks may trigger periods of industrial change in which a series of process innovations increases production and access to consumption markets, causing as Perkin (1969) put it *"a revolution in men's access to means of life"* (cited by Mokyr, 1999). In this section, we undertake two thought experiments. In both cases we assume that the economy is initially in an equilibrium that is characterized by low growth and low (or complete absence of) mass production, and analyze exogenous shocks triggering a process of transition toward a new steady state. In doing so, our analysis sheds light on the process by which demand and/or supply shocks generate a take-up of productivity growth and a transition of a society with high exclusion and low consumer-participation of the lower classes to a mass consumption society.

The first thought experiment is a demand shock generated by a major drop in inequality through an increase in θ. Assume that the economy is initially in a steady state characterized by high inequality and low mass production so that the initial balanced growth equilibrium is located on the upward sloping branch of the resource curve (see Figure 7). As we have seen in the last section, starting from such an equilibrium, a major drop in inequality leads to a new balanced growth path with higher growth and a higher extent of mass production. One potentially relevant situation from recent economic history is the substantial drop in inequality during the Great Depression and WWII that might help explain the boom in consumer durables in the U.S. of the post-war era. The second thought experiment relates to a positive productivity shock lowering the costs of process innovation, G. Such a shock may trigger an industrial revolution through which an initially stagnant economy of craftsmanship and high exclusion is transformed into an industrialized society with high consumer-participation and growth.

Notice that the two state variables that characterize the transition process are the total number of firms $N(t)$ and the number of mass producers $M(t)$. It turns out that, when the economy operates along the balanced growth path both variables grow pari passu. When the economy operates off this path, there are either only product innovations or only process innovations but not both. We summarize this result in

Proposition 4 *Suppose Assumption 1 holds and the economy features both product and process innovations. Then the economy is on the balanced growth path.*

Proof. *See Appendix D.* ■

The proposition has an important implication. We will see that, when the economy has too few mass producers $M(t)$, the transition process will be characterized by process innovations only. Similarly, if there are too few exclusive producers $N(t) - M(t)$, the transition process will be characterized only by product innovations. Hence all adjustments in the state variable $m(t) = M(t)/N(t)$ occur by a "bang-bang" rule. We will also see that this implies that the transition from an old to a new steady state will occur in finite time.

6.1 A major drop in inequality

An exogenous (and instantaneous) drop in inequality leads to transitional dynamics in our framework during which the fraction of firms that have invested in process innovation increases. One can think of the introduction of compulsory schooling, increasing relative productivity of the poor, or an extreme event such as a war lowering financial wealth inequality (such as during WWII), leading to such an adjustment.

Initial and final balanced growth equilibrium We assume that both in the initial and final balanced growth equilibrium conditions are such that exclusive producers sell (their high quality) only to the rich; and mass producers sell the high quality to rich and the low quality to poor households. In contrast to the analysis of the last section, we need to relax the assumption of identical endowment distributions. This is because the transition process will be characterized by a situation where the two types of households face different incentives to save and hence will accumulate wealth at unequal speed. In other words, in the transition process, the wealth distribution is no longer stationary invalidating the assumption $\theta_\ell = \theta_v = \theta$. Instead we need to account for the fact that $\theta_v(t)$ changes over time.

The initial and final balanced growth paths are still characterized by the equations from above, (3), (4), and (8). However, since θ_ℓ may not be equal to θ_v, equation (5) needs to be adjusted, as relative lifetime incomes of rich households now depend on the factor income distribution, i.e. on wages $w(t)$ and firm values $v(t)$. With a constant interest rate r and a constant growth rate g, the present value of household i's lifetime income (the right-hand-side of the household i's intertemporal budget constraint) equals $w(t)\ell_i/\rho + v_i(t)$. By normalization, the wage is equal to $w(t) = A(t) = N(t)\left(\psi + (1-\psi)m^\gamma\right)^{1/\gamma}$ and, from the zero-profit conditions

(3), we have $v(t)L = N(t)\,(F + mG)$. As the left-hand-side of a household's intertemporal budget constraint is unaffected by the more general specification of the endowment distributions, we can rewrite equation (5) as

$$\frac{mp_h + (1-m)p_e}{mp_l} = \xi(m), \tag{11}$$

where relative lifetime incomes $\xi(m)$ are now given by

$$\xi(m) \equiv \frac{\rho(1-\beta\theta_v)(F+mG) + (1-\beta\theta_\ell)L\,(\psi + (1-\psi)m^\gamma)^{1/\gamma}}{\rho(1-\beta)\theta_v(F+mG) + (1-\beta)\theta_\ell L\,(\psi + (1-\psi)m^\gamma)^{1/\gamma}},$$

with $\xi_m(m) > 0$ since $(\theta_v, \theta_\ell) < (1,1)$. Note also that $\xi(m)$ decreases in both θ_v and θ_ℓ.

We can solve this more general case in a similar way as above. First calculate $p_e = q_h\mu_R$ using equation (11) and no-arbitrage conditions (3). Then plug the resulting expression into the no-arbitrage condition for the exclusive producer to get a new no-arbitrage curve (7)

$$g = \frac{L(1-\beta)}{F}\left[q^h \frac{\beta a_l - (1-\beta)a_h G/F}{(q_h/m - q_l)\,/\,(\xi(m)-1) - (1-\beta)\,(q_l + q_h G/F)} - a_h\right] - \rho, \tag{12}$$

For a given growth rate g, raising θ_v or θ_ℓ increases m, since $\xi(m)$ is increasing in m as well as decreasing in θ_v and θ_ℓ. Lowering financial wealth or labor income inequality reduces exclusion. Hence, in much the same way as above, the inequality-growth relationship depends on the slope of the resource curve.

Transition Now consider a mean-preserving spread in the endowment distributions raising incomes of poor households at the expense of the rich, so that $(\theta_v'(t_0), \theta_\ell') > (\theta_v, \theta_\ell)$, in a balanced growth equilibrium at time $t = t_0$. Imagine that the introduction of compulsory schooling increases relative productivity of the poor,[20] $\theta_\ell' > \theta_\ell$, or shares in firms are redistributed (e.g. during a war) from rich to poor, $\theta_v'(t_0) > \theta_v$.

FIGURE 8

Figure 8 illustrates the transitional dynamics triggered by a drop in inequality. As a result of the shift in purchasing power, poor households increase consumption whereas the consumption of rich households initially stagnates. Since the economy has too few mass producers $M(t)$, demand for the mass production technology is high, and all R&D resources are temporarily directed towards process innovation. The economy reaches the new steady state in finite time at $t = t_2$ when product innovations become attractive again. The figure is drawn in such a

[20] Strictly speaking, introducing/increasing compulsory schooling leads to a more equal endowment distribution by changing not only the spread but also the mean of the labor endowment distribution. It is straightforward to see that an increase in L increases growth because the model exhibits a scale effect. Here our focus are distributional consequences, hence we consider mean-preserving spreads.

way that growth is higher in the final state, which is the case if inequality is sufficiently high in the initial state such that the resource curve is upward sloping ($g'(m) < 0$ see Proposition 3). The following proposition characterizes the transition process in detail:

Proposition 5 *Suppose Assumption 1 holds at all times. a) A fall in inequality at date* t_0*, from* (θ_v, θ_ℓ) *to* $(\theta'_v(t_0), \theta'_\ell)$*, triggers a transition period of finite duration* (t_0, t_2) *where* $\dot{N}(t) = 0$ *and* $\dot{M}(t) > 0$*. A new balanced growth equilibrium with* $m' > m$ *is reached at date* t_2*. b) During the entire transition period consumption of the rich stagnates at* $c_R(t) = q_h N(t_0)$*. c) When the initial reduction in inequality is substantial,* $c_P(t)$ *jumps to a higher level at date* t_0*. During a first transition period,* $t \in [t_0, t_1)$*,* $c_P(t) > q_l M(t)$*; during a second transition period,* $t \in [t_1, t_2)$*,* $c_P(t) = q_l M(t)$*. When the initial reduction in inequality is minor,* $c_P(t)$ *does not change discontinuously at date* t_0*, the first transition period does not exist and* $c_P(t) = q_l M(t)$ *for all* $t > t_0$*.*

