To be or not to be in monetary union: A synthesis*

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Abstract

Monetary union can benefit countries suffering from policy credibility problems if it eliminates the inflation bias and also allows for more efficient management of certain shocks. But it also carries costs as *some* stabilization may be feasible even in the absence of credibility, and this may be more than what an individual country can hope for in a monetary union. In this paper, we combine the stabilization and credibility branches of the currency union literature and construct a simple welfare criterion that can be used to evaluate alternative monetary arrangements. We produce examples where monetary union may be welfare improving even for low-modest levels of inflation bias (2-3%) as long as business cycles are not too a-synchronized across countries.

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Introduction

The literature on international monetary arrangements and, in particular, on the issue of the optimal degree of exchange rate volatility is quite large. Two general results seem to have emerged from the recent literature. First, at least some degree of exchange rate volatility is optimal. And second, the optimal degree of volatility may involve a trade off between balancing the volatility of the real (CPI based) exchange rate and that of the terms of trade (see Devereux and Engel, 2007).

In the light of the first result, it would appear that international monetary arrangements that eliminate all exchange rate variability, such as a currency union, would be hard to justify on economic welfare grounds alone. This, however, is not true as the literature on optimal exchange rate regimes abstracts from a factor that has played a key role in the formation of the European monetary union (EMU). Namely, the differences in the degree of credibility enjoyed by national monetary authorities. For instance, a country may participate in a currency union as a means of delegating the conduct of its monetary affairs to another country's authority. In abstracting from issues of policy credibility, the new literature on optimal international monetary arrangements is thus related to the branch of the earlier literature on currency union that compared flexible exchange rate regimes and currency unions on the basis of macroeconomic stability alone[1]. According to that branch, participation in monetary union is invariably a costly affair, because a country loses control over its monetary policy and cannot thus stabilize macroeconomic activity. The size of the cost depends on traditional optimum currency area criteria (see Tavlas, 1993, or de Grauwe, 2005) such as similarity in economic structure and shocks, labor mobility and so on.

There is, however, a second branch of the optimum currency area literature which assumes that the conduct of monetary policy does differ across the two monetary environments. This branch focuses on the possibility that the move to a new monetary arrangement is entirely motivated and accompanied by the adoption of a more efficient monetary practice. Hence, the loss of monetary control may carry benefits. The standard example involves a country that suffers from an inflation bias à la Barro and Gordon (1983), and which cannot gain credibility through national means (such as making the central bank independent). This country may eliminate its inflation bias overnight simply by joining a union whose monetary policy enjoys greater credibility (Giavazzi and Pagano, 1988). Many of the countries that joined EMU are perceived as having acted according to this logic. In this approach, monetary union participation represents an unambiguously welfare improving proposition as issues of stabilization never come into play in a country conducting its monetary affairs in a sub-optimal,

[1] Of course, unlike the earlier literature, the new one uses measures of macroeconomic stability that map into proper measures of welfare.

discretionary manner.

In this paper we argue that the complete abstraction from stabilization issues in the credibility motivated monetary union literature is not justified. Even monetary authorities that suffer from an inflation bias can do some stabilization. Based on the work of Woodford (2003) we demonstrate that this perhaps limited and not always efficient ability for macroeconomic stabilization may still be of value in ways that are emphasized by the first branch of the literature, namely the one focusing on macroeconomic stabilization. But abstracting from issues of credibility, as is done by the stabilization literature, is not justified either, as monetary authorities in different countries do seem to enjoy different levels of credibility. Consequently, both branches may be needed in order to produce a relevant assessment of the economic costs and benefits of alternative monetary arrangements. Our paper does just that, namely, it merges these two distinct approaches and makes it possible to study the decision to or not to participate in a monetary union when this decision involves meaningful trade offs[2].

Naturally, our paper is not the first one in the literature to attempt this. Prominent recent examples are the papers by Alesina and Barro (2002), and Cooley and Quadrini (2003). But Alesina and Barro do not use a fully specified macroeconomic model and also rely on an ad hoc Barro-Gordon objective function which is not a proper welfare criterion. Cooley and Quadrini, on the other hand, use a fully specified, dynamic, general equilibrium framework. Their model, though, is a model with flexible prices. There is a widely held view that price rigidity is an important feature of the real world and that the costs and benefits of alternative international monetary arrangements are likely to be affected significantly by the degree of price rigidity[3]. It is consequently worthwhile to revisit this issue in the context of the standard, sticky price, macroeconomic model used nowadays for monetary analysis, namely the New Keynesian (NK) model[4].

