Policy at the Zero Bound

Isabel Correia, Emmanuel Fahri, Juan Pablo Nicolini and Pedro Teles

Banco de Portugal, Universidade Catolica Portuguesa and CEPR;

Harvard University

Minneapolis Fed and Universidad Di Tella;

Banco de Portugal, Universidade Catolica Portuguesa and CEPR.

I. Introduction

Arbitrage between money and bonds restricts nominal interest rates from becoming negative. If interest rates are close to zero, they cannot be further reduced. At the zero bound, policy appears to be powerless in the event of a severe recession. If the objective of policy is to lower real rates of interest, how can that be accomplished when nominal rates cannot be used?

If nominal rates cannot be lowered, real rates can still be low, if expected inflation is high. But getting all prices in the economy to move together, in response to aggregate conditions, so that expected inflation is high, may be costly. It may also be hard to implement, if there are commitment issues. If the central bank has had a good record in fighting inflation, how can the public be convinced that it will be doing otherwise?. Are there alternative policies that can implement low real rates?

This issue received considerable attention following the prolongued recession that Japan experienced during the 90´s, while nominal interest rates were at their lower bound (see Eggertsson and Woodford (2003) and Eggertsson and Woodford, 2004a and 2004b). A lot more attention was placed on this issue in recent times, following the outbreak of the financial crisis in 2007-2008. Nominal interest rates have indeed been very close to zero in the US, the EMU, the UK and other countries. There has been work on public spending multipliers, showing that these can be very large at the zero bound (see Christiano, Eichenbaum, rebelo, 2009). There has also been work on tax policy. Eggertsson (2009) considers different taxes and assesses which ones are the most desirable. All this work is done in standard sticky price models, where the zero bound on interest rates can indeed be a serious challenge to policy. The zero bound is also a key component in the numerical work presented in Romer and Bernestein (2009) as well as in the reply by Coglan, Cwick, Taylor and Wieland (2010). It is always a main concern in Blanchard, Dell'Ariccia and Mauro (2010), in which they argue for a better integration between monetary and fiscal policy, an issue that is direclty addressed here.

In this paper, we move further, and show that the zero bound constraint on interest rates, is non-binding if consumption taxes are used to stabilize the economy. Since the zero bound is relevant only in exceptional circunstances, it is natural to think that consumption taxes can be used, as exceptional measures. The argument that consumption taxes neutralize the effects of the zero bound on the policy response is very simple. We mentioned that the objective of policy is to lower real rates, without creating inflation. But the inflation that is costly is producer price inflation. It may be hard to get all producers in the economy to raise all future prices uniformly, but inflation arrising from future consumption taxes is easy to achieve. Can be announced and implemented at zero cost, and will certainly bring down real interest rates.

We show that once monetary and fiscal policy are jointly considered, the zero bound on nominal interest rates is not a binding constraint on policy during a severe recession. We show in a particular example how the appropriate choice of consumption and labor income taxes can implement the same allocation that would be achieved if nominal interest rates could be reduced following a negative shock. This is true for any value of the nominal interest rate at the begining of the contraction, even if it is zero.

The result is an application of a general result in Correia, Nicolini, and Teles (2008), that sticky prices do not matter for policy, provided both monetary and fiscal policy are used for stabilization. Since the zero bound is irrelevant under flexible prices, it is also irrelevant under sticky prices. Here we derive it in a standard new-keynesian framework, which is not the case in that paper. It has been argued that fiscal instruments are not as flexible as monetary policy instruments. Whether this argument applies to stabilizion policy during a "great moderation" period could be argued about. However, we believe it does not apply to the either the recent crisis or to the Japanesse economy in the nineties, precisely because the need to use fiscal instruments is exceptional. There have been recent policy proposals in this direction by Bob Hall and Susan Woodward,[1] and earlier on, by Feldstein (2003), intended at Japan. And it is one of the messages in the discussion of the future of macroeconomic policy in Blanchard et. al. (2010).

Poterba, Rotemberg and Summers (1986) also approach the issue of how consumption taxes interact with sticky prices in a paper aimed at testing nominal rigidities.

II. Model

The model we analyze is very similar to the one studied by Eggertsson and Woodford (2003) and (2004b). The preferences of the households are described by:

$$U = E_0 \sum_{t=0}^{\infty} \beta^t \left[u\left(C_t, N_t, \xi_t\right) + vV\left(\frac{M_t}{\left(1+\tau_t^c\right) P_t}\right) \right] \tag{1}$$

where

$$C_t = \left[\int_0^1 c_{it}^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}, \theta > 1, \tag{2}$$

where c_{it} is consumption of variety $i \in [0,1]$, N_t is total labor, $\frac{M_t}{(1+\tau_t^c)P_t}$ are real money balances and ξ_t is a preference shock. The functions U and V are concave and ν is a parameter that controls the velocity of money. The separability assumption is an important assumption. Money is determined residually. For $v = 0$, money does not contribute to welfare.