See Appendix D for the technical details including a description of the procedure of numerical simulations. The discussion here is confined to the key dynamics to understand the main results and the intuition behind these results. If Assumption 1 holds for both (θ_v, θ_ℓ) and $(\theta'_v, \theta'_\ell)$, the balanced growth equilibrium before and after the transition corresponds to a situation where exclusive producers sell (their high quality) only to the rich and the mass producers sell the high quality to the rich and the low quality to the poor. A redistribution from top to bottom has two key effects. *First*, there is an effect on the direction of technical change as only process but no product innovations occur during transition. Redistributing income towards the poor raises their purchasing power and their willingness to pay relative to the one of the rich. Consequently, process innovations become temporarily strictly more attractive than product invention and all R&D activities are concentrated on the implementation of mass production technologies. During this period interest rates are constant and given by

$$r_1 G = \left[\frac{q_l}{q_h} - \frac{a_l}{a_h}\right] L\beta a_h. \tag{13}$$

The right-hand side is the incremental profit flow from a mass separating strategy, which must be equal to the current interest rate times the investment for process innovation.[21]

The *second* effect concerns the price setting behavior of exclusive producers. If the drop in inequality is substantial, it becomes attractive for exclusive producers to exploit the higher willingness to pay of the poor. An (endogenous) fraction of exclusive producers will set a price that equals the willingness to pay of the poor and sell temporarily to all households; and the remaining fraction of exclusive producers will still sell only to the rich at a price equal to their

[21] We have used condition (14) to eliminate the willingness-to-pay of rich and poor in the incremental profit flow, $L\left(q_l \mu_P(t) - (1-\beta) q_l \mu_R(t) - \beta a_l\right)$. The flow must be equal to $r_1 G$ since $V_P(t) = G$ and thus $\dot{V}_P(t) = 0$.

(high) willingness to pay.[22] During the first transition period $t \in (t_0, t_1)$ exclusive producers are indifferent between setting a low price and selling to all households and setting a high price and selling only to the rich, i.e. we must have

$$L\left(q_h \mu_P(t) - a_h\right) = L(1-\beta)\left(q_h \mu_R(t) - a_h\right). \tag{14}$$

It is also interesting to look at optimal consumption choices during transition. We need to adjust the Euler equation for the rich. Recall that consumption expenditures are $q_h \mu_R(t) N(t)$ and, since in transition $\dot{N}(t)/N(t) = 0$, consumption expenditures grow at rate $\dot{\mu}_R(t)/\mu_R(t)$. The Euler equation therefore determines the growth rate of the willingness to pay of the rich

$$\frac{\dot{\mu}_R(t)}{\mu_R(t)} = r_1 - \rho. \tag{15}$$

When the drop in inequality is substantial, poor households' consumption expenditures in the first transition period are $\mu_P(t) c_P(t)$ where $c_P(t)$ is the consumption aggregator for the poor households (see section 3.3). The Euler equation of a poor household therefore is

$$\frac{\dot{\mu}_P(t)}{\mu_P(t)} + \frac{\dot{c}_P(t)}{c_P(t)} = r_1 - \rho. \tag{16}$$

Because (14) must hold during the first transition phase, it must be that $\mu_P(t)$ increases at a smaller rate than $r_1 - \rho$.[23] Consequently, $\dot{c}_P(t)/c_P(t) > 0$. Denote by $N_P(t)$ the number of goods that the poor can afford. During the first period of transition we have $N_P(t) > M(t)$ and $c_P(t) = q_l M(t) + q_h(N_P(t) - M(t))$. Since $M(t)$ grows faster than $N_P(t)$, there is a date $t = t_1$ where we have reached $M(t_1) = N_P(t_1)$. From date t_1 onwards we have $c_P(t) = q_l M(t)$. The equal-profit condition (14) does not hold anymore and exclusive producers are strictly better off selling only to the rich. $\mu_R(t)$ continues to grow at rate $r(t) - \rho$, but $\mu_P(t)$ grows more slowly. Interest rates are no longer constant, but still determined by incremental profit flows and investment costs for process innovation.

The final law of motion comes from the resource constraint. Recalling that in the entire transition period we have $\dot{N}(t) = 0$ and $N(t) = N(t_0)$ we can write

$$\dot{M}(t)G/L = A(N(t_0), M(t)) - \beta\left[M(t)a_l + (N_P(t) - M(t))a_h\right] - (1-\beta)N(t_0)a_h. \tag{17}$$

Moreover, we have initial conditions $M(t_0) = mN(t_0)$ and $N(t_0)$, and transversality conditions for rich and poor households. At date t_2, the economy reaches the new balanced growth

[22] The fraction of exclusive producers that sell to all households depends on the extent to which the consumer budget of the poor exceeds the spending on mass consumption goods. In the transition, as the fraction of mass producers increases the share of exclusive producers that sell to all households decreases. By date t_1 the number of firms that have adopted mass production has increased sufficiently so that the optimal spending of the poor exactly coincides with spending on mass consumption goods only.

[23] From (14) it is straighforward to calculate $\dot{\mu}_P(t)/\mu_P(t) = \left[(1-\beta)\dot{\mu}_R(t) + \beta a_h\right] / \left[(1-\beta)\mu_R(t) + \beta a_h\right] < \dot{\mu}_R(t)/\mu_R(t) = r_1 - \rho$

equilibrium with $m(t) = m'$ in finite time as soon as product innovation becomes attractive again, $r(t)F = L(1-\beta)(q_h\mu_R(t) - a_h)$.

Note that in the opposite case of a decrease in (θ_v, θ_ℓ), raising inequality, one can show that innovation is purely directed to product innovation during the transition. A phase in which one engine of growth stops temporarily is not specific to our set-up. See Matsuyama (1999) for another example where in one phase product variety expansion stops, while the economy accumulates physical capital. In our framework, expansion of variety stops while the economy accumulates process innovation. In fact, this transition closely resembles the related work of directed technical change (see Proposition 1 of Acemoglu and Zilibotti, 2001), where only one type of innovation takes place outside the balanced growth equilibrium. Alternatively, Galor and Moav (2004, 2006) have developed models where in the early stages physical capital accumulation was the prime source of growth, while in latter stages human capital emerged as growth engine.

To sum up, a substantial drop in inequality may trigger a period of industrial change where innovation activity is purely directed towards process innovation. Such a transition could have been triggered by the substantial drop in inequality during the Great Depression and WWII, helping to explain the boom in consumer durables in the U.S. in the post-war era.

6.2 Positive productivity shock

Process innovations, such as the introduction of assembly lines, play an important role in the emergence of modern mass consumption markets. In this subsection we study an economy in a stagnant/low-growth state where process innovation initially is too expensive or not available at all (G prohibitively high). If a positive productivity shock lowers G sufficiently, the economy experiences a takeoff, transforming a stagnant (or low-growth) highly exclusive economy into an economy with high consumer-participation and growth.

Initial exlusive stage Suppose that, initially, the economy is characterized by a balanced growth equilibrium where process innovations are absent altogether. More precisely, assume initially G is too high to make process innovations sufficiently attractive. In such a steady state the economy invests only in product innovations. Active firms do not have access to the mass production technology. (Think of the high quality as goods produced by craftsmen. The poor households can only afford a very limited subset of these expensive, hand-crafted goods, e.g. one set of furniture which holds for a lifetime or one tailored suit.) Hence the initial equilibrium is characterized by a situation where a fraction $n_P = N_P(t)/N(t)$ of producers serve the entire customer base at price $q_h\mu_P$ and a fraction $1-n_P = (N(t)-N_P(t))/N(t)$ sells their product only to the rich at price $q_h\mu_R$. Lifetime income of houshold i still is $w(t)\ell_i/\rho + v_i(t)$. However, since the (initial) balanced growth equilibrium features $m = 0$, we have $w(t) = A(t) = N(t)\psi^{1/\gamma}$ and $v(t)L = N(t)F$. The relative budget constraint of a rich to a poor household (5) now becomes

$$\frac{(1-n_P)\mu_R + n_P\mu_P}{n_P\mu_P} = \xi(0) \equiv \frac{\rho(1-\beta\theta_v)F + (1-\beta\theta_\ell)L\psi^{1/\gamma}}{\rho(1-\beta)\theta_v F + (1-\beta)\theta_\ell L\psi^{1/\gamma}},$$

and the no-arbitrage and resource curves read

$$\begin{aligned} g &= \frac{L(1-\beta)}{F}\left[\frac{\beta a_h}{(1/n_P - 1)\,/\,(\xi(0)-1) - (1-\beta)} - a_h\right] - \rho, \text{ and} \\ g &= \frac{L[\psi^{1/\gamma} - (1-\beta+n_P\beta)a_h]}{F}. \end{aligned}$$

In this initial stage, the long-run growth performance of the economy is weak because technical progress is only fueled by product R&D whereas process R&D projects are not undertaken at all. As a result manufacturing acitivities and product invention is relatively unproductive (high $\tilde{F}(t)$ and $\tilde{a}_h(t)$). In this initial stage, raising inequality clearly is beneficial for growth as higher exclusion frees up resources for product R&D.

