Finance and Competition*

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Abstract

We investigate the role of financial constraints for product market competition in a general equilibrium model, where firms may differ in terms of own wealth and/or efficiency. We find that, in general, the amelioration of financial constraints increases competition (it lowers the Lerner index of markups) in financially dependent sectors even when other standard concentration indexes indicate otherwise. Our analysis implies that disruptions in financial markets –such as the recent financial crisis– may have adverse effects on competition in product markets, a cost that has not been identified before. JEL Codes: L1, E2.

Keywords: financial constraints, liberalization, market structure, product market competition, entry.

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1 Introduction

Does the existence of financial constraints hinder product market competition in financially dependent sectors? Do such constraints matter for the number and size of firms as well as for concentration indexes and markups in these markets? There is a strong presumption, based on the notion that financial constraints act as a barrier to entry/expansion in financially dependent activities, that a more developed financial system is conducive to greater product market competition. Surprisingly, though, there exists no theory linking asset market development to competition. And things are not better on the empirical front.

The empirical literature is quite meagre. Rajan and Zingales (1998), and Haber (2000) study the effects of financial development and Cetorelli and Strahan (2006) and Bertrand, Schoar and Thesmar (2007) the effects of changes in the degree of "competition" in the banking sector on various measures of market structure and performance. Rajan and Zingales find that financial development increases the number of firms while it has an ambiguous effect on average firm size in financially dependent industries. Haber compares the cotton industries in Brazil and Mexico during 1880-1930, a period of financial liberalization, and argues that concentration indexes decreased, in particular in Brazil, the country that underwent the most effective financial market reform. Cetorelli and Strahan study how market structure in nonfinancial sectors was affected by the increase in "competition" –lower bank concentration and looser state-level restrictions– following deregulation in state banking in the US. They find that the number of firms increased, average size (number of employees per establishment) and concentration decreased, and the share of establishments in the smallest size group also increased. Similarly, Bertrand, Schoar and Thesmar examine the effects of lower state intervention (deregulation) in the French banking industry on performance and competition in financially dependent sectors. They find an increase in firm entry and exit rates and also a reduction in the level of product market concentration. .

The theoretical literature on finance and product market competition is quite extensive but, at the same time, quite narrow in scope. It has two key features. First, it is partial equilibrium. Second, it is exclusively concerned with the *strategic* relationship between financial decisions and output market decisions when both financial and product markets are imperfectly competitive. One strand of the literature studies how investors (financial intermediaries) select financial contracts or instruments in order to influence the customer firm's – as well as its rivals' – competitive behavior: pricing decisions, the incentive to enter, the incentive to collude, the choice to compete in strategic complements vs substitutes, and so on.[1] The other strand studies the reverse question, namely, how firms select the financial contracts or instruments in order to influence the investor's incentives to finance other firms (whether or not to provide funds to potential entrants, to rivals of the firms and so on[2]).

The objective of this paper is to provide a general framework for the study of the relationship between financial constraints and the degree of competition in product markets. Our analysis contains four novel elements relative to the existing literature. First, it is general equilibrium. As such, it captures the macroeconomic effects of changes in the functioning of financial markets. Second, it allows financial markets to matter even when they are perfectly competitive. Third, it allows for heterogeneity in efficiency across firms. Forth, it allows for differences in the degree of financial constraints across firms.

We employ a general equilibrium, two-sector model, with heterogeneous agents. The agents may differ with respect to their productivity level as well as to their wealth holdings and thus their need for external finance. One sector (sector 1) is financially dependent in the sense that production depends on the amount of capital used and capital may need to be borrowed from the financial system.[3] This sector is also imperfectly competitive, with firms competing *a la* Cournot. In the other sector (sector 2), productivity is independent of capital and the market structure is perfectly competitive. There is free entry in both sectors.

We use this model to compute the general equilibrium with and without financial markets[4] and then address the following questions. How does the amelioration of financial constraints affect the economy's total output and its composition, as well as the quantity and prices of the goods produced in capital dependent sectors? How does it affect the number and size of firms in these sectors? What is the effect on market shares and markups? What is the role played by heterogeneity in wealth relative to

[1] Brander and Lewis (1986) is the pioneering work. See Cestone and White (2003) for a discussion of the literature and further references.

[2] See Cestone and White (2003).

[3] In order to keep the analysis tractable, we abstract from uncertainty, agency problems and so on. The sole role of the financial system in our model is to allocate funds from multiple savers toward investment projects.

[4] One could easily consider intermediate cases of financial market imperfections without affecting the main results.

that in ability? Is taking product market competition when analyzing financial frictions important?

We find that financial markets make the output of the financially dependent sector expand and its price drop. This is consistent with the finding of Rajan and Zingales (1998) that industries that are more dependent on external finance grow relatively faster in more financially developed countries. It arises from the fact that financial development expands *asymmetrically* the economy's production possibilities frontier.

The effect of the relaxation of financial constraints on the number and size of firms as well as on market shares is ambiguous. But the effect on the degree of product market competition (measured properly by the Lerner index) is, in general[5] positive[6]. These findings are important for two reasons. First, because there exists no reason to believe that in an economy with two frictions, the elimination of one of the frictions will in general ameliorate the other one (that is, that the effect of the frictions contains a positive interactive term). A contribution of the present paper is to show the prevalence –and quantitative significance– of such a positive interactive term between the financial and the product market frictions. And second, because they establish that the standard measures of competition commonly employed in the literature (such as the change in the number of firms or the change in the concentration indexes) may be not be a reliable indicator of the change in the degree of competition in an economy with financial imperfections.

The result that the Lerner index decreases and that this decrease represents an improvement in competitive conditions obtains under very general parametrization of the model. Nonetheless, the model is highly stylized, so before one accepts this result one may need additional assurances that the model represents an empirically relevant laboratory for the study of the question at hand. One way to address this concern is by examining whether the model can match the stylized facts regarding net firm entry, average firm size, and market concentration indexes reported above. Recall, that, the literature alleges that following the amelioration of financial constrains there is positive net firm entry, a decrease in average firm size and a decrease in market concentration indexes. We find that the model can match this pattern under the assumption that

[5] While there are cases where the Lerner index may *increase* following the relaxation of the financial constraints, such cases are confined to a small subset of the parameter space and are quite fragile.

[6] Note that even perfectly competitive firms would have a positive markup when they are constrained. Hence, a decline in the Lerner index following the removal of the financial friction does not necessarily indicate an improvement in the degree of competition. We show that a large share of the decline in the Lerner index reflects indeed an improvement in competition.

ability is homogeneous and the distribution of wealth in the model matches that observed in the data. But also under a broader set of parameterizations and assumptions about the joint cross sectional variation in ability and wealth, which is reassuring as we know little about how these two distributions are related.