Aggregate government purchases G_t,

$$G_t = \left[\int_0^1 g_{it}^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}, \tag{3}$$

are exogenous. The government minimizes the expenditure on the individual goods, for a given aggregate, and finances it with time varying taxes on consumption, τ_t^c, on labor income, τ_t^n, and on profits, τ_t^d, and by printing money, M_t. We assume that profits are fully taxed, $\tau_t^d = 1$, and that initial wealth is zero which is equivalent to assuming that it is also fully taxed. We also allow for lump-sum taxes, T_t, which is a residual variable that adjusts so that the government government budget constraint is satisfied.[2] This assumption is without loss of generality.

If we let

$$P_t = \left[\int_0^1 {p_{it}}^{1-\theta} di \right]^{\frac{1}{1-\theta}}, \tag{4}$$

then, the minimization of expenditure on the individual goods, implies

$$\frac{c_{it}}{C_t} = \left(\frac{p_{it}}{P_t} \right)^{-\theta}, \tag{5}$$

and

$$\frac{g_{it}}{G_t} = \left(\frac{p_{it}}{P_t} \right)^{-\theta}. \tag{6}$$

The budget constraints of the households can be written in terms of the aggregates as

$$M_{t+1} + \frac{1}{R_t}\overline{B}_{t+1}^h + \sum_{s^{t+1}/s^t} Q_{t,t+1}B_{t,t+1} = M_t - (1+\tau_t^c)\, P_t C_t + \overline{B}_t^h + B_{t-1,t}^h + (1-\tau_t^n)\, W_t N_t - T_t,\ t \geq 0$$

together with a no-Ponzi games condition, $lim_{T\to\infty} Q_{0,T}\left[\overline{B}_T + \sum_{s^{T+1}/s^T} Q_{T,T+1} B_{T,T+1}\right] \geq 0$. $B_{t,t+1}$ represent the quantity of state contingent bonds that pay one unit of money at time $t+1$, in state s^{t+1} and $\overline{B}^h_{t+1}$ are risk free nominal bonds. $Q_{t,t+1}$ is the price of the state contingent bond and $\frac{1}{R_t}$ is the price of the riskless bond - so R_t is the gross nominal interest rate. We are assuming a particular timing of transactions: The households get into the period with money holdings that they obtained the period before.

Households problem The first order conditions of the households problem include

$$-\frac{u_C\left(C_t, N_t, \xi_t\right)}{u_N\left(C_t, N_t, \xi_t\right)} = \frac{\left(1+\tau_t^c\right) P_t}{\left(1-\tau_t^n\right) W_t}$$

$$E_t\left[\frac{\nu V_M\left(\frac{M_{t+1}}{\left(1+\tau_{t+1}^c\right) P_{t+1}}\right)}{P_{t+1}\left(1+\tau_{t+1}^c\right)}\right] = (R_t - 1) E_t\left[\frac{u_C(\left(C_{t+1}, N_{t+1}, \xi_{t+1}\right))}{P_{t+1}\left(1+\tau_{t+1}^c\right)}\right]$$

for $\nu > 0$,[3] and

$$\frac{u_C\left(C_t, N_t, \xi_t\right)}{P_t\left(1+\tau_t^c\right)} = \beta R_t E_t \frac{u_C\left(C_{t+1}, N_{t+1}, \xi_{t+1}\right)}{P_{t+1}\left(1+\tau_{t+1}^c\right)}$$

Firms The production function of each good i, y_{it}, uses labor only, n_{it}, and is given by

$$y_{it} = A_t n_{it},$$

where A_t is an aggregate productivity shock.

We assume that prices are set as in Calvo (1983). Every period, a firm is able to revise the price with probability $1-\alpha$. Since there is a continuum of firms, $1-\alpha$ is also the share of firms that are able to revise prices. Those firms choose the price p_t to maximize profits

$$E_t \sum_{t=0}^{\infty} (\alpha\beta)^j Q_{t,t+j} \left[p_t y_{t+j} - \frac{W_{t+j}}{A_{t+j}} y_{t+j} \right]$$

where output y_{t+j} must satisfy the technology constraint and the demand function

$$y_{t+j} = \left(\frac{p_t}{P_{t+j}} \right)^{-\theta} Y_{t+j}.$$

obtained from (5) and (6), where $y_{t+j} = c_{t+j} + g_{t+j}$, and $Y_{t+j} = C_{t+j} + G_{t+j}$. The optimal price set by these firms is

$$p_t = \frac{\theta}{(\theta - 1)} \sum_{t=0}^{\infty} \eta_{t,j} \frac{W_{t+j}}{A_{t+j}}, \tag{7}$$

where $\eta_{t,j} = \frac{(\alpha\beta)^j \frac{U_C(t+j)}{\left(1+\tau_{t+j}^C\right)} (P_{t+j})^{\theta-1} Y_{t+j}}{E_t \sum_{t=0}^{\infty} (\alpha\beta)^j \frac{U_C(t+j)}{\left(1+\tau_{t+j}^C\right)} (P_{t+j})^{\theta-1} Y_{t+j}}$.