We use the standard NK model to derive a simple, empirically implementable criterion that relies on a standard parametrization and provides a welfare evaluation of alternative monetary arrangements for the model economy under consideration.

We apply our criterion under various scenarios. We produce examples where even a modest

[2]It is standard in the literature to assume that there is no inflation bias in a currency union. However, Chari and Kehoe (2007) argue that such a bias may arise if *fiscal* policies are not coordinated across the union members.

[3]Cooley and Quadrini admit that their main result regarding the welfare improving properties of a currency union owes much to the flexible price specification. They speculate that this superiority would be likely overturned under fixed prices (because of the greater significance of stabilization policy in that case). Our analysis shows that, under a plausible calibration of nominal rigidity and shocks, this may not be the case.

[4]Note that the NK model has been the standard vehicle of analysis in two related literatures. One that studies the optimal degree of exchange rate flexibility (Benigno and Benigno, 2003, Devereux and Engel, 2007). And another that deals with the benefits from international policy coordination. The latter literature has been preoccupied with issues of strategic interactions, rather than with the optimal choice of the exchange rate arrangement; see, for instance, Canzoneri, Cumby and Diba, 2005.

inflation bias (in the range[5] of 2-3%) would justify participation in the EMU on standard economic welfare grounds as long as the shocks were not too a-synchronized across countries. But even if they were a-synchronized, a currency union might still be preferable if cost-push shocks were more important than other shocks. In general, however, the ranking is ambiguous, so one would need to use a country specific DSGE model in order to evaluate the desirability of monetary union for that particular country under consideration[6].

The remaining of the paper is organized as follows. Section 1 presents the model. Section 2 considers alternative international monetary arrangements in turn and section 3 carries out the comparison of these arrangements. The last section contains the conclusions.

1 The model

We use the small open economy model of Galí and Monacelli (2005) with three modifications: First, we make all countries besides the one considered identical in all respects, so that the rest of the world can be considered as a single foreign economy; second, we introduce domestic and foreign cost-push shocks; and, third, we allow for the possibility of an inflation bias. For simplicity we assume, as Galí and Monacelli (2005) do in their welfare analysis, that utility from consumption is logarithmic and the elasticity of substitution between domestic and foreign goods is equal to unity[7].

1.1 The domestic economy

The domestic economy is a standard, small open New Keynesian economy that is linked to the rest of the world through trade in goods and assets, with a degree of openness given by α, $0 < \alpha < 1$. The economy is described by a set of log-linear equations. Following standard practice in the literature (see Woodford, 2003, ch. 6, Galí, ch. 5, 2008), the log-linearization of the model as well as the second-order approximation of the welfare measure are done in the neighborhood of the –distorted– zero-inflation steady state as optimal policy is found to require an equilibrium in its neighborhood (even in the case of large distortions).

The Phillips curve is given by:

$$\Delta p_{H,t} = \beta E_t \left\{ \Delta p_{H,t+1} \right\} + \kappa \widehat{x}_t + u_t \tag{1}$$

with $0 < \beta < 1$ and $\kappa > 0$. $p_{H,t}$ denotes the GDP deflator (in domestic currency) at date t, and

$$\widehat{x}_t \equiv x_t - x \tag{2}$$

[5]These seem to be below those present in Europe in the pre-EMU era, see section 3.

[6]Nonetheless, our analysis establishes that price rigidity does not per se create a presumption *against* monetary union.

[7]We later discuss whether deviations from these assumptions may impact on the results.

is the deviation at date t of the welfare-relevant output gap, x_t, from its zero-inflation steady-state value, x. $E_t\{.\}$ is the –rational– expectations operator conditional on information available at date t (which includes current variables and shocks). Δ is the first-difference operator. u_t is an exogenous cost-push shock occurring at date t. The welfare-relevant output gap is defined as

$$x_t \equiv y_t - y_t^e \tag{3}$$

where y_t is the actual and y_t^e the efficient level of output respectively (see the Appendix for the determination of the latter).