The model also has additional implications. One is that, for a given distribution of ability and wealth and level of financial development, poorer countries will exhibit less competitive markets that richer countries. Consequently, the gains in product market competition arising from financial liberalization will be inversely related to a country's average level of income.

There are also implications regarding the political economy of financial liberalization. A popular view is that the development of financial markets is hindered by the power of incumbents. This includes both incumbent financial institutions that are concerned about competition in the financial market, as well as incumbent firms that fear that a more competitive financial system will finance entrants into their sectors. We find that incumbency is not always sufficient to characterize preferences towards financial liberalization. There may be incumbents who will support liberalization (efficient but undercapitalized producers), as well as incumbents who may object to it (well capitalized firms). It is the fact that there are situations where the financial markets tend to favor disproportionately the most efficient but poorly capitalized producers that can account for such divisions within the class of incumbent firms.

The paper proceeds as follows. In the next section, we present our model. Section 2 describes the model. The main results are presented in section 3.

2 The Model

2.1 The Environment

Preliminaries The economy is populated by a finite number of individuals N. Individuals differ with respect to their ability level and private wealth holdings. Let $\mathcal{A}$ and $\mathcal{K}$ denote the corresponding sets of ability and wealth. We have $\mathcal{A} = \{a_j\}_{j=1}^{N}$ and $\mathcal{K} = \{\bar{k}_l\}_{l=1}^{N}$. Individual i is defined by a pair $(a_i, \bar{k}_i) \in \mathcal{A} \times \mathcal{K}$, $i = 1, \ldots, N$. $\bar{K}$ denotes the economy's global endowment of capital, given by $\sum_i \bar{k}_i$. Ability and wealth holdings are publicly observed and common knowledge. $F(a, k)$ denotes the joint function of ability and wealth in the economy.

Production There are two goods produced in this economy. Good 1 requires capital as an input. If individual i works in sector 1, its output q_i is

$$q_i = a_i k_i^{\beta} \tag{1}$$

with $\beta \in (0,1)$. k_i is the amount of capital individual i invests in production.

Output in sector 2 is independent of ability and, moreover, it does not require the use of any capital. If individual i chooses to work in sector 2, he produces A units of good 2.

In the presence of financial markets, individuals may borrow capital if their desired scale of operations exceeds their individual capital holdings. Without financial markets, individual investment is constrained to satisfy:

$$k_i \leq \bar{k}_i \tag{2}$$

Since the ability to borrow affects the scale of individual production in sector 1 but not in sector 2, sector 1 is said to be a financially dependent sector.

Preferences Individuals have utility defined over two goods. Given a consumption vector (c_1, c_2), total utility is

$$u(c_1, c_2) = \log(c_1) + \gamma \log(c_2)$$

for $\gamma > 0$. Let p be the relative price of good 1 in terms of good 2.

Wealth endowments, $\bar{k}_l \in \mathcal{K}$, are expressed in units of good 2. The income, l_i, of an individual i who chooses to operate in sector 1 is:

$$l_i = \bar{k}_i + (pq_i(k_i) - k_i).$$

and that of one who operates in sector 2:

$$l_i = A + \bar{k}_i$$

Individuals buy (or sell) the difference between $\bar{k}_i$ and k_i at the price of one.

Aggregate Demand The budget constraint for i is:

$$pc_i^1 + c_i^2 = l_i$$

Given an income level l_i, the demand for goods 1 and 2 is:

$$c_i^1 = \frac{1}{p}\frac{l_i}{1+\gamma}, \quad c_i^2 = \frac{\gamma}{1+\gamma} l_i$$

Since the Engel curves are straight lines from the origin, we have a representative agent economy. Aggregate demand depends only on aggregate income (the sum of individual income across individuals) and not on how it is distributed. Define

$$I \equiv \sum_{i=1}^{N} l_i$$

so that I is aggregate income. Aggregate demand for good j, denoted C^j, is then

$$C^1 = \frac{1}{p}\frac{I}{1+\gamma}, \quad C^2 = \frac{\gamma}{1+\gamma} I.$$

The inverted demand curve for good 1 is:

$$p = \frac{1}{C^1}\frac{I}{1+\gamma} \tag{3}$$

Due to the log preference specification the (absolute value of the) elasticity of demand of good 1 with respect to its relative price p is equal to 1. The relative demand schedule of good 1 in terms of good 2 is:

$$\frac{C^1}{C^2} = \frac{1}{p}\frac{1}{\gamma} \tag{4}$$

Next, we examine the economy without financial constraints.

2.2 Financially Unconstrained Economy

We first describe how a firm that has chosen sector 1 selects its optimal level of production. We then describe how firms choose their sector of activity.

2.2.1 Optimal Choice of Level of Production in Sector 1

Consider an individual i, with ability level a_i. The profits from operating in sector 1 are:

$$\pi_i^1 = p\left(Q_1\right) q_i - k_i = p\left(Q_1\right) q_i - \left(\frac{q_i}{a_i}\right)^{\frac{1}{\beta}} \tag{5}$$

where q_i is the quantity produced by individual i, k_i the amount of capital used, Q_1 the total output of good 1 produced, and $p\left(Q\right)$ the inverse demand curve for good 1. We assume that sector 1 is characterized by quantity competition *a la* Cournot, while sector 2 is perfectly competitive.

Under Cournot competition, each firm chooses output q_i taking the quantities of the remaining firms as given. The first-order condition for firm i is:

$$p\left(1-\frac{q_i}{Q_1}\right)=MC\left(q_i\right) \tag{6}$$

where $MC\left(q_i\right)$ indicates firm i's marginal cost. We thus obtain the familiar result that the price to marginal cost ratio, the markup, equals $\left(1-q_i/Q_1\right)^{-1}$.

Equation (6) defines firm i's optimal quantity q_i in terms of the relative price p, total market output Q_1, and level of efficiency, a_i. Optimal quantity q_i is strictly increasing in ability. Holding p and Q_1 constant, more able firms will have greater market shares and higher markups than less able ones. Since the marginal cost declines with ability, more able firms are more profitable than less able ones.

Equation (6) allows us to solve for the optimal quantity produced by firm i, q_i^*. It can be written as:

$$q_i^*=q_i^*\left(\overset{+}{p},\overset{+}{Q_1},\overset{+}{a_i}\right) \tag{7}$$

Using the production function, equation (7) can be rewritten in terms of capital:[7]

$$k_i^*=k_i^*\left(\overset{+}{p},\overset{+}{Q_1},\overset{+}{a_i}\right) \tag{8}$$

It follows that more able firms choose a larger scale of production.