The price level can be writen as

$$P_t = \left[(1-\alpha) p_t^{1-\theta} + \alpha P_{t-1}^{1-\theta} \right]^{\frac{1}{1-\theta}}. \tag{8}$$

Equilibria In equilibrium, it must be that

$$c_{it} + g_{it} = A_t n_{it}$$

and

$$N_t = \int n_{it} di. \tag{9}$$

Using the demand functions (5), (6), it follows that

$$C_t + G_t = \left[\int_0^1 \left(\frac{p_{it}}{P_t} \right)^{-\theta} di \right]^{-1} A_t N_t. \tag{10}$$

An equilibrium for $\{C_t, N_t\}$, $\{P_t, W_t\}$, and $\{R_t, \tau_t^c, \tau_t^n\}$ is characterized by

$$-\frac{u_C(C_t, N_t, \xi_t)}{u_N(C_t, N_t, \xi_t)} = \frac{(1+\tau_t^c) P_t}{(1-\tau_t^n) W_t} \tag{11}$$

$$\frac{u_C(C_t, N_t, \xi_t)}{(1+\tau_t^c) P_t} = E_t \left[R_t \frac{\beta u_C(C_{t+1}, N_{t+1}, \xi_{t+1})}{(1+\tau_{t+1}^c) P_{t+1}} \right] \tag{12}$$

$$p_t = \frac{\theta}{(\theta - 1)} E_t \sum_{t=0}^{\infty} \eta_{t,j} \frac{W_{t+j}}{A_{t+j}}, \tag{13}$$

$$P_t = \left[(1-\alpha) p_t^{1-\theta} + \alpha P_{t-1}^{1-\theta} \right]^{\frac{1}{1-\theta}}$$

$$C_t + G_t = \left[\sum_{j=0}^{t+1} \varpi_j \left(\frac{p_{t-j}}{P_t} \right)^{-\theta} \right]^{-1} A_t N_t \tag{14}$$

ϖ_j is the share of firms that have set prices j periods before, $\varpi_j = (\alpha)^j (1-\alpha)$, $j = 0, 2, ..., t$, and $\varpi_{t+1} = (\alpha)^{t+1}$, which is the share of firms that have never set prices, so that they charge an exogenous price p_{-1}.

We do not need to keep track of the budget constraints, since lump sum taxes adjust to satisfy the budget.

Steady state Let $\xi_t = 1$, $G_t = G$, $A_t = 1$. There is a steady state in this economy, with zero producer price inflation, such that: $R_{t+1} = R$, $1+\tau_t^c = (1+\tau^c)\left(\gamma^{\tau^c}\right)^t$, $1-\tau_t^n = (1-\tau^n)\left(\gamma^{\tau^n}\right)^t$,

$\gamma^{\tau^c} = \gamma^{\tau^n} = \gamma$, and $C_t = C$, $N_t = N$, $P_t = P$, $p_t = P$, $W_t = W$. The steady state is restricted by

$$-\frac{u_C(C,N,\xi)}{u_N(C,N,\xi)} = \frac{(1+\tau^c)P}{(1-\tau^n)W} \tag{15}$$

$$R = \frac{\gamma}{\beta} \tag{16}$$

$$P = p = \frac{\theta}{(\theta-1)}\frac{W}{A}, \tag{17}$$

$$C + G = AN. \tag{18}$$

If the consumption taxes do not grow, $\gamma = 1$, then the steady state will have the nominal interest rate equal to the steady state real interest rate, $R = \beta^{-1}$.

The linearized model with preference shocks If we assume away productivity shocks ($A_t = 1$), consider the cashless case, $v = 0$, assume constant government consumption, $G_t = G$, and assume that $u_C(C_t, N_t, \xi_t) = u_C(C_t, N_t)\xi_t$, so that preference shocks do not affect the consumption-leisure margin, the following equations provide a log linear approximation[4] to the model above

$$\widehat{y}_t = E_t\widehat{y}_{t+1} - \sigma(i_t - E_t\pi_{t+1} - r_t) + \sigma(E_t\widehat{\tau}^c_{t+1} - \widehat{\tau}^c_t), \tag{19}$$

$$\pi_t = \kappa\widehat{y}_t + \kappa\psi\left(\widehat{\tau}^n_t + \widehat{\tau}^c_t\right) + \beta E_t\pi_{t+1}, \tag{20}$$

where $\pi_t = \ln\frac{P_t}{P_{t-1}}$, $i_t = \ln R_t$, $\widehat{y}_t = \ln\frac{Y_t}{Y}$, $\widehat{\tau}^c_t = \ln\frac{(1+\tau^c_t)}{(1+\tau^c)}$, $\widehat{\tau}^n_t = \ln\frac{(1-\tau^n_t)}{(1-\tau^n)}$, and $r_t = \ln\beta^{-1} + \ln\xi_t - E_t\ln\xi_{t+1}$. Note that i_t and r_t are in levels, while the other variables are in deviations to the steady state. That is only for the convenience of defining the lower bound.