The IS equation is given by:

$$\widehat{x}_t = E_t\{\widehat{x}_{t+1}\} - (r_t - E_t\{\Delta p_{H,t+1}\} - rr_t) \tag{4}$$

where r_t denotes the short-term nominal interest rate at date t. The natural rate of interest rr_t is :

$$rr_t = \rho + E_t\{\Delta a_{t+1}\} \tag{5}$$

with $\rho > 0$. a_t is an exogenous productivity shock in period t.

The terms-of-trade equation, which is derived from the law of one price and an international risk sharing equation (see Galí and Monacelli, 2005), can be written as:

$$y_t = y_t^* + e_t + p_t^* - p_{H,t} \tag{6}$$

where y_t^* is the level of foreign output at date t, e_t is the nominal exchange rate (*i.e.* the price of the foreign currency in terms of the domestic currency) and p_t^* is the foreign price level (in terms of foreign currency).

The interest rate parity (UIP) equation is:

$$r_t = r_t^* + E_t\{\Delta e_{t+1}\} \tag{7}$$

with r_t^* denoting the foreign, short-term, nominal interest rate at date t.

1.2 The foreign economy

The foreign economy (the rest of the world) is essentially a "closed" economy in the sense that it is too big relative to the domestic economy to be affected by anything other than foreign developments. It is characterized by standard Phillips curve and IS equations, log-linearized in the neighborhood of the zero-inflation steady state.

The Phillips curve takes the form:

$$\Delta p_t^* = \beta E_t\left\{\Delta p_{t+1}^*\right\} + \kappa \widehat{x}_t^* + u_t^* \tag{8}$$

where

$$\widehat{x}_t^* \equiv x_t^* - x^* \tag{9}$$

denotes the deviation at date t of the foreign welfare-relevant output gap, x_t^*, from its zero-inflation steady-state value x^*. u_t^* is an exogenous cost-push shock in period t. The foreign welfare-relevant output gap is defined as

$$x_t^* \equiv y_t^* - y_t^{e*} \tag{10}$$

where y_t^{e*} is the efficient foreign output level.

The IS equation takes the form:

$$\widehat{x}_t^* = E_t\left\{\widehat{x}_{t+1}^*\right\} - \left(r_t^* - E_t\left\{\Delta p_{t+1}^*\right\} - rr_t^*\right) \tag{11}$$

where the natural rate of interest rr_t^* is written:

$$rr_t^* = \rho + E_t\left\{\Delta a_{t+1}^*\right\} \tag{12}$$

and a_t^* is an exogenous productivity shock.

1.3 The shocks

All four shocks are assumed to follow AR(1) processes:

$$u_t = \rho_u u_{t-1} + \varepsilon_t^u$$

$$a_t = \rho_a a_{t-1} + \varepsilon_t^a$$

$$u_t^* = \rho_{u^*} u_{t-1}^* + \varepsilon_t^{u^*}$$

$$a_t^* = \rho_{a^*} a_{t-1}^* + \varepsilon_t^{a^*}$$

where $0 \leq \rho_u < 1$, $0 \leq \rho_a < 1$, $0 \leq \rho_{u^*} < 1$, $0 \leq \rho_{a^*} < 1$, and ε_t^u, ε_t^a, $\varepsilon_t^{u^*}$, $\varepsilon_t^{a^*}$ are *i.i.d.* shocks with variances V_u, V_a, V_{u^*} and V_{a^*} respectively. We assume that $E\left\{\varepsilon_t^u \varepsilon_{t+k}^a\right\} = E\left\{\varepsilon_t^u \varepsilon_{t+k}^{a^*}\right\} = E\left\{\varepsilon_t^{u^*} \varepsilon_{t+k}^a\right\} = E\left\{\varepsilon_t^{u^*} \varepsilon_{t+k}^{a^*}\right\} = 0$ for all $k \in \mathbb{Z}$ and $E\left\{\varepsilon_t^u \varepsilon_{t+k}^{u^*}\right\} = E\left\{\varepsilon_t^a \varepsilon_{t+k}^{a^*}\right\} = 0$ for all $k \in \mathbb{Z}^*$, and denote $\mu_a \equiv E\left\{\varepsilon_t^a \varepsilon_t^{a^*}\right\}$ and $\mu_u \equiv E\left\{\varepsilon_t^u \varepsilon_t^{u^*}\right\}$.