Transition Consider an exogenous positive productivity shock, $G' < G$, lowering investment costs of process innovations sufficiently such that

$$rG' < \left[\frac{q_l}{q_h} - \frac{a_l}{a_h}\right] L\beta a_h < rG. \tag{18}$$

The incremental profit flow of having implemented process innovation must be greater than prevailing interest rates times the investment amount, G'.[24] Process innovations become attractive once productivity gains, a_l/a_h, sufficiently outweigh quality discounts, q_l/q_h. Such a positive supply shock triggers an industrial revolution in which a series of process innovation transforms the initial exclusive society into a modern mass consumption society.

FIGURE 9

Figure 9 displays the evolution of the economy around the transition from an exclusive to a mass consumption society. After the economy experiences a positive productivity shock lowering G at time t_0, product innovation temporarily halts as firms focus on innovating their manufacturing processes. In this phase, consumption of the rich stagnates, whereas the product range of the poor grows as they shift their consumption towards goods at lower prices and quality once available. After all mass producers have innovated their manufacturing process, product invention activities resume once the economy reaches the new balanced growth equilibrium with higher growth and lower exclusion in finite time. The transition process resembles the one following a shift in inequality from above (see Appendix E):

[24] Similarly to the first example of a transition, we have used condition (4) to eliminate the willingness-to-pay of rich and poor in the additional profit flow, $L\,(q_l\mu_P(t) - (1-\beta)q_l\mu_R(t) - \beta a_l)$, which initially must hold.

Proposition 6 *Suppose Assumption 1 holds after the shock: a) A substantial drop in process innovation costs, $G' < G$, at $t = t_0$ such that condition (18) holds, triggers a transition of finite duration $t \in (t_0, t_2)$ with $\dot{N}(t) = 0$ and $\dot{M}(t) > 0$. From t_2 onwards, the economy is in a new steady state with $m > 0$. b) Consumption of the rich stagnates at $c_R(t) = q_h N(t_0)$ during the entire transition. c) Consumption of the poor jumps to $c_P(t_0) = q_h N_P(t_0)$ at date t_0. During a first phase of the transition, $t \in (t_0, t_1)$, $c_P(t) = q_l M(t) + q_h(N_P(t) - M(t))$ grows at a rate lower than $\dot{M}(t)/M(t)$. During a second phase of transition $t \in (t_1, t_2)$, $c_P(t)$ and $M(t)$ grow pari passu.*

If poor households immediately stopped consuming higher quality goods, consumption of the poor would need to drop to zero, since immediately after the shock no firm is able to offer the low quality yet. This cannot be the case due to infinite marginal utility at zero consumption. Hence, there is an initial phase with $M(t) < N_P(t)$ corresponding to the initial phase following a drop in inequality, characterized by the dynamic system (14)-(17) with initial conditions $M(t_0) = 0$ and $N(t_0) > 0$, and transversality conditions. During the first phase $t \in (t_0, t_1)$ poor households purchase both high-quality goods produced with the inefficient technology and low-quality goods produced with the new mass production technology. From date t_1 onwards, only firms that have made the process innovation sell to the poor. In this second transition phase all R&D activity still consists of process innovation and only when the new balanced growth level of $m = M(t)/N(t)$ has been reached, firms start developing new products. Given the stagnant consumption of rich households, $\dot{N}(t) = 0$, prices for exclusive goods increase relative to mass goods until product innovation becomes attractive again, and the economy reaches the new balanced growth equilibrium, corresponding to the one of Proposition 2 with Assumption 1 holding. Growth is higher in the new balanced growth equilibrium if process innovation is sufficiently important for technical progress and productivity growth (if ψ and γ are not too high).

Process innovations are able to transform the initial stagnant/low-growth economy burdened by high exclusion into a modern mass consumption society characterized by significantly higher growth and lower exclusion. Notice that our results are quite different from those in Matsuyama (2002) who also studies the transition to a mass consumption society. In contrast to the learning-by-doing formulation of Matsuyama, where competitive firms experience technical progress due to past production experience, in our case intentional innovation activities drive the adoption of mass production technologies and the introduction of mass consumption goods. Hence, under certain parameter values, our analysis may feature a situation where, in the initial exclusive society, inequality is unambiguously beneficial for growth, while after the transition to a new steady state, the inequality-growth relationship may be turned upside down. Once mass production technologies break even, a more egalitarian society increases mass consumption markets fostering process innovation and brings the economy on a steeper long-run growth path.

7 Conclusion

In this paper we presented an endogenous growth model where firms invest both in product and process innovations. Product innovations (that open up completely new product lines) satisfy the luxurious wants of the rich. Subsequent process innovations (that decrease costs per unit of quality) transform the luxurious products of the rich into conveniences of the poor. A prototypical example for such a product cycle is the automobile. Initially an exclusive product for the very rich, the automobile became affordable to the middle class after the introduction of Ford's Model T, the car that "put America on wheels". We argue that recent economic history is full of examples where consumer durables followed a similar product cycle.

Our analysis shows that the extent of economic inequality in a society generates substantially different incentives for product and process innovation. An egalitarian society creates strong incentives to adopt mass production technologies that allow the production of low-quality low-cost versions of existing luxuries (such as Model T). In contrast, an unequal society creates strong incentives for product innovations (new luxuries). Depending on which type of innovative activity drives technical progress, economic inequality is harmful or beneficial for long-run growth. This distinct role of product and process innovations goes in an important way beyond standard R&D based growth models, in which process innovations and product inventions are often mathematically similar (Acemoglu, 2009). To investigate the role of income inequality, one must deviate from the standard homothetic preferences. If the wealthy upper class consumes both more and better goods than the large majority of poorer households, in line with both casual observation and empirical evidence, inequality shapes product markets and thus relative incentives for product versus process innovation.

Our framework is sufficiently simple and tractable so that we can characterize not only balanced growth paths but also transition processes. Studying transitional dynamics is not only interesting from a methodological point of view but is relevant to better understand episodes in recent economic history. For instance, our analysis has shown that a major redistribution of economic resources such as the fall in U.S. income inequality between the Great Depression and WWII may help to explain the post-war boom in consumer durables. Our analysis shows that a demand shock arising from a major income redistribution temporarily generates very strong incentives for process innovations and the introduction of mass consumption goods. Similarly, major technological inventions, such as the assembly line, also give temporary strong incentives to implement mass production technologies so that existing sectors – one after the other – adopt mass production technologies, leading to a trickle-down process from which the poor benefit disproportionately.

For the sake of simplicity and tractability, our model reduced the income distribution to two groups of households. A more general income distribution would smooth the product cycle with penetration levels following logistic Engel curves in the aggregate (rather than a jump as in the stylized case of two groups of consumers). A new producer would start out serving only the richest households and then, by setting lower prices, expand the market step-by-step (in the case of a discrete number of distinct groups) or continuously (in the case of a continuous endowment distribution). Once a certain "cut-off" date has been reached, the producer would invest in process innovation. However, apart from generating more realistic dynamics of product penetration, such a generalization – while substantially complicating the formal analysis – would add little additional economic insight to the model.

Our model has abstracted from continuous quality improvements of existing goods. It was assumed that quality adjustments occur only once – when the process innovation is made and the mass production technology together with a low-quality version of an existing luxury good is implemented. However, continuous quality improvements both of luxuries and mass consumption goods are important features of reality. Our model could be easily adapted to account for exogenous quality increases. If q_h and q_l increased at an exogenous rate, all features of our model would remain the same. At a more general level, understanding how the quality upgrading of existing products interacts with the degree of inequality in society is an interesting direction of future research.