We also assume that monetary policy follows an interest rate rule that explicitly takes into account the lower bound on nominal interest rates

$$i_t = \max\{0, r_t + \phi_\pi \pi_t + \phi_y \widehat{y}_t\}. \tag{21}$$

III. Avoiding a recession

A shock that lowers the real rate of interest Under flexible prices, the shock to preferences ξ_t would have no effects on the real allocation, it would only affect the real rate of interest. In the first best benchmark of the economy considered above, firms are competitive, prices are flexible, and distortionary taxes are not used. We look at equilibria with zero inflation, $\frac{P_{t+1}}{P_t} = 1$, but inflation is irrelevant in that economy. The economy would be characterized by the following two static conditions, for consumption, C_t, and labor, N_t, and the intertemporal condition for R_t:

$$-\frac{u_C(C_t, N_t, \xi_t)}{u_N(C_t, N_t, \xi_t)} = \frac{1}{A_t}, \tag{22}$$

$$C_t + G_t = A_t N_t, \tag{23}$$

$$R_t = \frac{u_C(C_t, N_t, \xi_t)}{E_t\left[\beta u_C\left(C_{t+1}, N_{t+1}, \xi_{t+1}\right)\right]}, \tag{24}$$

provided $\frac{u_C(C_t, N_t, \xi_t)}{E_t\left[\beta u_C\left(C_{t+1}, N_{t+1}, \xi_{t+1}\right)\right]} \geq 1$, meaning that the real interest rate is not negative.

Assuming the shock to preferences is multiplicative, then the efficient allocations are only a function of the shocks A_t and G_t, not the shock to preferences, ξ_t. The shock to preferences only affects the real rate of interest. If the shock was such that the real interest rate should be negative, under flexible prices, that just means that the efficient allocation would be implemented with inflation.

Under sticky prices inflation is costly. And a shock to preferences, that should not affect real allocations may indeed result in a downturn, if the nominal and real rates cannot be lowered as they should. If this was the case there would be a cost associated with the lower bound.

In the economy we analyze, under sticky prices, there are multiple distortions: the mark up distortion, due to monopolistic competition; the sticky prices and the distortionary taxes. Still for constant taxes, and for standard preferences, it may still be optimal that the allocations do not react to the preference shock as in the first best.

Using monetary policy to avoid a recession

Consider the case where fiscal policy is not used, $\widehat{\tau}_t^c = 0$ and $\widehat{\tau}_t^n = 0$. As long as the lower bound does not bind, movements in the nominal interest rate can fully offset the preference shock ξ_t. Indeed, the interest rate rule is defined so as to fully insulate output and inflation from this shock, so that in equilibrium it may be that $\widehat{y}_t = 0$, and $\pi_t = 0$. The intuition is simple: shocks to the real interest rate should be absorbed one to one by changes in the nominal interest rate. In this way, the shock does not affect prices and therefore there is no change in output. This would be monetary policy in normal times.

Note, on the other hand, that if the interest rate is zero and there is a negative shock to the real interest rate ($r_t < 0$), this could result in deflation and, given the price frictions, output would drop. For given expectations of inflation and output, $E_t\pi_{t+1}$ and $E_t\widehat{y}_{t+1}$, this would be the solution of the equations above, (19) and (20). This is why the zero bound on interest rates can be a cost to policy.

Using fiscal policy

Fiscal policy can also be used to respond to the shock, and fully stabilize the economy. Suppose the outcome of the interest rate rule is that the nominal interest rate is zero, $i_t = 0$. From (19), it is clear that there will be a conditional growth rate of the consumption tax,

$$E_t \widehat{\tau}^c_{t+1} - \widehat{\tau}^c_t = r,$$

that will satisfy the equation for $\widehat{y}_t = E_t \widehat{y}_{t+1} = 0$ and $E_t \pi_{t+1} = 0$. From (20), there is an adjustment on the labor income tax,

$$\widehat{\tau}^n_t = -\widehat{\tau}^c_t,$$

that will satisfy the condition for $\widehat{y}_t = 0$ and $\pi_t = E_t \pi_{t+1} = 0$. The interest rate rule, (21), is satisfied.

How can fiscal policy be implemented? Suppose the objective is to have the real rate be -4%. Then the future consumption should be four percentual points higher than the tax rate today. This can be accomplished with a cut in the present consumption tax rate of two percentual points, and with an increase in the future tax of two percentual points, or many other possible combinations. If the shock today is persistent, then there would be a need to keep on raising the consumption tax, for as many periods as the real rate were to stay below zero.

To illustrate, in more detail, the way fiscal policy can be managed, let us assume a simple deterministic downturn.[5] Let us assume the economy is in a steady state with $i = r$, $\pi = 0$, and

$\widehat{y} = 0$, and let

$$r_t = \{ \begin{array}{c} -r = \left(\widehat{\xi}_t - \widehat{\xi}_{t+1}\right) < 0 \text{ for } t = 0, 1, 2, ..T-1 \\ 0 \text{ for } t = T, T+1, T+2, .. \end{array}$$

We will now construct a sequence of consumption and labor tax changes such that $\pi_t = \widehat{y}_t = 0$ for all t, and with constant nominal interest rates. We call this policy the "complete" insulation policy. In order to achieve this, the aggregate supply equation (20) implies

$$\widehat{\tau}_t^n + \widehat{\tau}_t^c = 0 \text{ for all } t \tag{25}$$

and the aggregate demand (19) implies

$$0 = r_t + (\widehat{\tau}_{t+1}^c - \widehat{\tau}_t^c) \text{ for all } t \tag{26}$$

where the expectations have been removed since the solution is deterministic.