2 Alternative international monetary arrangements

2.1 Social welfare loss functions

The period t utility of the representative domestic household is given by

$$U(C_t, N_t) = logC_t - \frac{N_t^{1+\varphi}}{1+\varphi}$$

As shown in the Appendix, Part A, under the assumption that the zero inflation steady state is characterized by a sufficiently small distortion, the social welfare loss function for the domestic economy takes the form[8]:

$$L_t = E_t \left\{ \sum\nolimits_{k=0}^{+\infty} \beta^k \left[\left(\Delta p_{H,t+k}\right)^2 + \delta \left(\widehat{x}_{t+k} - \chi\right)^2 \right] \right\}, \tag{13}$$

where χ and δ are functions of the parameters of the model. In particular, if one period represents one quarter and the inflation rate is measured at a quarterly rate, $\delta = \frac{\kappa}{\epsilon}$ where ϵ denotes the elasticity of substitution between varieties produced in the domestic economy. χ is given (see the Appendix, Part A) by

$$\chi \equiv \frac{\epsilon\left(1-\tau\right)\left(1-\alpha\right) - \left(\epsilon - 1\right)}{\left(\epsilon-1\right)\left(1+\varphi\right)}. \tag{14}$$

α is the degree of openness (the share of foreign goods in domestic consumption). τ is an exogenous proportional tax or subsidy. If χ is positive (negative) then the model will exhibit an inflation (deflation) bias under discretion (see equation 16). Given values for the other parameters, the appropriate choice of τ –and thus χ– can generate an inflation bias of a particular size. We establish later that for realistic values of the inflation bias, χ is sufficiently small as to make the minimization of this quadratic loss function subject to the linearized structural equations lead to a correct linear approximation of the socially optimal monetary policy.

As in Galí and Monacelli (2005) the domestic social loss function (13) does not differ from that which would have obtained in a closed economy. This is due to the assumption of unitary elasticity of intertemporal and intratemporal substitution. De Paoli (2009) shows that the welfare function would also include the real exchange rate if one deviated from this assumption. We later discuss if and how a more general specification might impact on our main results.

The social welfare loss function for the –essentially– closed foreign economy takes the form:

$$L_t^* = E_t \left\{ \sum\nolimits_{k=0}^{+\infty} \beta^k \left[\left(\Delta p_{t+k}^*\right)^2 + \delta \left(\widehat{x}_{t+k}^* - \chi^*\right)^2 \right] \right\}, \tag{15}$$

where χ^* is a function of the parameters of the model.

2.2 Flexible exchange rate with optimal discretionary policy

In order to motivate participation in a monetary union with the foreign country, we will assume that the *conduct* of monetary policy differs significantly across the domestic and the

[8] Equivalently, we could write this social welfare loss function in a form that makes the term linear in $\widehat{x}_{t+k}$ appear explicitly, as in Galí (2008, chap. 5) by expanding the second quadratic term. Namely, $L_t = E_t \left\{ \sum_{k=0}^{+\infty} \beta^k \left[\left(\Delta p_{H,t+k}\right)^2 + \delta \left(\widehat{x}_{t+k}\right)^2 - 2\delta\chi\widehat{x}_{t+k} \right] \right\} + t.i.p.$, where "$t.i.p. = \delta\chi^2/(1-\beta)$" stands for "terms independent of policy".

foreign economies. In particular, policy is conducted in a discretionary fashion in the domestic economy, while there is policy commitment in the foreign economy[9]. If the zero inflation steady state level of output falls short of the efficient level then there is an inflation bias at home while no such bias exists abroad.

The variables $\Delta p_{H,t}$, $\widehat{x}_t$ and r_t are determined by equations (1), (4), (5) and the domestic interest-rate rule, while the nominal exchange rate e_t is residually determined by equation (6), given the foreign country equilibrium.

The domestic policymaker therefore chooses r_t at each date t in order to minimize (13) subject to (1), (4) and (5). As shown in the Appendix, Part B, this results in the following solution for domestic inflation and the output gap:

$$\Delta p_{H,t} = \frac{\kappa\delta\chi}{\kappa^2 + \delta(1-\beta)} + \frac{\delta u_t}{\kappa^2 + \delta(1-\beta\rho_u)} \tag{16}$$

$$\text{and } \widehat{x}_t = \frac{\delta(1-\beta)\chi}{\kappa^2 + \delta(1-\beta)} - \frac{\kappa u_t}{\kappa^2 + \delta(1-\beta\rho_u)}, \tag{17}$$