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Appendix A: Proof of Proposition 1

Taking labor as the numeraire so that $w(t) = A(t)$, we can rewrite marginal costs $w(t)\tilde{a}_k(t) = a_k$ for $k \in \{l, h\}$ given spillovers. A mass producer selling the high quality to the rich and the low quality to the poor faces the following profit maximization problem:

$$\max_{p_h, p_l} \left[L(1-\beta)(p_h - a_h) + L\beta(p_l - a_l)\right],$$

s.t. (i) $p_h \leq q_h\mu_R$, (ii) $p_l \leq q_l\mu_P$, (iii) $q_h\mu_R - p_h \geq q_l\mu_R - p_l$, and (iv) $q_l\mu_P - p_l \geq q_h\mu_P - p_h$,

The constraints are based on the first-order conditions of households (2). (i) and (ii) ensure that households purchase the good (rationality constraints), and (iii) and (iv) ensure that rich households prefer to buy the high quality and poor the low (incentive constraints). Notice that a firm cannot separate the rich into the low quality and the poor into the high given the higher willingness to pay of the rich, $\mu_R > \mu_P$.[25]

Constraint (iii) and $\mu_R > \mu_P$ imply $q_h\mu_R - p_h \geq q_l\mu_R - p_l > q_l\mu_P - p_l$. Hence if constraint (ii) were inactive, so would be (i). But then the firm could increase both prices by the same amount without violating (iii) and (iv). Hence constraint (ii) must be active, $q_h\mu_R - p_h \geq q_l\mu_R - p_l > q_l\mu_P - p_l = 0$, which implies that constraint (iii) must be active, too. Otherwise the firm could increase the price of the high quality without violating constraints (iii) and (i). Since constraint (iii) is active, $q_h\mu_R - p_h = q_l\mu_R - p_l > q_l\mu_P - p_l = 0$, constraint (i) cannot be active. Rewritting the active constraint (iii), $p_h - p_l = q_h\mu_R - q_l\mu_R > q_h\mu_P - q_l\mu_P$ shows that constraint (iv) is not active as well. Hence constaints (ii) and (iii) are active, $p_l = q_l\mu_P$ and $q_h\mu_R - p_h = q_l\mu_R - p_l$, and a separating mass producer optimally sets prices $p_l = q_l\mu_P$ and $p_h = q_l\mu_P + (q_h - q_l)\mu_R$.

Recall that a mass producer has four other options besides separating the rich into the high quality and the poor into the low (h, l): sell the high quality only to rich $(h, 0)$ or to all households (h, h), or sell the low quality only to rich $(l, 0)$ or to all households (l, l). The five options yield the following profit flows:

$$\begin{aligned}
\pi_{h,0} &= L(1-\beta)(q_h\mu_R - a_h), \\
\pi_{h,h} &= L(q_h\mu_P - a_h), \\
\pi_{h,l} &= L\beta(q_l\mu_P - a_l) + L(1-\beta)((q_h - q_l)\mu_R + q_l\mu_P - a_h), \\
\pi_{l,l} &= L(q_l\mu_P - a_l), \\
\pi_{l,0} &= L(1-\beta)(q_l\mu_R - a_l).
\end{aligned} \tag{19}$$

It is easy to verify that if Assumption 1 holds, separating households (h, l) is an optimal

[25] Incentive constraints of $q_l\mu_R - p_l \geq q_h\mu_R - p_h$ and $q_h\mu_P - p_h \geq q_l\mu_P - p_l$ would require $(q_h - q_l)\mu_P \geq p_h - p_l \geq (q_h - q_l)\mu_R$, which cannot hold.

strategy for mass producers. Condition (i) $(q_h - q_l)\,\mu_R > a_h - a_l$ ensures that selling the low quality to all households (l, l) yields lower profits. Condition (iii) $q_l\mu_P - (1-\beta)\,q_l\mu_R - \beta a_l \geq 0$ ensures that selling only the high quality to rich households $(h, 0)$ yields equal or lower profits. And since condition (ii) $(1-\beta)\,(q_h\mu_R - a_h) \geq (q_h\mu_P - a_h)$ ensures that exclusive producers (weakly) prefer selling the high quality only to rich households instead to all, selling the high quality to all households (h, h) must generate lower profits for mass producers, as well. And finally, condition (i) also ensures that selling the low quality only to rich households $(l, 0)$ is inferior (to selling the high quality only to rich households and thus to separating households). Similarly for exclusive producers which can only supply the high quality, condition (ii) ensures that selling only to rich households is an optimal strategy.

If conditions (ii) and (iii) in Assumption 1 hold with strict inequality, exclusive producers sell only to the rich generating $\pi_e = \pi_{h,0}$, and mass producers separate households generating $\pi_m = \pi_{h,l}$, proofing part (a) of Proposition 1. When condition (ii) holds with equality, exclusive firms are indifferent between selling only to rich and to all households, $\pi_e = \pi_{h,0} = \pi_{h,h}$, proofing part (b). And when condition (iii) holds with equality, mass producers are indifferent between selling only to rich and selling to all, separating households, $\pi_m = \pi_e$, proofing part (c).

If Assumption 1 does not hold, it might be more profitable for exclusive producers to sell the high quality to all households and/or for mass producers to sell only one quality either only to rich or to all households. Appendix B takes into account the general equilibrium to say more about the different outcomes.

Appendix B: Proof of Proposition 2

In a first step we prove the following lemma stating the possible equilibrium outcomes:

Lemma 1 *In a balanced growth equilibrium, four outcomes are possible: (1) some firms sell the high quality only to rich while others the high quality to rich and the low quality to poor, (2) some firms sell the high quality only to rich while others the low quality to all, (3) all firms only sell the high quality, some only to rich while others to all, and (4) all firms only sell the low quality, some only to rich while others to all.*

Proof. In any equilibrium, among the five options of firms (see Appendix A), two are equilibrium strategies: some firms sell to all households since otherwise poor households would consume nothing, and some firms sell only to rich households.[26] This leaves six combinations of two strategies of which two can be ruled out:

Compare profit flows (equations 19) to see that "$(q_h - q_l)\mu_P > a_h - a_l$" $\Longrightarrow$ "$\pi_{h,h} > \pi_{l,l}$", "$(q_h - q_l)\mu_R > a_h - a_l$" $\Longrightarrow$ "$\pi_{h,0} > \pi_{l,0}$" and "$(q_h - q_l)\mu_R > a_h - a_l$" $\Longrightarrow$ "$\pi_{h,l} > \pi_{l,l}$". Since $\mu_R > \mu_P$, we have "$\pi_{h,h} \geq \pi_{l,l}$" $\Longrightarrow$ "$\pi_{h,0} > \pi_{l,0}$", "$\pi_{l,0} \geq \pi_{h,0}$" $\Longrightarrow$ "$\pi_{l,l} > \pi_{h,h}$", and "$\pi_{h,0} > \pi_{l,0}$" $\Longleftrightarrow$ "$\pi_{h,l} > \pi_{l,l}$". Hence we can rule out outcomes where some firms separate and other firms sell the low quality only to rich households. We can also rule out outcomes where some firms sell the high quality to all households and other firms the low quality only to rich households, which require $\pi_{l,0} - \pi_{h,0} \geq \pi_{l,l} - \pi_{h,h}$, implying $(q_h - q_l)\mu_P\,[1/\beta - (1-\beta)\mu_P/\beta/\mu_R] \geq a_h - a_l$. But from above we know that "$\pi_{l,0} \geq \pi_{h,0}$" $\Longrightarrow$ "$\pi_{l,l} > \pi_{h,h}$", implying $(q_h - q_l)\mu_P < a_h - a_l$, which contradicts the inequality in the previous sentence, since the term in the square bracket is smaller than one ($\mu_R > \mu_P$, and $\beta < 1$). ■

Note that in any balanced growth equilibrium one of the four outcomes prevails. On transitional equilibrium paths we have shown that, if $m(t)$ is too low, three strategies co-exist for general parameter values. Focusing on balanced growth paths, let us characterize these four outcomes in more detail starting with the one of the main text proofing Proposition 2.

A balanced growth equilibrium determined by (7) and (8) exists if Assumption 1 holds with strict inequalities. From Proposition 1 we know that exclusive producers sell only to the rich and mass producers separate households if Assumption 1 holds with strict inequalities. Hence one needs to compute μ_R and μ_P for a given set of parameters, using equations (7) and (8) and the expressions for prices of Section 4.3, and test whether Assumption 1 holds. If this

[26] Firms which sell to all households cannot charge the entire willingness to pay of the rich (even when separating households they need to leave an "informational rent" to incentivize rich households to buy the high quality). Hence rich would have no binding first-order condition (i.e. would not exhaust their budgets) if all firms sold to all households. We can rule out such equilibrium outcomes, since rich households would have an infinite willingness to pay and thus firms would have an incentive to sell only to rich households instead.

is the case, no firm has an incentive to deviate, and the outcome is indeed an equilibrium.[27] Computations have shown that Assumption 1 holds in a balanced growth equilibrium if the quality gap $q_h - q_l$ is sufficiently high relative to the cost gap $a_h - a_l$ and process innovation costs G; and if inequality is sufficiently high, i.e. the group of poor β is sufficiently large as well as θ not too high. If Assumption 1 is violated, alternative outcomes prevail (see below).