2.2.2 Optimal Choice of Sector of Activity

Individuals choose to work in the sector that generates the highest income. Let us consider the minimum amount of capital, k_{min} that makes an individual of ability level a indifferent between the two sectors. $k_{\min}(a,p)$ is thus determined by the equation:

$$pa_ik_{\min}^{\beta}-k_{\min}=A \tag{9}$$

Equation (9) defines a relationship between ability and capital. It can be verified that $k_{\min}(a,p)$ is decreasing and convex in ability. Moreover, since a higher price raises profits for given levels of ability and capital, an increase in p shifts the $k_{\min}\left(\cdot\right)$ schedule downwards in (a,k) space.

[7]The strictly monotonic relationship between ability and capital depends on an additional condition that no firm holds a market share in excess of 50% of Q_1. This will be the case if there is not a single super-able agent.

Under appropriate assumptions on the distribution of ability and the parameters of the model, there exists a level of ability, $\tilde{a}$, such that

$$k_{\min}(p, \tilde{a}) = k^*(p, \tilde{a}, Q_1) \tag{10}$$

In a financially unconstrained economy, the ability level $\tilde{a}$ is the threshold determining the separation of entrepreneurs into activities: those whose ability exceeds $\tilde{a}$ work in sector 1 whereas the remaining work in sector 2. This threshold is implicitly defined by equation (10) as a function of p and Q_1:

$$\tilde{a} = \tilde{a}\left(p, Q_1\right) \tag{11}$$

(9) are depicted in Figure **??**.

Once the choice of activity has been made and production completed, the aggregate supply of good 1, Q_1, is given by

$$Q_1 = \sum_{a_i \geq \tilde{a}(p,Q_1)}^{\bar{a}} q_i^* \left(\overset{+}{p}, \overset{+}{a_i}, \overset{+}{Q_1}\right). \tag{12}$$

Total input demand for good 1 production, K^*, is:

$$K^* = \sum_{a_i \geq \tilde{a}(p,Q_1)}^{\bar{a}} \left(\frac{q_i^*}{a_i}\right)^{\frac{1}{\beta}}. \tag{13}$$

Let Q_2 denote the total quantity of good 2 produced in the economy:

$$Q_2 = \sum_{a_i \leq \tilde{a}(p,Q_1)} A. \tag{14}$$

The quantity of good 2 available for consumption is

$$C_2 = Q_2 + \bar{K} - K^* \tag{15}$$

where $\bar{K}$ is the economy's initial endowment of capital and K^* the amount used up in production in sector 1.

2.2.3 The Equilibrium

Definition 1 *An equilibrium in the financially unconstrained economy is a triple* $\left(\tilde{a}, p, \{q_i\}_{i=1}^N\right)$, *with* $Q_1 = \sum_{i=1}^N q_i$, *such that:*

i) q_i *satisfies* (6) *for* $a_i \geq \tilde{a}$ *and is equal to zero for* $a_i < \tilde{a}$*;*

ii) $C_1 = Q_1$, $C_2 = Q_2 + \bar{K} - K^*$*;*

iii) Equation (4) *is satisfied.*

In order to construct the equilibrium one can proceed as follows. Use (10) to solve for $\tilde{a} = \tilde{a}\,(p, Q_1)$. Substituting this expression into (12), (13) and (14) yields Q_1, Q_2 and K^*, respectively, as functions of p. Substituting Q_1 for C_1 and $Q_2 + \bar{K} - K^*$ for C_2 in (4) determines p.[8]

Note that in the special case of equal ability $a_i = a, \ \forall i$, the equilibrium involves a triple (n, p, q) such that: for $i \leq n, \ \ q_i = q$ and for $i < n, \ \ q_i = 0$; and conditions (ii) and (iii) in the definition of the equilibrium (definition 1) are satisfied with $\sum_{a_i \geq \tilde{a}}$ being replaced by $i \leq n$. Hence, n identical firms operate in sector 1 and $N - n$ identical agents work in sector 2.

2.3 Financially constrained economy

We now discuss the determination of the equilibrium in this economy in the *absence* of financial markets.[9] The superscript c is used to indicate equilibrium values in the constrained economy. Let q_i^c denote individual i 's output in the constrained environment. If this entrepreneur cannot use external funds, then his production in sector 1 cannot exceed the level that could have been financed by his own initial capital stock, $\bar{k}_i$:

$$q_i^c = min\{a_i \bar{k}_i^{\beta}, a_i (k_i^*)^{\beta}\} \tag{16}$$

or, equivalently, in terms of capital

$$k_i^c = min\{\bar{k}_i, k_i^*\} \tag{17}$$

where k_i^* is determined by equation (8).

Unlike the financially unconstrained economy, the choice of activity now depends on both individual ability and individual wealth. In general, it is no longer the case that there exists a threshold ability level for the choice of activity in the financially constrained economy, $\tilde{a}^c$, or a threshold amount of capital, $k_{\min}^c(\tilde{a}, p)$, such that all individuals with ability $a \geq \tilde{a}^c$ or those who have wealth exceeding $k \geq k_{min}^c(a)$ will operate in sector 1. In particular, the following condition must be satisfied for individual i to operate in sector 1:

$$p a_i (k_i^c)^{\beta} - k_i^c \geq A \tag{18}$$

[8] For the cases considered in this paper, we confirm numerically that the unconstrained equilibrium exists and is unique. We do not know whether our model has this property in general. While the $\log(\cdot)$ preference format generates strictly convex indifference curves, the presence of Cournot imperfect competition may lead to segments on the production possibilities frontier with decreasing opportunity cost.

[9] While for reasons of simplicity we study an economy without any asset trade, our analysis is applicable to more general environments with asset markets but with restricted asset trade.

Those who are not simultaneously able and rich enough to operate in sector 1, will work in sector 2.

2.3.1 The Equilibrium

Definition 2 *An equilibrium in the financially constrained economy is a set of agents J such that:*

i. For $i \in J$ condition 18 is satisfied

ii. $C_1 = Q_1$, $C_2 = Q_2 + \bar{K} - K^c$ with $Q_1^c = \sum_{i \in J} q_i^c, K^c = \sum_{i \in J} k_i^c$

iii. Equation (4) is satisfied.