Assume also, as some argue, that consumption taxes cannot be negative. Then, if τ_0^c is the initial value of the consumption tax, then $\widehat{\tau}_0^c > -\tau_0^c$.

In describing a "complete" policy, we distinguish two cases.

Consider first the case in which $r \leq \tau_0^c$. For $t = 0$, set $\widehat{\tau}_0^c = -r$, so $\widehat{\tau}_1^c = 0$ from (26). Then, using (25) set $\widehat{\tau}_0^n = -\widehat{\tau}_0^c = r$. For $t = 1$, set $\widehat{\tau}_2^c = r$ so (26) is satisfied. Given that $\widehat{\tau}_1^c = 0$, let $\widehat{\tau}_t^n = 0$. For $t > 1$, $(\widehat{\tau}_{t+1}^c - \widehat{\tau}_t^c) = r$ and $\widehat{\tau}_{t+1}^n - \widehat{\tau}_t^n = -r$.

Thus, in the first period, consumption taxes go down. This adjusts to the real interest rate shock, by lowering the price of consumption today, relative to tomorrow. Labor income taxes must go up, so as not to affect the labor/consumption choice. In period 2, there is no adjustment in current consumption taxes, but future consumption taxes go up. This also reduces the current price of consumption, relative to future consumption. Future labor taxes must be reduced, so as not to affect the labor/consumption decision, as before. While the downturn lasts, consumption taxes grow at a rate equal to the absolute value of the shock in the real interest rate, and labor income taxes go down, so as not to modify the labor/consumption choice.

Consider now the case in which $r > \tau_0^c$. For $t = 0$, set $\widehat{\tau}_0^c = -\tau_0^c$,[6] and $\widehat{\tau}_1^c = r - \tau_0^c$ so (26) is satisfied. Then, using (25) set $\widehat{\tau}_0^n = -\widehat{\tau}_0^c$. For $t \geq 1$, $(\widehat{\tau}_{t+1}^c - \widehat{\tau}_t^c) = r$ and $\widehat{\tau}_{t+1}^n - \widehat{\tau}_t^n = -r$.

In this case, the reduction in consumption taxes when the shock becomes negative is not enough to fully adjust for the real interest rate shock. Then, is is required that future taxes increase. It should be clear from the examples above, that the policy is not unique. Note that there are redundant instruments. In particular, one could set $\widehat{\tau}_0^c = 0$, and still obain - in this case the unique - a complete policy. This would be a policy that makes the adjustment in relative prices just by increasing future taxes.

An interest rate rule that implements zero inflation and zero interest rates Is there an interest rate rule, that implements both zero nominal rates and zero producer price inflation? There is one, provided the nominal rate can be negative off equilibrium. Let the interest rate rule be

$$i_t = \phi \pi_t$$

in which the nominal interest rate reacts to deviations in inflation net of consumption taxes, with a coefficient that is greater than one, $\phi > 1$. The policy rule, together with the intertemporal condition,

$$i_t = r_t + E_t \pi_{t+1}^c,$$

gives

$$r_t + E_t\pi_{t+1} + E_t\widehat{\tau}^c_{t+1} - \widehat{\tau}^c_t = \phi\pi_t$$

If fiscal policy is such that

$$E_t\widehat{\tau}^c_{t+1} - \widehat{\tau}^c_t = -r_t,$$

then, the difference equation is

$$E_t\pi_{t+1} = \phi\pi_t,$$

and the locally determinate solution is $\pi_t = 0$, implying $i_t = 0$.

IV. Equivalence between nominal interest rates and consumption taxes

Note that more generally, it is possible to prove that the interest rate is a redundant policy instrument. Consider again the system of equations

$$\pi_t = \kappa\widehat{y}_t + \kappa\psi\left(\widehat{\tau}^n_t + \widehat{\tau}^c_t\right) + \beta E_t\pi_{t+1}$$

and

$$\widehat{y}_t = E_t\widehat{y}_{t+1} - \sigma(i_t - E_t\pi_{t+1} - r_t) + \sigma(E_t\widehat{\tau}^c_{t+1} - \widehat{\tau}^c_t)$$

which can be rearranged as

$$\pi_t + \kappa\widehat{y}_t - \beta E_t\pi_{t+1} = \kappa\psi\left(\widehat{\tau}^n_t + \widehat{\tau}^c_t\right)$$

$$\frac{\widehat{y}_t - E_t\widehat{y}_{t+1}}{\sigma} - E_t\pi_{t+1} = (E_t\widehat{\tau}^c_{t+1} - \widehat{\tau}^c_t - i_t + r_t)$$

If we set $i_t = 0$ for all t, and $\widehat{\tau}_0^c = 0$, one can let $E_t\widehat{\tau}_{t+1}^c = -r_t$, and $(\widehat{\tau}_t^n + \widehat{\tau}_t^c) = 0$ and fully stabilize inflation and the output gap.

In Correia, Nicolini and Teles (2008), it is shown that the equivalence result does not depend on the linearization. It is also shown that it extends to many other environments and to many other shocks.