The solution has four important properties. First, if χ is greater than zero then there exists an inflation bias (the first term in (16)). This term would have been absent in the presence of policy commitment. The reason for the inflation bias is that $\chi > 0$ means that the net effect of the various distortions present in the model (imperfect competition, distortionary tax, terms of trade externality[10]) is to make actual output fall short of its efficient level. A policymaker who cannot commit will systematically try to close this gap and this will generate a positive rate of actual and expected inflation as in Barro and Gordon (1983). Second, the IS shocks do not matter for inflation and the output gap. This is because these shocks do not generate a trade off between inflation and output gap variability. That is, limiting the variability of one also limits the variability of the other. As the same result would have obtained under policy commitment this suggests that discretionary policy involves an efficient response to some types of shocks[11]. Third, the domestic variables are not affected by foreign shocks. This is due to the assumption of a unitary elasticity of intertemporal and intratemporal substitution. And fourth, the response of the economy to a domestic cost push shock differs from that under policy commitment (see below). As is well known, the response under discretion is less efficient than that under commitment because the policymaker cannot rely on credibility to spread out (smooth) the reaction to a current shock. Woodford (2003) contains a detailed discussion of this point.

[9] In this paper we take it for granted the existence or absence of credibility in monetary policy as its exact source does not matter for our analysis. There is a very large literature -mostly from the late 80s and early 90s- that deals with the sources of and remedies for lack of policy credibility.

[10] See the discussion in section 3 on the role of these distortions in generating an inflation/deflation bias in the conduct of discretionary policy.

[11] Note that for the same reason the response to IS shocks would remain efficient also in a more general version of the model where foreign shocks entered the domestic IS curve.

Using (13), (16) and (17) we obtain the following expression for the unconditional mean of the loss function:

$$L^{flex} = \frac{\kappa^2\delta\left(\kappa^2+\delta\right)\chi^2}{(1-\beta)\left[\kappa^2+\delta(1-\beta)\right]^2} + \frac{\delta\left(\kappa^2+\delta\right)V_u}{(1-\beta)\left(1-\rho_u^2\right)\left[\kappa^2+\delta(1-\beta\rho_u)\right]^2}. \tag{18}$$

2.3 Monetary union

We now consider the implications of monetary union for domestic welfare. We will assume that the central bank in charge of monetary affairs in the union can credibly pre-commit and in particular acts according to Woodford's (2003) timeless perspective.

Given the equilibrium in the foreign economy (see the Appendix, Part C), the domestic variables $\Delta p_{H,t}$ and $\widehat{x}_t$ are determined by equations (1), (2), (3), and (6) together with

$$e_t = e \tag{19}$$

and r_t is determined by (7) and (19). It is instructive to focus on the special case with zero autocorrelation in the shocks, $\rho_{u^*} = \rho_{a^*} = \rho_u = \rho_a = 0$ (the solution for the case of autocorrelated shocks can be found in the Appendix, Part D). In this case the equilibrium in the domestic small economy is given by:

$$\begin{aligned}\Delta p_{H,t} &= -\kappa\theta\varepsilon_t^a + \kappa(1-\theta)\sum_{k=1}^{+\infty}\theta^k\varepsilon_{t-k}^a \\ &\quad +\theta\varepsilon_t^u - (1-\theta)\sum_{k=1}^{+\infty}\theta^k\varepsilon_{t-k}^u \\ &\quad +\kappa\theta\varepsilon_t^{a^*} - \kappa(1-\theta)\sum_{k=1}^{+\infty}\theta^k\varepsilon_{t-k}^{a^*} \\ &\quad +\sum_{k=0}^{+\infty}\left[(1-\theta)\theta^k - (1-\omega)\omega^k\right]\varepsilon_{t-k}^{u^*}\end{aligned} \tag{20}$$

$$\begin{aligned}\text{and } \widehat{x}_t &= -(1-\kappa\theta)\varepsilon_t^a + \kappa\sum_{k=1}^{+\infty}\theta^{k+1}\varepsilon_{t-k}^a - \sum_{k=0}^{+\infty}\theta^{k+1}\varepsilon_{t-k}^u \\ &\quad +(1-\kappa\theta)\varepsilon_t^{a^*} - \kappa\sum_{k=1}^{+\infty}\theta^{k+1}\varepsilon_{t-k}^{a^*} \\ &\quad -\sum_{k=0}^{+\infty}\left[\frac{\kappa}{\delta}\omega^{k+1} - \theta^{k+1}\right]\varepsilon_{t-k}^{u^*}\end{aligned} \tag{21}$$