In order to determine existence of a positive growth equilibrium, denote the horizontal m-axis intercepts of the NA- and RC-curve as m_{NA} and m_{RC} (see Figure 4). If $m_{RC} < m_{NA}$ a positive balanced growth equilibrium must exist. The left hand side of (8) is increasing in ψ for $m < 1$. Hence, the RC-curve shifts downwards when ψ decreases. Thus, $m_{RC}|_{\psi>0} < m_{RC}|_{\psi=0} = (1-\beta)a_h/\,(1-\beta a_l)$, by using (8). Since $(1-\beta)a_h/\,(1-\beta a_l) < 1$ (otherwise RC and NA could not cross at $m < 1$ for $\psi = 0$ thereby violating assumption 1), the RC-curve (8) is fulfilled for $g > 0$ if $m = 1$. We derive a sufficient condition for $m_{RC} < m_{NA}$

$$(1-\beta)a_h/\,(1-\beta a_l) <$$
$$\theta\left(q_l/q_h + (1-\theta)G/F + (1-\theta)\left[\beta a_l - (1-\beta)a_h G/F\right]/\left[\rho F/L + (1-\beta)a_h\right]\right)^{-1} = m_{NA}.$$

Note further that the condition $m_{RC} < m_{NA}$ trivially holds if the RC-curve has a vertical axis intercept in the positive (m, g)-quadrant, which is true whenever $\psi^{1/\gamma} > (1-\beta)a_h$.

The balanced growth equilibrium is necessarily unique if the NA-curve is upward sloping (which holds true if $a_l\beta/(1-\beta) > a_h G/F$). The NA-curve is always convex in m. To see this, note that $\partial^2 q_h\mu_R/\partial m^2 = 2\zeta(1-\theta)\left[a_l\beta/(1-\beta) - a_h G/F\right]\left[\theta/m - \zeta\right]^{-3} > 0$ with $\zeta \equiv q_l/q_h + (1-\theta)G/F$. The definition of (6) requires that the nominator and the denominator have the same sign such that $q_h\mu_R > 0$. The RC-curve is concave when it is upward sloping, this holds true as $(\psi + (1-\psi)m^\gamma)^{1/\gamma}$ is a concave function. Hence, the curves can cross only once in only once in the positive (m, g)-quadrant as long as the horizontal m-axis intercept of an upward sloping NA-curve lies to the right of the RC-curve. For $m_{RC} \geq m_{NA}$ or a downward sloping NA-curve, a positive growth equilibrium exists as well but it is not necessarily unique.

Alternative equilibrium outcomes When one of the conditions in Assumption 1 is violated, alternative outcomes will arise. We briefly discuss these outcomes. *First*, mass producers may supply only the low quality to both poor and rich households while exclusive producers sell only to the rich. Along the lines of the main text, we can derive prices, a

[27] Furthermore, one can show that the equilibrium is unique by checking that firms have incentives to deviate in every alternative equilibrium outcome (see below).

no-arbitrage curve and a resource curve, respectively, for such an outcome,

$$
\begin{aligned}
p_l &= q_l\mu_P = (1+G/F)(1-\beta)\left(q_h\mu_R - a_h\right) + a_l, \\
p_e &= q_h\mu_R = \frac{a_l - (1+G/F)(1-\beta)a_h}{(1/m-1)\,/\,(\xi(m)-1) - (1-\beta)(1+G/F)}, \\
g &= \frac{L(1-\beta)}{F}\left[\frac{a_l - (1+G/F)(1-\beta)a_h}{(1/m-1)\,/\,(\xi(m)-1) - (1-\beta)(1+G/F)} - a_h\right] - \rho, \\
g &= \frac{L\left[(\psi + (1-\psi)m^\gamma)^{1/\gamma} - (1-m)(1-\beta)a_h - m a_l\right]}{F+Gm}.
\end{aligned}
$$

This outcome is qualitatively similar to the one we focus on with one difference: Even in the case of $A(t) = N(t)$, i.e. $\psi = 1$, inequality may be harmful for growth. For a sufficiently small a_l, the resource curve may be increasing in m. An increase in the fraction of mass producers m may set free resources for product R&D, as mass producers only use the less laborious process. Even though more goods are produced, less production labor is needed. If a_l is not sufficiently small, the results are analogous to the main text. The conditions for this case are that exclusive producers prefer selling only to rich, $(1-\beta)(q_h\mu_R - a_h) > (q_h\mu_P - a_h)$, and mass producers prefer selling the low quality to all households, $a_h - a_l > (q_h - q_l)\mu_R$ and $(q_l\mu_P - a_l) > (1-\beta)(q_l\mu_R - a_l)$.

Second, if process innovation costs are too high, no firm invests in mass production and firms either sell to rich or to all households. Such an outcome corresponds to the initial stage in Section 6.2, and the equilibrium is characterized by the equations presented there. Recall that in this equilibrium outcome, inequality unambiguously is beneficial for growth as the resource curve is downward sloping in n_P given the absence of process innovation. Process innovation costs are too high if $G > \max(\pi_{h,l}/\,(g+\rho) - F,\ \pi_{l,l}/\,(g+\rho) - F,\ \pi_{l,0}/\,(g+\rho) - F)$.

Third, the last outcome arises in the opposite case where the mass production technology is too attractive. The resulting outcome is qualitatively equivalent to the previous one (initial stage in Section 6.2), substituting $(F+G, a_l, q_l, \xi(0), 1)$ for $\left(F, a_h, q_h, \xi(1), \psi^{1/\gamma}\right)$, and arises if $F > \max(\pi_{h,0}/\,(g+\rho)\,, \pi_{h,h}/\,(g+\rho))$ and $a_h - a_l > (q_h - q_l)\mu_R$, that is if process innovation costs G are sufficiently low, and the quality gap $q_h - q_l$ relatively low compared to the cost gap $a_h - a_l$. Also in this equilibrium outcome, inequality is unambiguously beneficial for growth as the resource curve is downward sloping since all firms, even the one only selling to rich households, invest in the low-quality process innovation, $m = 1$.

Appendix C: Product cycles

Given the symmetry in preferences and technology in our model, firms are indifferent about the timing of process innovation as discussed in the section on R&D. The individual product cycle is indeterminate. There are two natural extensions to our model, either adjusting preferences or technology, which break this symmetry and thus replicate the empirically observed product cycles.

Hierarchic preferences Both intuitively and empirically, it makes sense that there is a *hierarchy of needs* as opposed to the symmetric preferences of the main model. Certain more *basic* goods have priority:

$$u(t) = \int_0^{N(t)} \xi(j)x(j,t)q(j,t)dj,$$

where we have added a *hierarchy weight* $\xi(j)$ to felicity which is strictly monotonically decreasing in j. Hence low-j goods get a higher weight than high-j goods, and thus households have a higher willingness to pay for low-j than for high-j goods. Product innovation R&D would focus on the lowest-j goods not yet invented. For balanced growth, the hierarchy weight needs to be a power function, $\xi(j) = j^{-\eta}$ (see Bertola, Foellmi, and Zweimüller, 2006, Chapter 12). The process innovation timing problem becomes:

$$\begin{aligned}
\max_{\Delta} V(j,t) &= \int_t^{t+\Delta} \pi_e(j,s)\exp(-rs)ds + \int_{t+\Delta}^{\infty} \pi_m(j,s)\exp(-rs)ds - G\exp(-r\Delta), \\
\pi_e(j,s) &= L(1-\beta)\left[j^{-\eta} q_h \mu_R(s) - a_h\right], \\
\pi_m(j,s) &= L\left[\beta\left(j^{-\eta} q_l \mu_P(s) - a_l\right) + (1-\beta)\left(j^{-\eta}\left((q_h - q_l)\mu_R(s) + q_l\mu_P(s)\right) - a_h\right)\right].
\end{aligned}$$

Profit flows depend on hierarchy levels and on time, as $\mu_R(t)$ and $\mu_P(t)$ are increasing at rate ηg.[28] Hence, the difference between profit flows from mass and exclusive strategies grows.[29] In equilibrium, firms start out being exclusive producers. As the difference narrows to $\pi_m(j,s) - \pi_e(j,s) = G$, it becomes optimal for firms to switch to mass strategy, Δ units of time after product innovation (using Leibniz rule). The size makes low-j goods more attractive to sell in mass consumption markets than high-j goods. Note that if we let $\eta \to 0$, the hierarchic preferences formulation converges to the symmetric case of the main text but with a determinate product cycle.

[28] In order that the no-arbitrage condition holds, the initial present value of every newly set up firm must equal F. Hence the hierarchy-independent part of the willingness-to-pay $\mu_i(t)$ must rise at $-\partial/\partial t\left(j^{-\eta}\right) = \eta g$ over time, in order that the overall willingness to pay for a good only depends on the time span since inception, and not on time.