It is also interesting to emphasize, that the policy described implements exactly the same allocation that can be implemented with the nominal interest rate. As it is the same allocation, it satisfies the implementability constraint in a fully specified Ramsey problem.[7] Therefore, it is revenue neutral. This means that, contrary to a policy of increasing government expenditures,[8] the policy we propose has no impact on the government budget constraint.

REFERENCES

[1] Blanchard, O., G. Dell'Ariccia and P. Mauro. "Rethinking Macroecocomic Policy", IMF Staff Position Note, 2010.

[2] Calvo, G.. 1983. "Staggered Prices in a Utility-Maximizing Framework." *Journal of Monetary Economics* 12: 383-398.

[3] Correia, I., J. P. Nicolini and P. Teles. 2008. "Optimal Fiscal and Monetary Policy: Equivalence Results." *Journal of Political Economy* 168 (1): 141-170.

[4] Christiano, L., M. Eichenbaum, and S. Rebelo, 2009,

[5] Eggertsson, G. B., 2009, What Fiscal Policy is Effective at Zero Interest Rates?, Federal Reserve Bank of New York.

[6] Eggertsson, G. B., and M. Woodford, 2003, The Zero Bound on Interest Rates and Optimal Monetary Policy, Brookings Papers on Economic Activity 1, 212-219.

[7] Eggertsson, G. B., and M. Woodford, 2004a, Policy Options in a Liquidity Trap, American Economic Review 94, 2: 76-79.

[8] Eggertsson, G. B., and M. Woodford, 2004b, Optimal Monetary and Fiscal Policy in a Liquidity Trap, ISOM conference volume.

[9] Feldstein, M, 2003, Rethinking Stabilization, Kansas City Fed.

[10] Poterba, J. M., J. J. Rotemberg, and L. H. Summers, 1986, A Tax-Based Test for Nominal Rigidities, American Economic Review, 76, 4, 659-675.

V. Appendix: The log-linearized version of the model

As productivity shocks play no particular role, we assume that $A_t = 1$ for all t, so (13) becomes

$$\frac{p_t}{P_t} = \frac{\theta}{(\theta - 1)} E_t \sum_{t=0}^{\infty} \eta_{t,j} \frac{W_{t+j}}{P_t}$$

The steady state has

$$\begin{aligned} C_t &= C,\, N_t = N,\, \xi_t = 1, \tau_t^c = \tau^c, \tau_t^n = \tau^n \\ P_t &= p_t = P, R = \beta^{-1} \end{aligned}$$

so that

$$\eta_{t,j} = (1 - \alpha\beta)(\alpha\beta)^j \text{ , and } \frac{\theta}{(\theta - 1)} W = P.$$

We, first, log-linearize equation (12) , using (**??**) to replace labor. We first write the equation as

$$\frac{u_C\left(C_t, N_t, \xi_t\right)}{\left(1+\tau_t^c\right)} = E_t\left[R_t \frac{\beta u_C\left(C_{t+1}, N_{t+1}, \xi_{t+1}\right)}{\left(1+\tau_{t+1}^c\right)} \frac{P_t}{P_{t+1}}\right]$$

The log-linearization is given by

$$\lambda \widehat{C}_t + \Gamma \widehat{\xi}_t - \widehat{\tau}_t^c \simeq i_t - \ln \beta^{-1} - E_t \pi_{t+1} + \lambda E_t \widehat{C}_{t+1} + \Gamma E_t \widehat{\xi}_{t+1} - E_t \widehat{\tau}_{t+1}^c \tag{27}$$

where

$$\begin{aligned}
\lambda &= \frac{C}{uc}(u_{CC} + u_{CN}),\ \Gamma = \frac{\xi}{u_C} u_{C\xi} = \frac{u_{C\xi}}{u_C} = 1 \text{ if } \ \xi_t \text{ is multiplicative} \\
\widehat{C}_t &= \ln \frac{C_t}{C},\ \widehat{\xi}_t = \ln \xi_t,\ \widehat{\tau}_t^c = \ln \frac{\left(1+\tau_t^c\right)}{\left(1+\tau^c\right)},\ \pi_{t+1} = \ln \frac{P_{t+1}}{P_t},\ i_t = \ln R_t
\end{aligned}$$

Linearization of the aggregate resource constraint yields

$$C_t + G_t = \left[\sum_{j=0}^{t+1} \varpi_j \left(\frac{p_{t-j}}{P_t}\right)^{-\theta}\right]^{-1} A_t N_t$$

assuming that government consumption is constant, delivers

$$\frac{C}{C+G} \widehat{C}_t = \widehat{y}_t$$

So, if we let $g^{-1} = \frac{C}{C+G}$, then

$$\widehat{C}_t = g \widehat{y}_t,$$

If we also assume that the shock ξ_t is multiplicative, so $\Gamma = 1$, we can write equation (27) as

$$\lambda g \widehat{y}_t + \widehat{\xi}_t - \widehat{\tau}_t^c \simeq i_t - \ln \beta^{-1} - E_t \pi_{t+1} + \lambda g E_t \widehat{y}_{t+1} + E_t \widehat{\xi}_{t+1} - E_t \widehat{\tau}_{t+1}^c$$

or, letting $\sigma = 1/\lambda g$,

$$\widehat{y}_t \simeq E_t \widehat{y}_{t+1} + \sigma \left[i_t - E_t \pi_{t+1} - \left(\ln \beta^{-1} + \widehat{\xi}_t - E_t \widehat{\xi}_{t+1} \right) \right] - \sigma \left(E_t \widehat{\tau}_{t+1}^c - \widehat{\tau}_t^c \right)$$