3 The effects of financial markets

3.1 Measures of competition

Concentration Indexes

Let us order the firms by size so that 1 represents the largest firm, 2 the second largest firm and so on. Then,

Definition 3 *The H_j index of the market share of the j largest firms is defined as:*

$$H_j \equiv \sum_{i=1}^{j} \frac{q_i}{Q_1}.$$

Markups

Definition 4 *The markup of firm i, μ_i is given by $\mu_i = (p - MC_i)/p$. When a firm is unconstrained, its markup is determined from the first-order condition, equation (6):*

$$\mu_i \equiv \frac{p - MC_i}{p} = \frac{q_i}{Q_1} \tag{19}$$

And when constrained and producing suboptimal quantity $q(\bar{k})$ by:

$$\mu_i \equiv \frac{p - MC_i}{p} = \frac{p - MC\left(q(\bar{k})\right)}{p} \tag{20}$$

Note that when a firm is unconstrained then the markup reflects only the imperfect competition friction. But when it is constrained, it reflects both the imperfect competition and the imperfect capital markets friction. That is, in this case the markup is positive even under perfect competition.

Definition 5 *The Lerner index of monopoly, σ, is defined as*

$$\sigma \equiv \sum_{i \in J} \mu_i \frac{q_i}{Q_1} \tag{21}$$

In the presence of financial constraints, a lower value of σ signifies either more competition or/and a less severe financial constraint. A main focus of our paper is to assess the interaction of the two frictions for the lerner index.

3.2 Implications for sectoral output and prices

Proposition 1 *The amelioration of financial constraints leads to a lower relative price and a higher quantity of the financially dependent good.*

Proof. See the appendix.

3.3 Implications for competition, net firm entry, firm size and concentration indexes

Having established that the amelioration of financial constraints leads to an increase in the output and to a decrease in the price of the financially dependent sector we now turn to the main subject of this study, namely the effects of financial constraints on the Lerner index of competition in product markets. For the reasons offered in the introduction, namely, a concern about the overall empirical properties of our model, we also examine the implications of the model for entry/exit, the number and size of firms and concentration indexes. As a means of establishing the robustness of the results and in order to also highlight the role played by cross sectional variation in ability and wealth, we have chosen to proceed stepwise. First we present the case where agents are perfectly homogeneous with regard to both ability and wealth. Then we introduce wealth heterogeneity (maintaining a common level of ability), and finally, we allow for heterogeneity in both ability and wealth.[10]

3.3.1 Homogeneous ability and wealth

With identical ability, $a_i = a, \ \forall i$, and initial wealth, $\bar{k}_i = k, \ \forall i$, all firms operating in sector 1 produce the same level of output and have the same profits. In an interior

[10]The case of homogeneous wealth and heterogeneous ability is straightforward. We comment on this briefly below.

equilibrium, whether in a financially constrained economy or not, the agents must be indifferent between the two sectors. That is, the equilibrium satisfies the condition

$$pak^{\beta} - k = A \tag{22}$$

where $k = \bar{k}$ in the constrained economy, and $k = k^*$ in the unconstrained economy.

Proposition 2 *For β close to unity*[11] *(constant returns to scale), the financially unconstrained economy has a smaller number of firms than the constrained one.*

Proof. See the appendix.

Consequently, when Proposition 2holds, the lifting of financial constraints leads to net firm exit, and to an increase to the average size of firms and concentration. Nevertheless, markups and the Lerner index decline, as $p^u < p^c$ and $MC^u > MC^c$.

We draw two conclusions from these findings. First, an increase in market concentration and/or a reduction in the number of firms does not necessarily imply a worsening of competitive conditions (a common belief that underlies policy in this area), at least in the presence of additional distortions (such as financial imperfections); price-cost margins are a more reliable indicator. And second, this version of the model cannot deliver results consistent with the empirical evidence (namely net entry and reduction in concentration) in the absence of some sort of heterogeneity[12]. Furthermore, a fully homogeneous environment produces a degenerate distribution of firm sizes, also an unpalatable outcome. In the next subsection we examine how heterogeneity in wealth affects the properties of the model.

3.3.2 Homogeneous ability, heterogeneous wealth

We maintain the assumption of identical ability, $a_i = a$, $\forall i$ but allow initial capital holdings to vary across individuals according to the function $\bar{k}_i = f(i)$. There is some information in the literature regarding the distribution of wealth which can be used to specify a suitable f function. We used a function that is convex in the index of

[11]While we can only prove analytically this proposition for β close to one, in the numerical exercises and for all the parametrizations used in the paper we always find this to be the case, independent of the value of β.

[12]A possible and feasible extension of the model would involve examining the role played by preferences for the entry-exit patterns. For, instance, using a CES instead of a log specification. We are not reporting such exercises in order not to clutter the present analysis.

agents that perfectly captures[13] the wealth distribution in the US reported in Table 2 of Cagetti and De Nardi (2006). When solving the model[14] we have used more or less arbitrary values for the other parameters of the model (they are reported below the Tables) but have also conducted extensive robustness analysis. The results are very robust to alternative parameterizations.

Table I presents results for various values of aggregate wealth (initial total capital[15]). As expected, the size of the financially dependent sector increases and the price of its good drops when the financial constraint is lifted. Moreover, competition as measured by the Lerner index *always* increases with the amelioration of the financial constraint (the index declines).

Does this specification satisfy the stylized facts discussed above? The model generates net entry when the economy wide capital stock is sufficiently high[16]. It also has no trouble producing a decrease in concentration indexes. It can also generate a decrease in average firm size[17] if the total stock of capital is large enough and a large fraction of output is produced by firms that are not financially constrained.

[13]The results reported correspond to a specification with individuals suitably clustered at the values reported by Cagetti and De Nardi. Using a smoother function based on interpolation made no difference for the results.

[14]The solution procedure is as follows. First, the agents are sorted in terms of their capital stock. In this case, the ordering is $\bar{k_1} > \bar{k_2} > ... > \bar{k_N}$. We first solve the model and compute firm profits when only agents 1 and 2 are in sector 1. Then we continue adding firms –in the order of their capital stock– and solve the model with the corresponding fixed number of firms and compute profits for the firms. The procedure stops –the optimal number of firms is reached –when a candidate firm has profits in equilibrium that fall short of its opportunity cost in the other sector. Because initial capital is monotonic in a suitably sorted index of agents, the procedure leads to a unique solution.

[15]Note that for low levels of $\bar{K}$, the capital employed in sector 1 in the unconstrained economy is larger than the capital endowment. This can happen without at the same time undermining the bite of the financial constraint in the constrained economy if we have a static economy and we require that agents produce with their endowment before they can go out and buy good 2. We are grateful to a referee for pointing this out.

[16]To be more precise: when the ratio of the aggregate capital stock to aggregate ability is high. When this ratio is low, a large fraction of firms is financially constrained, in which case there is no entry in the model. We explain this pattern below.