[29] The revenues of mass producers must be higher in equilibrium. Otherwise, firms would never switch to mass strategies given process innovation costs. Note also that both revenue streams grow at the same rate. It follows that $\pi_m(j,s) - \pi_e(j,s)$ grows over time.

Hierarchic preferences generate a product cycle where firms initially sell goods exclusively to rich households given their high willingness to pay for new goods even if they are low on their priority list. After a certain period of time, firms invest in process innovation to tap mass consumption markets as their goods have climbed the *relative* hierarchic ladder, being transformed from luxuries into necessities.

Learning-by-doing Process innovation costs $G(t)$ are likely to differ across firms, and decrease with individual manufacturing experience, in contrast to the main model. Individual learning-by-doing lowers $G(j,t)$. In fact, instead of modelling process innovation as an intentional investment of $G(j,t)$ depending on manufacturing experience, it is instructive to analyze the case of process innovation as a pure (passive) by-product of manufacturing:

$$a(j,t) = (1-\Lambda(j,t))a/N(t), \quad \Lambda(j,t) = \int_{-\infty}^{t} \delta x(j,s) \exp(-\delta(t-s))ds,$$

where δ is the speed of learning as well as the depreciation rate of learning capital, and $a(j,t)$ and $x(j,s)$ productivity and production level of firm j (see Matsuyama, 2002). Further let us assume that there is only one quality level, $q=1$. Individual productivity of a firm increases due to individual cumulative manufacturing experience, as well as through spillovers from product innovation. In equilibrium, mass consumption markets are more attractive for higher productivity levels due to market size effects. Hence firms start out exclusively producing for rich households, and eventually become producers for the mass markets, after a determined time interval Δ:

$$\max_{\Delta} \int_0^{\Delta} (1-\beta)L\left[p_h - (1-\Lambda(j,t))\right]\exp(-rt)dt + \int_{\Delta}^{\infty} L\left[p_l - (1-\Lambda(j,t))\right]\exp(-rt)dt = F/a,$$

where p_h is the price charged by "exclusive producers", and p_l by "mass producers", and we set $w(t) = N(t)/a$ (numéraire). The maximized present value needs to be equal to set-up costs, $\tilde{F}(t)w(t) = F/a$ (given spillovers $\tilde{F}(t) = F/N(t)$), generating a no-arbitrage condition. The optimal period of time Δ for being an exclusive producer is determined by $p_l = (1-\beta)p_h + \beta\left[1 - L(1-\beta)\left[1-\exp(-\delta\Delta)\right] - \delta L/(r+\delta)\right]$,[30] and the fraction of mass producers by Δ:

$$m = 1 - \int_{-\Delta}^{0} gN(0)\exp(gt)dt/N(0) = \exp(-g\Delta).$$

The equilibrium can be analyzed by combining these equations with the Euler equation (4) and the relative budget constraint, which in this case is $\xi(m) = \left((1-m)p_h + mp_l\right)/mp_l$, to form a no-arbitrage curve in m and g, as above. The resource curve is determined by the resource constraint:

$$L = gF + \frac{aL}{N(t)}\left[\int_0^{mN(t)} (1-\Lambda(j,t))dj + (1-\beta)\int_{mN(t)}^{N(t)} (1-\Lambda(j,t))dj\right].$$

[30] Use Leibniz rule and the fact that $\Lambda(j,t) = L(1-\beta)\left[1-\exp(-\delta t)\right]$ if $t \leq \Delta t$, and $\Lambda(j,t) = L\left[1-\exp(-\delta t)\right] - \beta L\left[\exp(-\delta(t-\Delta t) - \exp(-\delta t)\right]$ if $t > \Delta t$.

Computations show that the resource curve may be rising or falling in m, depending on the strength of learning-by-doing (LBD). Raising inequality, decreasing θ, increases prices and decreases mass consumption markets, m, which tends to reduce resources required in manufacturing. However, by lowering aggregate manufacturing, economy-wide LBD is reduced. Either effect may dominate. Inequality hurts growth if LBD is the dominant driver of productivity growth in the economy, otherwise inequality is beneficial for growth. Hence, our results hold also in the case of the continuous LBD process innovation.

Appendix D: Transitional dynamics

When the economy operates off the balanced growth path, there are either only product innovations or only process innovations but not both:

Proof of Proposition 4 Suppose the economy is in an equilibrium but not necessary the steady state where both product and process innovation occur. Since $V_N(t) = F$ and $V_M(t) = G$ hold, the instantaneous interest rate is given by

$$r(t) = L\left[q_l\mu_P(t) - (1-\beta)q_l\mu_R(t) - \beta a_l\right]/G = (1-\beta)L(q_h\mu_R(t) - a_h)/F. \tag{20}$$

The Euler equations of rich and poor, and the resource constraint read

$$\begin{aligned}\dot{\mu}_R(t)/\mu_R(t) &= r(t) - \rho - \dot{N}(t)/N(t), \quad \dot{\mu}_P(t)/\mu_P(t) = r(t) - \rho - \dot{M}(t)/M(t),\\ \dot{M}(t)G + \dot{N}(t)F &= L\left(\psi N(t)^\gamma + (1-\psi)M(t)^\gamma\right)^{1/\gamma} - L\beta M(t)a_l - L(1-\beta)N(t)a_h.\end{aligned}$$

We reduce this system of differential equations to get a single equation in $\mu_R(t)$ and $M(t)/N(t)$. Rewrite the resource constraint

$$\frac{\dot{M}(t)}{M(t)}\frac{M(t)}{N(t)}G + \frac{\dot{N}(t)}{N(t)}F = L\left(\psi + (1-\psi)\left(\frac{M(t)}{N(t)}\right)^\gamma\right)^{1/\gamma} - L\beta\frac{M(t)}{N(t)}a_l - L(1-\beta)a_h \equiv \phi\left(\frac{M(t)}{N(t)}\right)$$

Rearranging (20) we get $q_l\mu_P(t) = (1-\beta)\left(q_l + q_hG/F\right)\mu_R(t) + \beta a_l - (1-\beta)a_hG/F$. We take the derivative and insert this into the Euler equation of the poor to get

$$\frac{\dot{\mu}_R(t)}{\mu_R(t) + \left[\beta a_l - (1-\beta)a_hG/F\right]/\left[(1-\beta)\left(q_l + q_hG/F\right)\right]} =$$
$$(1-\beta)\frac{L}{F}(q_h\mu_R(t) - a_h) - \rho - \left(\frac{M(t)}{N(t)}G\right)^{-1}\left(\phi\left(\frac{M(t)}{N(t)}\right) - \frac{\dot{N}(t)}{N(t)}F\right),$$

and use the Euler equation of the rich to form

$$\frac{\dot{\mu}_R(t)}{\mu_R(t) + \left[\beta a_l - (1-\beta)a_hG/F\right]/\left[(1-\beta)\left(q_l + q_hG/F\right)\right]} + \frac{\dot{\mu}_R(t)}{\mu_R(t)}\frac{F}{GM(t)/N(t)} =$$
$$(1-\beta)\frac{L}{F}(q_h\mu_R(t) - a_h) - \rho - \left(\frac{M(t)}{N(t)}G\right)^{-1}\left(\phi\left(\frac{M(t)}{N(t)}\right) - (1-\beta)L(q_h\mu_R(t) - a_h) + \rho F\right).$$

We see that $\dot{\mu}_R(t)$ is monotonically increasing in $\mu_R(t)$. Denote the steady state level of $\mu_R(t)$ by μ_R^{SS}. Therefore, if $\mu_R(t) > (<)\mu_R^{SS}$, $\mu_R(t)$ will grow (fall) without bound. Hence, there is only one equilibrium: $\mu_R(t)$ must immediately adjust to μ_R^{SS}. As $\mu_P(t)$ and $\mu_R(t)$ are monotonically related through (20) the analoguous holds true for $\mu_P(t)$ as well. We conclude that in the presence of both process and product innovations the economy is in steady state.