On the other hand, linearization of (7) , delivers

$$\ln p_t \simeq \ln \frac{\theta}{(\theta - 1)} + \ln E \sum_{t=0}^{\infty} \eta_{t,j} W_{t+j}$$

But

$$W_t = \frac{(1 + \tau_t^c) P_t}{(1 - \tau_t^n)} \left[- \frac{u_C (C_t, N_t, \xi_t)}{u_N (C_t, N_t, \xi_t)} \right]^{-1}$$

so

$$\ln p_t \simeq \ln \frac{\theta}{(\theta - 1)} + \ln E_t \sum_{t=0}^{\infty} \eta_{t,j} \frac{\left(1 + \tau_{t+j}^c \right) P_{t+j}}{\left(1 - \tau_{t+j}^n \right)} \left[- \frac{u_C \left(C_{t+j}, N_{t+j}, \xi_{t+j} \right)}{u_N \left(C_{t+j}, N_{t+j}, \xi_{t+j} \right)} \right]^{-1}$$

or

$$\ln p_t - \ln P_t \simeq \ln \frac{\theta}{(\theta - 1)} + \ln E_t \sum_{t=0}^{\infty} \eta_{t,j} \frac{\left(1 + \tau_{t+j}^c \right) \frac{P_{t+j}}{P_t}}{\left(1 - \tau_{t+j}^n \right)} \left[- \frac{u_C \left(C_{t+j}, N_{t+j}, \xi_{t+j} \right)}{u_N \left(C_{t+j}, N_{t+j}, \xi_{t+j} \right)} \right]^{-1}$$

The log-linearization of the second term in the right hand side is given by

$$\ln E_t \sum_{t=0}^{\infty} \eta_{t,j} \frac{\left(1 + \tau_{t+j}^c \right) \frac{P_{t+j}}{P_t}}{\left(1 - \tau_{t+j}^n \right)} \left[- \frac{u_C \left(C_{t+j}, N_{t+j}, \xi_{t+j} \right)}{u_N \left(C_{t+j}, N_{t+j}, \xi_{t+j} \right)} \right]^{-1} \simeq (1 - \alpha\beta) E_t{}_{j=0}^{\infty} (\alpha\beta)^j \left[\Omega_{t+j} \right]$$

where

$$\Omega_{t+j} = \widehat{\tau}^c_{t+j} + \widehat{\tau}^n_{t+j} + \pi_t(j) + \phi\widehat{C}_{t+j} - \gamma\widehat{\xi}_{t+j}$$

where

$$\begin{aligned} \pi_t(j) &= \ln\frac{P_{t+j}}{P_t} \\ \widehat{\tau}^n_t &= \ln\frac{(1-\tau^n_t)}{(1-\tau^n)} \end{aligned}$$

and

$$\begin{aligned} \phi &= (-1)\frac{C}{U_C\left(-U_N\right)}\left[\left(U_{CC}+U_{NC}\right)\left(-U_N\right) - U_C\left(U_{NC}+U_{NN}\right)\right] \\ \gamma &= \frac{-1}{U_N^2}\left[U_{C\xi}U_N - U_C U_{N\xi}\right] \end{aligned}$$

Note that if, as we will assume, $u\left(C_{t+j}, N_{t+j}, \xi_{t+j}\right) = u\left(C_{t+j}, N_{t+j}\right)\xi_{t+j}$, then $\gamma = 0$. Note also that $\phi > 0$.

Thus, we can write

$$\begin{aligned} \widehat{p}_t &\simeq (1-\alpha\beta)E_t\sum_{t=0}^{\infty}(\alpha\beta)^j\left[\widehat{\tau}^c_{t+j} + \widehat{\tau}^n_{t+j} + \pi_t(j) + \phi\widehat{C}_{t+j} - \gamma\widehat{\xi}_{t+j}\right] \\ &\simeq (1-\alpha\beta)\left[\begin{array}{c} \left[\widehat{\tau}^c_t + \widehat{\tau}^n_t + \phi\widehat{C}_t - \gamma\widehat{\xi}_t\right] \\ +\left(\alpha\beta\right)E_t\sum_{t=0}^{\infty}\left(\alpha\beta\right)^j\left[\widehat{\tau}^c_{t+j+1} + \widehat{\tau}^n_{t+j+1} + \pi_t(j) + \phi\widehat{C}_{t+j+1} - \gamma\widehat{\xi}_{t+j+1}\right] \end{array}\right] \end{aligned}$$

where $\widehat{p}_t = \ln\frac{p_t}{P}$. But note that

$$\pi_t(j) = \ln\frac{P_{t+j}}{P_t} = \ln\frac{P_{t+1}}{P_t}\frac{P_{t+j}}{P_{t+1}} = \ln\frac{P_{t+1}}{P_t} + \ln\frac{P_{t+j}}{P_{t+1}} = \pi_{t+1} + \pi_{t+1}(j-1)$$