[17]In the literature, firm size is often measured by the number of employees. We measure firm size by output which is related to the level capital. Unfortunately, even if we were to introduce –fixed– labor in the model and interpret k as the capital-labor ratio we would still not be able to link firm size to employment.

Table I: Heterogeneous wealth, homogeneous ability, Cournot

$\bar{K}$	n	K	Q_1	Q_2	p	σ	HH(4)	size	n_c	q_c	W
Convex wealth, Constant ability											
50	12	62.38	52.90	88	1.43	0.09	0.33	4.41			26
50	20	40.00	35.33	80	2.55	1.05	0.55	1.77	20	1	
100	15	84.46	71.05	85	1.42	0.071	0.27	4.74			9.9
100	20	67.96	58.02	80	1.93	0.497	0.49	2.90	19	0.77	
300	26	173.51	143.51	74	1.40	0.040	0.15	5.52			2.0
300	20	164.79	131.11	80	1.64	0.180	0.40	6.56	15	0.50	
500	37	262.84	216.05	63	1.39	0.028	0.11	5.84			2.4
500	20	253.22	193.75	80	1.69	0.165	0.37	9.69	15	0.54	

Note: The lines containing bold characters correspond to the economy without financial constraints. $\theta_k = 0.98$, $a = 1$, $\beta = 0.9$, $A = 1$, $\gamma = 1$. The number of agents, N is set equal to 100. $\bar{K}$ = economy's endowment of capital, n = equilibrium number of firms, K = capital input, Q_1 = output in sector 1, Q_2 = output in sector 2, p = relative price, σ = Lerner index, $HH(4)$ = share of 4 largest firms, size=average firm size, n_c = number of financially constrained firms, q_c = share of industry 1 output produced by constrained firms,W = welfare consumption equivalent of losses from financial friction. It is computed as the percentage increase in total consumption of good 1 that would make the society indifferent to the lifting of the financial constraint.

The key to understanding the properties of the model regarding entry is in the behavior of the incumbent firms. Table I indicates that the magnitude of net firm entry depends negatively on the share of constrained firms. When many firms operate below their optimal capacity (because of the financial constraint), sectoral output expansion can take place with existing firms expanding along the intensive margin. The presence of an operative intensive margin makes it more likely that fewer new firms will be needed in order to meet the higher demand for good 1. Net entry is smaller and can even be negative. Additionally, the unconstrained firms -which are larger than the constrained-shrink with the lifting of the financial constraints (because of the decrease in the price of good 1). The larger their share in the population, the fewer the incumbent, constrained firms and hence the bigger the chunk of output to be produced by new firms. Convexity in initial capital ownership works in favor of entry because it implies that the firms that shrink are large relative to the firms that expand and also that their contraction is large relative to the expansion of the small firms (while concavity has the opposite implication) creating room for firm entry. Note also that the welfare gains from the amelioration of the financial constraint as captured by the decrease of the Lerner index depend positively on the share of constrained firms.

The Lerner index measures deviations of prices from marginal costs. But as discussed in section 3.1, a part of these deviations is due to the financial constraint and is *not* related to the existence of Cournot competition. That is, such deviations would be present even under perfect competition[18]. The remaining part comes from imperfect competition as well as from the interaction between imperfect competition and financial constraints. A question of interest is whether this interaction matters –quantitatively and qualitatively – for the properties of the model. In order to address this question we solve the model under the assumption of perfect competition. Table II reports the results. One can evaluate the importance of the interaction between the two frictions as follows: Consider the case of $\bar{K} = 300$. The effect of the financial friction that is independent of the degree of imperfect competition is 0.08 (the decline in the Lerner index following the elimination of the financial constraint under perfect competition). The effect of imperfect competition that is independent of the financial friction (the difference in the value of the Lerner index between imperfect and perfect competition in the case of a zero financial friction) is 0.04. Consequently, the interaction between

[18]This is due to the fact that a binding financial constraint implies the existence of a non-zero markup even under perfect competition as a competitive, constrained firm cannot adjust its quantity in order to equate its marginal cost to the price.

imperfect competition and imperfect capital markets accounts for 0.06 $(0.18-0.08-0.04)$ of the Lerner index, that is about 33% (0.06 out of a total of 0.18). This is quantitatively substantial.

A similar picture regarding the importance of this interaction emerges from inspection of the output, Q_1, figures across Tables I and II. Consider again the case $\bar{K} = 300$. The combined effect of removing both imperfect competition and the financial friction on the level of output Q_1 is 13 $(144.09 - 131.11)$ units. Of this, the pure effect of the financial constraint is 7.54 $(144.09 - 136.45)$ while the pure effect of imperfect competition is 0.58 $(144.09 - 143.51)$. Hence the interaction accounts for 4.88 $(13 - 7.54 - 0.58)$ units, that is 37% of the output gap arising from the frictions.

Naturally, the relative contribution of the interaction depends on the relative severity of the two frictions. When the financial friction is very large (for instance, when all of the firms are severely, financially constrained, which occurs when capital is scarce relative to ability) then the contribution of the interaction is very small. For instance, in the case of $\bar{K} = 300$, the contribution of the interactive term to the Lerner index and the output gap is 18% and 14% respectively.

It is also worth noting that, in the presence of financial frictions, commonly used measures of change in competition, such as the change in the number of firms or their size, are not particulary informative for determining whether there has been an improvement in competitive conditions or not. This can be seen from the fact that the number of firms may decline following the removal of the financial friction even under perfect competition (see Table II). And also from the fact that under imperfect competition and following the amelioration of the financial friction, the correlation between changes in the number-size of firms and changes in the Lerner index is ambiguous. In order to judge the change in competitive conditions one needs to decompose the change in the Lerner index as described above.

Finally, the comparison of Tables I-II reveals that market structure matters not only quantitatively but also qualitatively (for some of the results). For instance, the removal of the financial constraint leads to firm entry under imperfect competition but to firm exit under perfect competition in the case of $\bar{K} = 300$. Or, in the case of $\bar{K} = 100$, it is associated with the an increase in the HH(4) index under perfect competition but a decline under imperfect competition. These findings reinforce our argument of the unreliability of commonly used measures of competition.