Hence, a change in parameter values leading to a balanced growth equilibrium with a higher $m = M(t)/N(t)$ and where Assumption 1 holds, triggers a sequence of adjustment with at most two phases with only process innovations and no product innovations. If the variety of consumption of the poor jumps to $N_P(t_0) > M(t_0)$ after a shock (which is necessarily the case if $M(t_0) = 0$), the economy enters phase 1:

Phase 1 (t_0, t_1) The laws of motion governing the initial phase (equations 13-17) are repeated and simplified here for convenience:

$$\begin{aligned}\dot{\mu}_R(t)/\mu_R(t) &= r_1 - \rho, \\ \dot{\mu}_P(t)/\mu_P(t) &= r_1 - \rho - \left[\dot{M}(t)q_l + \left(\dot{N}_P(t) - \dot{M}(t)\right) q_h\right] / \left[M(t)q_l + (N_P(t) - M(t))\, q_h\right], \\ \dot{M}(t)G/L &= A(N(t_0), M(t)) - \beta\left[M(t)a_l + (N_P(t) - M(t))a_h\right] - (1-\beta)N(t_0)a_h,\end{aligned}$$

and $\dot{N}(t) = 0$, with $r_1 = [q_l/q_h - a_l/a_h]\, L\beta a_h/G$, and $\mu_P(t) = (1-\beta)\mu_R(t) + \beta a_h/q_h$.

Moreover, we have initial values for the state variables, $N(t_0) > 0$, and $M(t_0) \geq 0$. The equal-profit and transversality conditions for rich and poor households fix initial values for the costate variables, $\mu_R(t_0)$ and $\mu_P(t_0)$ (and $N_P(t_0)$). Numerically, we solve the system by backward integration starting in the final balanced growth equilibrium and letting time run backward. Initial values of costate variables thus can be fixed by using final balanced growth equilibrium values as boundary conditions.

Since prices of mass and exclusive goods evolve differently, wealth inequality, $\theta_v(t)$, changes during the transition,

$$\begin{aligned}\dot{v}_R(t) &= r_1 v_R(t) + (1-\beta\theta_\ell)/(1-\beta)A(N(t_0), M(t)) - [N(t_0) - N_P(t)]\, q_h\mu_R(t) - \\ &\quad [N_P(t) - M(t)]\, q_h\mu_P(t) - M(t)\left[(q_h - q_l)\mu_R(t) + q_l\mu_P(t)\right], \\ \dot{v}_P(t) &= r_1 v_P(t) + \theta_\ell A(N(t_0), M(t)) - [N_P(t) - M(t)]\, q_h\mu_P(t) - M(t)q_l\mu_P(t).\end{aligned}$$

Initial wealth inequality can be fixed using final values as boundary conditions if we know final wealth inequality, θ_v'. If we know initial wealth inequality instead, $\theta_v(t_0)$, we guess final wealth inequality, shoot backward, and check whether the resulting initial wealth inequality corresponds to the true value. This process is reiterated with new guesses until a sufficiently close value is found (see below).

Finally, the economy exits phase 1 and enters phase 2 as soon as $N_P(t) = M(t)$.

Phase 2 $[t_1, t_2)$ If all mass producers have invested in process innovation, $N_P(t) = M(t)$, we need to adjust the laws of motion as follows:

$$\begin{aligned}\dot{\mu}_R(t)/\mu_R(t) &= r(t) - \rho, \\ \dot{\mu}_P(t)/\mu_P(t) &= r(t) - \rho - \dot{M}(t)/M(t), \\ \dot{M}(t)G/L &= A(N(t_0), M(t)) - \beta M(t)a_l - (1-\beta)N(t_0)a_h,\end{aligned}$$

and $\dot{N}(t) = 0$, with $r(t) = [q_l\mu_P(t) - (1-\beta)q_l\mu_R(t) - \beta a_l]\, L/G$. Since all mass producers have innovated, the equal profits equation does not need to hold anymore, and interest rates are no longer constant. Initial values of state variables are given by the values at the end of phase 1,

$N(t_0)$ and $M(t_1)$. Final conditions using backward integration fix the level of costate variables, $\mu_R(t)$ and $\mu_P(t)$. Wealth accumulation is

$$\begin{aligned}
\dot{v}_R(t) &= r(t)v_R(t) + (1-\beta\theta_\ell)/(1-\beta)A(N(t_0), M(t)) - \\
&\quad [N(t_0) - M(t)]\, q_h\mu_R(t) - M(t)\left[(q_h - q_l)\mu_R(t) + q_l\mu_P(t)\right], \\
\dot{v}_P(t) &= r(t)v_P(t) + \theta_\ell A(N(t_0), M(t)) - M(t)q_l\mu_P(t).
\end{aligned}$$

The economy exits phase 2 once product innovation becomes attractive again, $r(t)F = (1-\beta)L(q_h\mu_R(t) - a_h)$, and enters the new balanced growth equilibrium (given Proposition 4).

Note that an economy never skips phase 2 in a transition to a higher m', directly entering the new balanced growth path after phase 1 (i.e. every such transition contains phase 2). Since in phase 1, exclusive producers make equal profits selling only to rich or to all households, and this is not the case in the final steady state (given Assumption 1 with strict inequalities), there needs to be a phase where prices adjust accordingly (as costate variables cannot jump expectedly).

A note on stability Finally, transitional dynamics and numerical simulations allow us to analyze the stability of the balanced growth equilibrium of Section 5. For most parameter values, the equilibrium is globally saddle path stable.[31] If the economy starts with a too low m, we enter a transitional phase characterized above with no product innovation and only process innovation. In contrast, if the economy starts with a too high m, *mutatis mutandis*, society goes through a phase without process innovations and only product innovations, reaching the balanced growth equilibrium in finite time (with an initial phase where mass and exclusive producers earn equal profit flows, and some mass producers do not use the mass production process and only sell the high quality to rich households).

Some notes on the numerical simulation procedure

> "[F]or yourself, sir, shall grow old as I am, if like a crab you could go backward."
> Hamlet to Polonius, William Shakespeare, *Hamlet*, 2.2,200-201

We use backward integration (Brunner and Strulik, 2002) to tackle transitional dynamics, analyzing the dynamic system numerically with the Mathematica procedure "NDSolve". However, since transition is finite and has different phases, we need to make adjustments to the standard procedure. Let us briefly outline the key steps:

1. We start by solving the final and the initial balanced growth equilibrium.

[31] For some special parameter values there are multiple balanced growth equilibria with a high and an intermediate growth equilibrium which is unstable.

2. Using the differential equations derived above, we let time run backward by multiplying the right-hand side of the ordinary differential equation system with the scalar (-1).

3. Hence, we start in phase 2, solve for the path of state and costate variables, then solve phase 1, using "NDSolve".

4. To determine at what point the economy switches to the preceding phase we keep track of the no-arbitrage conditions. As an example, going backward in phase 2, as soon as the mass high strategy becomes attractive, we have reached the start of phase 2, and thus the end of phase 1. The values of state/co-state variables at the calculated point of time serve as ending values for the preceding phase.

5. If we are in phase 1, time running backwards, as soon as $m(t)$ hits the initial value, we know that we are at the time of the shock, t_0.

6. Having programed all phases, we need to take the final balanced growth equilibrium state/co-state variables and let time run backward. Note that since in our model the transition period is finite, we do not need to perturb final balanced growth path values slightly as would be the case in the standard procedure if convergence were asymptotic. We simply need to start with the dynamic system of phase 2 using the exact values of the final balanced growth path variables.

7. If we know final wealth inequality θ_v', we can track wealth levels backward (using the wealth accumulation equations), computing θ_v at time t_0. If we know initial wealth inequality $\theta_v'(t_0)$ instead, we must guess final wealth inequality, shoot backward, and check whether the resulting initial wealth inequality corresponds to the true value. This process must be reiterated with new guesses until one is sufficiently close to the true value.