so we can write the equation as

$$
\begin{aligned}
\widehat{p}_t &\simeq (1-\alpha\beta)\left[\begin{array}{c} \left[\widehat{\tau}_t^c + \widehat{\tau}_t^n - \phi\widehat{C}_t - \gamma\widehat{\xi}_t\right] + \\ (\alpha\beta)\, E_t{}_{j=0}^{\infty} (\alpha\beta)^j \left[\widehat{\tau}_{t+j+1}^c + \widehat{\tau}_{t+j+1}^n + \pi_{t+1} + \pi_{t+1}(j-1) + \phi\widehat{C}_{t+j+1} - \gamma\widehat{\xi}_{t+j+1}\right] \end{array}\right] \\
&= (1-\alpha\beta)\left[\begin{array}{c} \left[\widehat{\tau}_t^c + \widehat{\tau}_t^n + \phi\widehat{C}_t - \gamma\widehat{\xi}_t\right] + (\alpha\beta)\, E_t \sum_{t=0}^{\infty} (\alpha\beta)^j \left[\pi_{t+1}\right] + \\ (\alpha\beta)\, E_t \sum_{t=0}^{\infty} (\alpha\beta)^j \left[\widehat{\tau}_{t+j+1}^c + \widehat{\tau}_{t+j+1}^n + \pi_{t+1}(j-1) + \phi\widehat{C}_{t+j+1} - \gamma\widehat{\xi}_{t+j+1}\right] \end{array}\right] \\
&= (1-\alpha\beta)\left[\widehat{\tau}_t^c + \widehat{\tau}_t^n + \phi\widehat{C}_t - \gamma\widehat{\xi}_t\right] + (\alpha\beta)\, E_t\left[\pi_{t+1}\right] + (\alpha\beta)\, E_t\widehat{p}_{t+1}
\end{aligned}
$$

But the log linearization of (8) delivers

$$\ln P_t \simeq \alpha \ln P_{t-1} + (1-\alpha)\ln p_t^*$$

so

$$\ln P_t - \alpha \ln P_t \simeq \alpha \ln P_{t-1} + (1-\alpha)\ln p_t^* - \alpha \ln P_t$$

or

$$\widehat{p}_t \simeq \frac{\alpha}{1-\alpha}\pi_t$$

Replacing above

$$\frac{\alpha}{1-\alpha}\pi_t \simeq (1-\alpha\beta)\left[\widehat{\tau}_t^c + \widehat{\tau}_t^n + \phi\widehat{C}_t - \gamma\widehat{\xi}_t\right] + (\alpha\beta)\, E_t\left[\pi_{t+1}\right] + (\alpha\beta)\, E_t\frac{\alpha}{1-\alpha}\pi_{t+1}$$

or

$$\pi_t \simeq (1-\alpha\beta)\frac{1-\alpha}{\alpha}\left[\widehat{\tau}_t^c + \widehat{\tau}_t^n + \phi\widehat{C}_t - \gamma\widehat{\xi}_t\right] + \frac{1-\alpha}{\alpha}(\alpha\beta)\, E_t\left[\pi_{t+1}\right] + (\alpha\beta)\, E_t\pi_{t+1}$$

so

$$\pi_t \simeq (1-\alpha\beta)\frac{1-\alpha}{\alpha}\left[\widehat{\tau}_t^c + \widehat{\tau}_t^n + \phi\widehat{C}_t - \gamma\widehat{\xi}_t\right] + \beta E_t \pi_{t+1}$$

Finally, recall that

$$\widehat{C}_t = g\widehat{y}_t,$$

so

$$\pi_t \simeq (1-\alpha\beta)\frac{1-\alpha}{\alpha}\left[\widehat{\tau}_t^c + \widehat{\tau}_t^n + \phi g\widehat{y}_t - \gamma\widehat{\xi}_t\right] + \beta E_t \pi_{t+1}$$

Letting

$$\begin{aligned}\kappa &= (1-\alpha\beta)\frac{1-\alpha}{\alpha}\phi g \\ \psi &= (\phi g)^{-1}\end{aligned}$$

we obtain

$$\pi_t \simeq \kappa\psi\left(\widehat{\tau}_t^c + \widehat{\tau}_t^n\right) + \kappa\widehat{y}_t - \kappa\psi\gamma\widehat{\xi}_t + \beta E_t \pi_{t+1}$$

We assume that the shock ξ_t is multiplicative, so $\gamma = 0$. If we let $r_t = \left(\ln\beta^{-1} + \widehat{\xi}_t - E_t\widehat{\xi}_{t+1}\right)$, the system can be written as

$$\pi_t \simeq \kappa\widehat{y}_t + \kappa\psi\left(\widehat{\tau}_t^n + \widehat{\tau}_t^c\right) + \beta E_t \pi_{t+1}$$

$$\widehat{y}_t \simeq E_t\widehat{y}_{t+1} - \sigma(i_t - E_t\pi_{t+1} - r_t) + \sigma(E_t\widehat{\tau}_{t+1}^c - \widehat{\tau}_t^c)$$

$$i_t \geq 0$$