Table II: Heterogeneous wealth, homogeneous ability, perfect competition

$\bar{K}$	n	K	Q_1	Q_2	p	σ	HH(4)	size	n_c	q_c	W
Convex wealth, Constant ability											
50	7	67.74	53.98	93	1.39	0.00	0.57	7.71			28
50	20	40	35.33	80	2.55	1.05	0.55	1.77	20	1	
100	9	90.47	71.83	91	1.40	0.00	0.44	7.98			8.8
100	16	74.60	60.68	84	1.80	0.33	0.60	3.79	16	1.00	
300	19	180.47	144.09	81	1.39	0.00	0.21	7.58			2.2
300	20	172.76	136.45	80	1.52	0.08	0.42	6.82	19	0.88	
500	30	270.00	216.74	70	1.38	0.00	0.13	7.22			2.5
500	20	266.72	202.53	80	1.55	0.06	0.39	10.13	15	0.52	

Note: The lines containing bold characters correspond to the economy without financial constraints. $\theta_k = 0.98$, $a = 1$, $\beta = 0.9$, $A = 1$, $\gamma = 1$. The number of agents, N is set equal to 100. $\bar{K}$ = economy's endowment of capital, n = equilibrium number of firms, K = capital input, Q_1 = output in sector 1, Q_2 = output in sector 2, p = relative price, σ = Lerner index, $HH(4)$ = share of 4 largest firms, size=average firm size, n_c = number of financially constrained firms, q_c = share of industry 1 output produced by constrained firms, W = welfare consumption equivalent of losses from financial friction.

While the model with a convex function of wealth performs well under the assumption of homogeneous ability, it is of interest to investigate the role of ability heterogeneity for several reasons. First, entrepreneurial ability is likely to vary in the population and its presence may undermine the success of the model obtained under homogeneity. And second, this version of the model leads to a degenerate distribution of firm sizes which deprives the model from additional implications. We include ability heterogeneity in the following subsections.

3.4 Heterogeneous ability, homogeneous wealth

The results in this case are straightforward as it is the largest, most efficient firms that face the most severe financial constraints. The relaxation of the constraint allows these firms to expand, forcing less efficient firms to exit. Concentration and average firm size increase, but even in this case, the Lerner concentration index declines.

3.5 Heterogeneous ability, heterogeneous wealth

We maintain the assumption of a convex function of capital holdings that satisfies the US wealth distribution as in section 3.3.2 but now allow ability to vary across individual according to some function $a_i = g(i)$.[19]

Table III reports some representative results.[20] In the first four lines the ability function g is convex in the index of agents. In the first two lines, the correlation of wealth and ability is assumed to be positive while in the third and forth lines it is assumed to be negative. Higher ability means a higher efficient scale of production and thus higher demand for capital. When the correlation with wealth is positive, this higher demand may be met through own resources so the financial constraint may or may not be binding. Under the specification used, it turns out that most of the firms that operate in this sector are constrained (of the 8 firms, only the largest one is not constrained). Its lifting leads to output expansion for the constrained firms along the intensive margin

[19] The evidence on the joint correlation between ability and wealth is sparse and rather inconclusive. Evans and Jovanovic (1989) find a small negative correlation between ability and wealth. Xu (1998) argues that the Evans and Jovanovic data under-represents wealth and leads to a downward bias in the estimation of that parameter. He estimates this correlation with a different data set and finds instead a small and positive value. We take a conservative approach and experiment with different specifications of the $g(\cdot)$ function. See in the appendix for the particular specifications used.

[20] Additional results obtained under various alternative linear and non linear specifications of ability as well as arbitrary correlations between ability and wealth exhibit the patterns described below.

and output contraction for the unconstrained largest firm. The net effect is small firm exit.

In the third and forth lines, it is the poorest that are the most able. In this case, all the firms are constrained and the constraint is tighter relative to the previous case, because of the negative correlation of ability and wealth. There is large expansion along the intensive margin, with significant crowding out of firms.

In lines five through eight ability is assumed to be a concave function with either a positive (the fifth and sixth line) or a negative correlation with wealth (the seventh and eight line). Finally the ninth line has linear ability with a positive correlation with wealth.

As in the case with homogeneous ability, the key consideration for determining the entry-exit pattern is the incidence of the financial constraint over the distribution of ability. If it is the more able that are severely constrained then the amelioration of the constraint is likely to lead to net firm exit. If it is the less able, then the likely outcome is net firm entry. This is due to the fact that the efficient firms have a larger optimal scale. When their constraint is loosened, these firms expand, crowding out smaller firms as well as potential entrants. But if they are not constrained, then the industry expansion occurs through new entrants.

The incidence of the financial constraint over the distribution of ability and thus the net entry-exit pattern depends both on the correlation of ability and wealth in the population and on the –relative curvatures– of the wealth and ability functions. In general, low variability in ability relative to that in wealth and a positive correlation between ability and wealth make net entry more likely as they imply that the brunt of financial constraints is felt *mostly* by the poorer and less efficient individuals. There is some presumption that this is the pattern characterizing the real world.

Table III: Heterogeneous wealth, heterogeneous ability

Abil.	$\bar{K}$	n	K	Q_1	Q_2	p	σ	HH(4)	size	n_c	q_c	cor(a,k)	W
Convex wealth													
CV	100	5	79.3	51.2	95	2.26	0.33	0.92	10.2				12.6
	100	8	60.9	40.2	92	3.26	0.97	0.83	5.0	7	0.48	0.74	
CV	100	10	84.8	65.0	90	1.62	0.12	0.48	6.5				528.4
	100	44	11.0	8.9	56	16.25	11.62	0.13	0.2	44	1.00	-0.30	
CC	100	13	84.8	68.6	87	1.49	0.08	0.34	5.3				10.1
	100	19	68.0	56.4	81	2.01	0.51	0.52	3.0	18	0.75	0.30	
CC	100	8	78.0	46.6	92	2.44	0.33	0.14	5.8				92.2
	100	18	42.4	19.8	82	7.04	2.65	0.29	1.1	17	0.96	-0.74	
LI	100	11	84.9	65.3	89	1.59	0.10	0.43	5.9				9.8
	100	16	67.0	52.9	84	2.21	0.58	0.58	3.3	15	0.69	0.37	

Note: The lines containing bold characters correspond to the economy without financial constraints. CV = convex ability, CA = concave ability, LI = linear ability. $\beta = 0.9$, $A = 1$, $\gamma = 1$. The number of agents, N is set equal to 100. $\bar{K}$ = economy's endowment of capital, n = equilibrium number of firms, K = capital input, Q_1 = output in sector 1, Q_2 = output in sector 2, p = relative price, σ = Lerner index, $HH(4)$ = share of 4 largest firms, size=average firm size, n_c = number of financially constrained firms, q_c = share of industry 1 output produced by constrained firms, W = welfare, consumption equivalent of losses from financial friction.

We have also the solved the model under perfect competition for the specifications reported in Table 3.5. With regard to the relative contribution of the interaction of the two frictions we find basically the same results and insights as those presented in the previous section under homogeneous ability.