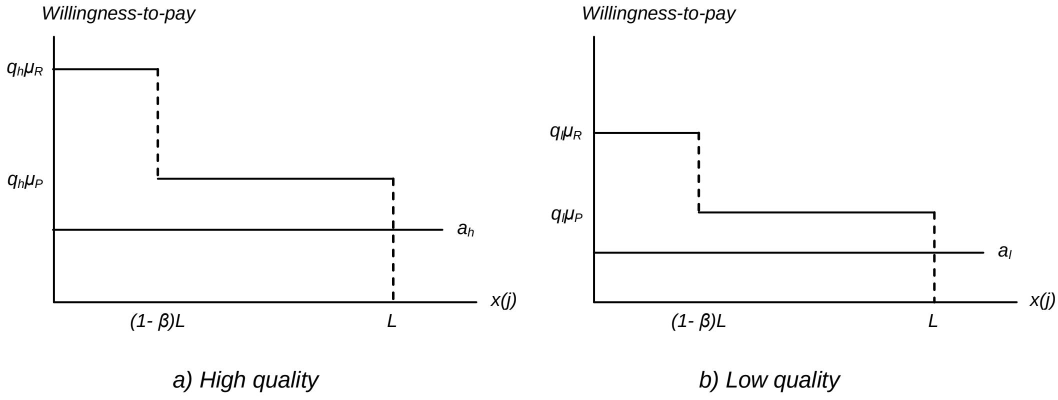


Figure 3: Market demand for high and for low quality

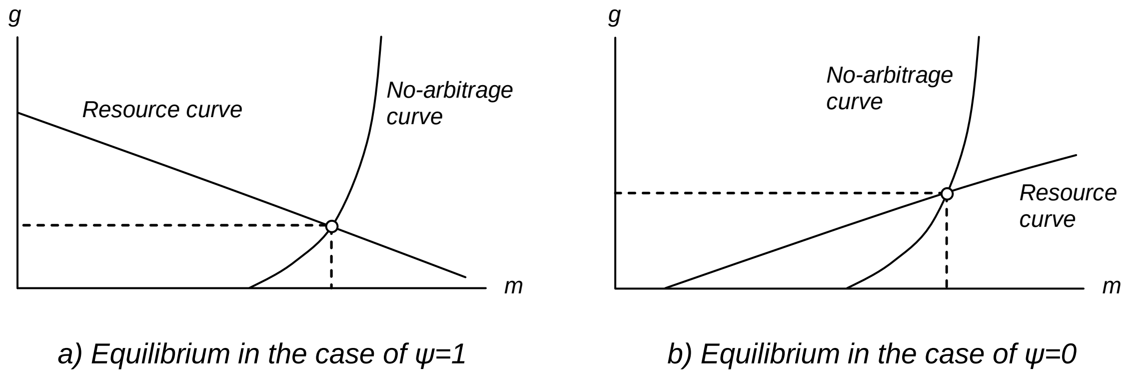


Figure 4: Balanced growth equilibrium

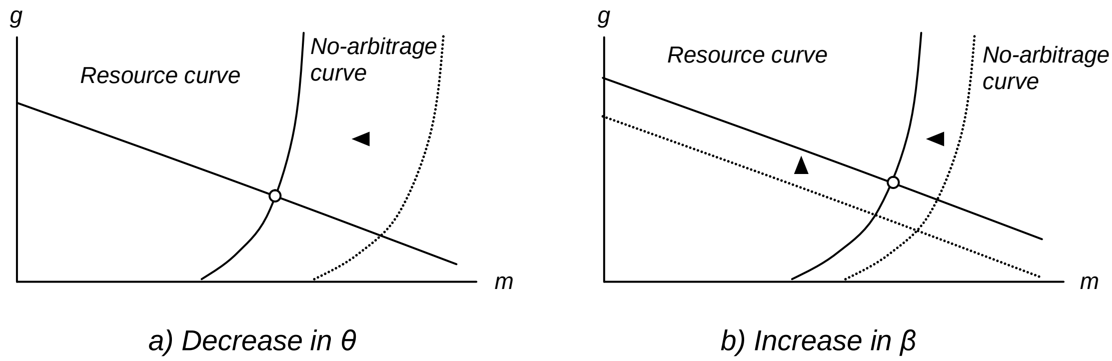


Figure 5: Impact of inequality in the case of $\psi = 1$

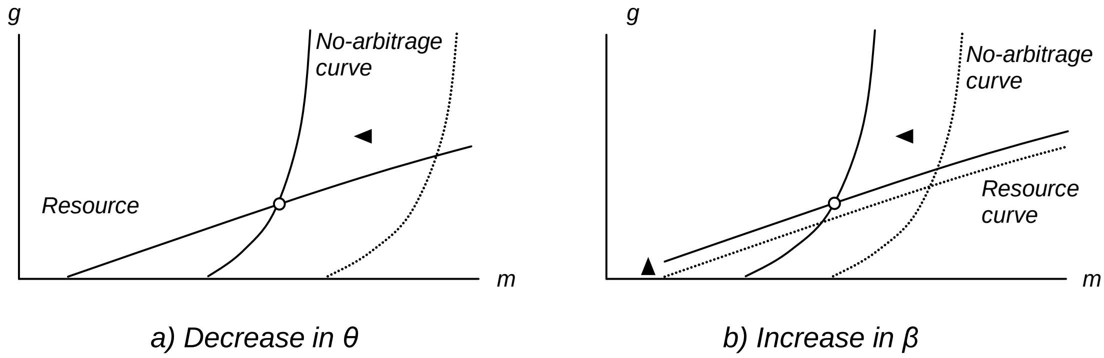


Figure 6: Impact of inequality in the case of $\psi = 0$

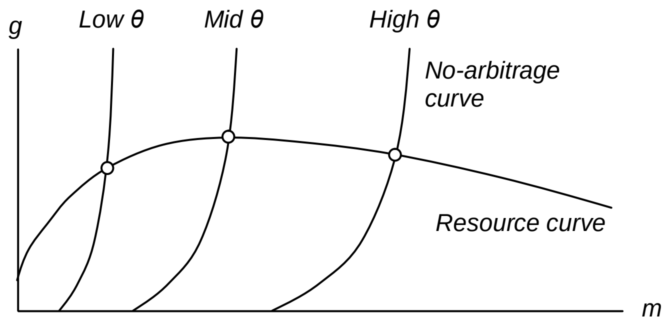
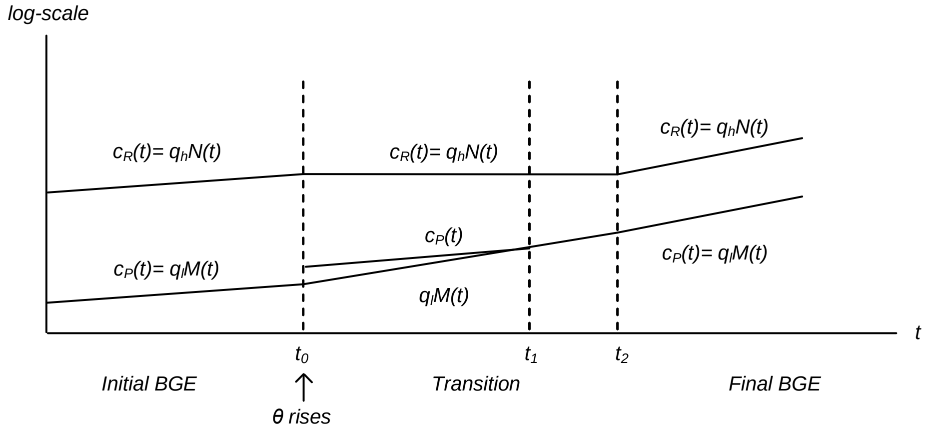


Figure 7: Impact of inequality in the case of $\psi \in (0, 1)$

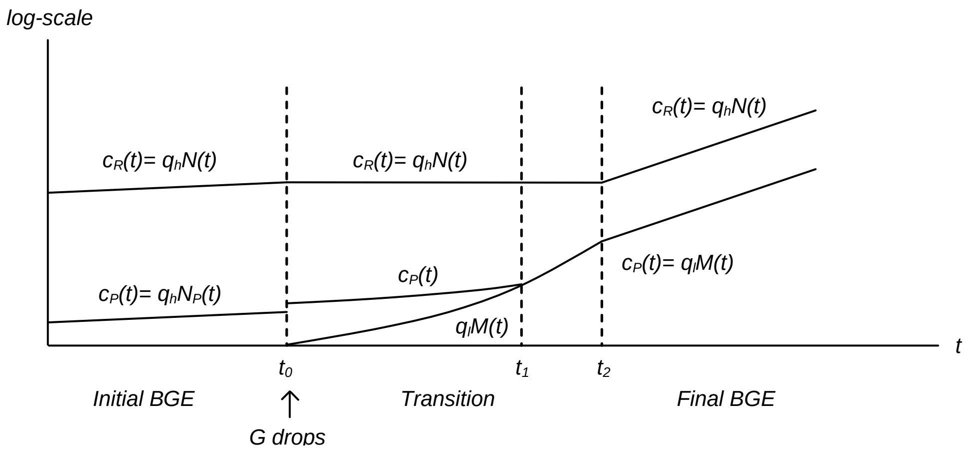


$q_lM(t)$ ... Consumption of mass goods of poor households

$c_P(t)= q_lM(t)+q_h[N_P(t)-M(t)]$ ... Consumption of poor households

$c_R(t)= q_hN(t)$ ... Consumption of rich housholds

Figure 8: Drop in inequality



$q_lM(t)$ ... Consumption of mass goods of poor households

$c_P(t)= q_lM(t)+q_h[N_P(t)-M(t)]$ ... Consumption of poor households

$c_R(t)= q_hN(t)$ ... Consumption of rich housholds

Figure 9: Positive productivity shock