3.6 Can the Lerner index *increase* with the amelioration of the financial constraint

In the specifications used so far, the Lerner index has always decreased following the amelioration of the financial constraint. There are two elements that support this tendency. The price of good 1 drops. And the level of output increases which, in a homogeneous economy, would increase average marginal cost. But our economy is heterogeneous so it is conceivable that the average marginal cost could move in the opposite direction, due to relocation of production from high to low marginal cost firms. This could potentially offset the first effect. We demonstrate this possibility below with an example.

Consider the following situation. In the constrained equilibrium, there are some large firms that are either completely unconstrained or slightly constrained. And some smaller firms that are more efficient than the large ones and which are severely constrained. The latter firms have larger markups than the former because their marginal cost is lower due to both higher ability and lower levels of production (recall that marginal cost is increasing in the level of output). Following the amelioration of the constraint, the small, more able firms expand and gain market share at the expense of the large firms. Their markups decrease due to the lower price and the higher level of production. The mark ups of the large firms may well decrease too. Nonetheless the "average" mark up in the economy (the Lerner index) may well increase due to the change in the market shares of the two groups in favor of the group with the high markups.

To keep things simple, let us assume that firms are homogeneous within each of these two groups. Let $s^i_j, i = c, u, \qquad j = H, L$ denote the share of industry output of firms of type j (H indicates high and L low ability) under financial regime i ("c" refers to the case with and "u" to the case without financial frictions). And μ^i_j the corresponding markups. The respective Lerner indexes, σ^c and σ^u are given by $\sigma^c = s^c_L\mu^c_L + s^c_H\mu^c_H$ and $\sigma^u = s^u_L\mu^c_L + s^u_H\mu^u_H$. Because $\mu^c_H > \mu^u_H$ (and the same is likely for the L-type due to the lower price of good 1), there is a natural predisposition for the Lerner index to be lower in the unconstrained economy. In order to counter this propensity so that $\sigma^c < \sigma^u$ it is necessary that the able firms see a large increase in their output share that is accompanied by a small decline in their markups[21]. This is theoretically possible and the example in the Appendix demonstrates this.

While a negative relationship between the financial friction and the Lerner index can indeed emerge, such an outcome requires, in general, finely engineered specifications of the distributions of ability and wealth. Unlike the positive relationship obtained in the previous sections which is very robust, the negative relationship is quite tenuous in the sense that even slight alterations in the distribution of ability and/or wealth reverse it. So such examples may establish the theoretical possibility of a negative relationship and also tease out the requirements for such a relationship, but they also reveal how special these cases may be.

3.7 Other implications of the model

The model has implications for the relationship between the level of development, the size distribution of firms and the degree of competition in product markets. For similar ability and wealth functions across countries the model implies that poorer countries are likely to not only have smaller firms but also less competitive markets than richer countries. They are thus more likely to benefit from further advancements in financial markets.

The model can also be used to study the political economy of financial liberalization/development. In particular, the relationship between incumbency and opposition to liberalization? An interesting implication of our analysis is that incumbency does not necessarily imply opposition to liberalization. For instance, in section 3.3.2 we presented cases (for instance, when all the large firms were constrained) where –at least some– incumbents would have favored liberalization. It is also quite easy to construct an example in which the firms operating in sector 1 do not speak with a single voice on issues of financial liberalization.

Before concluding it is worth raising an intriguing possibility associated with the recent disruption of the functioning of the financial markets as well as the proposed tightening of financial regulation. To the extent that financial disruption and financial regulation accentuate credit constraints, the degree of competition in product markets could be an additional –and previously unidentified– casualty of the recent financial crisis.

[21]The markups would not change much if the price of good 1 did not decrease significantly if the marginal cost curve were relatively flat. The larger $\beta < 1$ and "a", the flatter the marginal cost curve.

4 Conclusion

The effects of improvement in the functioning of asset markets (financial development, liberalization, deepening...) on the allocation of resources, economic growth and welfare have been extensively studied in the literature. There is one aspect, though, that has received scant attention in spite of the commonly held view that it is of great importance for economic performance and welfare. This is the relationship between finance and competition in product markets.

In this paper we have taken a first step in characterizing the effects of financial constraints on competition. We have used a general equilibrium model with heterogeneity in ability and wealth (and hence in the degree to which financial constraints may bind). We find that the amelioration of the financial constraints in general increases competition in the product markets of the financially dependent sectors. This result is very robust to the specification/parametrization of the model and also occurs independently of what happens the standard market concentration indexes. Moreover, we find that the model has good and robust overall empirical properties. In particular, it can replicate some important stylized facts pertaining to the effects of the lifting of financial constraints on the distribution of firms and entry/exit patterns. Namely, it can deliver net firm entry, a decrease in average firm size and a decrease in the standard concentration indexes. Consequently, the model seems to represent a reliable vehicle for the study of the relationship between finance and competition.

The analysis has been carried out in a framework that has been restricted in order to make it feasible to study such a complicated issue. For instance, the nature of the financial constraints has not been modelled. They correspond more closely to unspecified costs of asset trade than to the elaborate agency problems typically discussed in the literature; dynamics have been abstracted from; and so on. Consequently, there is a number of demanding but important extensions awaiting.[22] One could involve the incorporation of dynamics, so that both the cross section and time series properties of the distribution of firms in financially dependent sectors could be derived. Another might involve making financial constraints endogenous.

[22] An obvious but simple extension would be to consider a more general specification for the utility function in order to allow for variable expenditures.

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5 Appendix

Proposition 1: *The financially unconstrained economy has a lower relative price and a higher quantity of the financially dependent good.*

Proof. Let us remove the financial constraints and hold p fixed. Borrowed capital now allows the –previously– financially constrained firms to expand output. The output of the financially unconstrained firms (if there are any) will also increase due to equation (6). Hence, C_1 is higher. The increased use of good 1 as capital in sector 1 implies a lower level of C_2. Consequently, the production of good 1 relative to that of good 2 increases as well. Compared to the equilibrium prevailing before the removal of the financial constraints, there is now an excess supply of good 1 relative to that of good 2. In order for equilibrium to be restored p must decrease.

Could Q_1 end up being lower in the unconstrained equilibrium? Given that a lower p requires C_1/C_2 to increase, this would require a decrease in C_2. But C_2 can decrease only if either more of good 2 is used as capital in sector 1 or (and) if fewer agents operate in sector 2. In either case, that implies a higher $C_1(Q_1)$. ■

Proposition 2: *Under homogeneous ability and wealth, the financially unconstrained economy has a smaller number of firms than the constrained one when β is close to one.*

Proof. Let N be the total number of agents and n the number of firms in sector 1. Equation (4) can be re-written as $\gamma p C_1 = C_2 = (N-n)A + \bar{K} - nk$. Multiplying both sides of (22) by n and substituting for nk in the previous expression leads to

$$pC_1 = \frac{NA + \bar{K}}{1+\gamma}$$

Consequently, pC_1 and C_2 are independent of the existence of financial constraints. Hence,

$$pC_1 = p^c q^c n^C = p^c a(k^c)^\beta n^c = p^u a(k^u)^\beta n^u = p^u q^u n^c$$

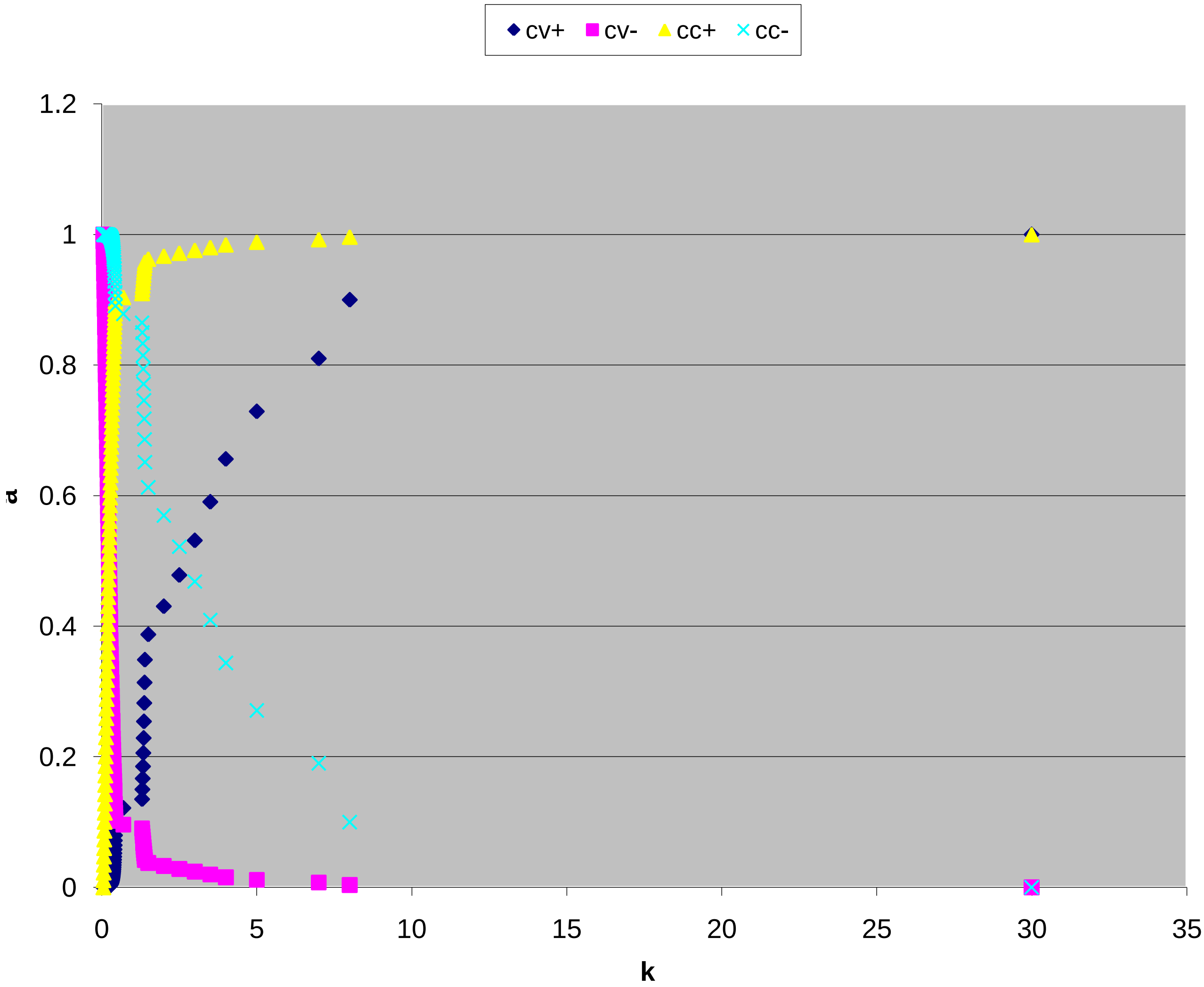
Let $\beta = 1$. We then have that $n^c > n^u$ iff $p^c k^c < p^u k^u$.

From equation (22) we have

$$A = ap^c k^c - k^c = ap^u k^u - k^u$$

Equation 5 can be rewritten as $a(p^c k^c - p^u k^u) = k^c - k^u$, which implies that $p^c k^c < p^u k^u$) iff $k^c < k^u$. But equation 5 can also be written as $(ap^c - 1)k^c = (ap^u - 1)k^u$ which implies that $k^c < k^u$ iff $p^c > p^u$. Hence, $n^c > n^u$.

Figure 1: Ability and initial capital holdings



CV signifies convex and CC concave ability. A '+' signifies positive correlation of ability with wealth and a '-' a negative correlation.

Negative correlation between the friction and the Lerner index

Table IV: Negative correlation between the friction and the Lerner index

$\bar{K}$	n	K	Q_1	Q_2	p	σ	HH(4)	size	n_c	q_c	cor(a,k)	W
37	3	15.92	32.18	7	0.87	0.644	1	10.73				25
37	4	15.79	26.60	6	1.02	0.627	1	6.65	1	0.11	0.38	

Note: The first row corresponds to the economy without financial constraints. $\beta = 0.9$, $A = 1$, $\gamma = 1$. The number of agents, N is set equal to 10. For variable definitions see the notes to the previous tables

Table V: Initial wealth and ability

Agent	1	2	3	4	5	6	7	8	9	10
$\bar{k}_i$	10	10	10	1	1	1	1	1	1	1
a_i	2	2	1.5	1	1	1	1	1	1	3

N=10, Correlation($\bar{k}_i, a_i$) = 0.38. **Note**: The negative relationship between the friction and σ obtained in this example is very fragile. For instance, changing agent 9's level of ability from 1 to 3 overturns the negative correlation ($\sigma^c = 0.92, \quad \sigma^u = 0.48$).

Table VI: Properties of the equilibrium

Agent	1	2	3	10
	Constrained			
q_i	9.43	9.43	4.72	3
dC/dq_i	0.66	0.66	0.84	0.37
q_i/Q	0.35	0.35	0.18	0.12
μ_i	0.55	0.55	0.21	1.76
	Unconstrained			
q_i	8.21	8.21		15.75
dC/dq_i	0.65	0.65		0.44
q_i/Q	0.26	0.26		0.48
μ_i	0.34	0.34		0.96