where ω, θ and the polynomial $P(X)$ are defined in the Appendix, parts C and D. The corresponding unconditional mean of the loss function of the small open economy is then

$$\begin{aligned}L^{mu} &= \frac{\delta\chi^2}{1-\beta} + \frac{V_a + V_{a^*} - 2\mu_a}{1-\beta}\left[\frac{2\kappa^2\theta^2}{1+\theta} + \delta - 2\kappa\delta\theta + \frac{\kappa^2\delta\theta^2}{1-\theta^2}\right] \\ &\quad + \frac{\theta^2 V_u}{(1-\beta)(1+\theta)}\left[2 + \frac{\delta}{1-\theta}\right] + \frac{V_{u^*}}{1-\beta}\left[\frac{1-\theta}{1+\theta} + \frac{\delta\theta^2}{1-\theta^2}\right. \\ &\quad \left. + \frac{1-\beta\omega^2}{1+\omega} - \frac{2(1-\theta)(1-\omega)}{1-\theta\omega} - \frac{2\kappa\theta\omega}{1-\theta\omega}\right] \\ &\quad - \frac{2\theta\mu_u}{1-\beta}\left[\frac{2\theta}{1+\theta} + \frac{\delta\theta}{1-\theta^2} - \frac{\omega(2-\theta-\omega)}{1-\theta\omega} - \frac{\kappa\omega}{1-\theta\omega}\right]\end{aligned} \tag{22}$$

3 Discussion

Equations (18) (with $\rho_u = 0$) and (22) can be used to study the factors that favor monetary union over a flexible exchange rate system. Some special cases will help shed light on these factors.

Positive inflation bias, no shocks

Consider first a situation where the inflation bias is positive and there are no shocks. This is typically the case analyzed in the branch of the optimum currency area literature that focuses on issues of credibility. In this case, the second term in (18) as well as all the terms in (22) except for the first one involve variances and covariances of the shocks and are thus equal to zero. The comparison of the two monetary arrangements reduces to comparing the first term in (18) to that in (22). By combining these two expressions it can be seen that monetary union dominates if and only if $\frac{2\beta-1}{(1-\beta)^2} > \frac{\delta}{\kappa^2}$ which is always the case as long as the discount factor, β, is sufficiently high[12]. Moreover, the advantage of participation in monetary union is increasing in the size of the inflation bias (the value of χ) and decreasing in the weight attached to stabilization of the output gap in the welfare function (δ). The value of this weight is determined chiefly by the slope of the short run Phillips curve (κ).

All this is to be expected. In the absence of shocks there is no need for national output gap stabilization, so there is no value to independent (national) monetary policy. So let us now compare the two regimes under the assumption that there is *no* inflation bias but there are shocks. Is there a presumption that one regime would perform better than the other?

Shocks, no inflation bias

As argued above, for some shocks, stabilization under discretion in a flexible exchange rate regime is efficient. This is true for IS shocks, because the policymakers do not face any trade offs between conflicting objectives (namely, output vs inflation stabilization) when reacting to them. As a result, these shocks do not enter the optimal levels of inflation and output and the objective function under discretion. For IS shocks, monetary union lowers domestic welfare unless the domestic and foreign shocks are perfectly positive correlated (i.e. $V_a + V_{a^*} - 2\mu_a = 0$) in which case these shocks do not enter the welfare function (22). The deterioration in welfare is higher, the smaller the correlation between domestic and foreign IS shocks.

But for other shocks, such as the Phillips curve shock, which create a trade off between inflation and output stability, the comparison of the two regimes is less straightforward. On the one hand, the response to the domestic cost push shock under discretion and flexible exchange rates is inefficient relative to that under commitment. On the other hand, the monetary

[12]Under Woodford's timeless perspective approach there exist paradoxical cases where commitment is dominated by the discretionary equilibrium. This result is well known (see, for instance, Loisel, 2008). Commitment would always dominate if we were to use a stronger version of commitment technology.

authority in the currency union may only pay limited attention to that shock as it is pre-occupied with the union wide shock. There is thus a tension between an inefficient response and an efficient but potentially limited response. Again the correlation between domestic and foreign shocks plays an important role for evaluating the relative desirability of a currency union. If domestic and foreign shocks are positively correlated, then there is an indirect appropriate reaction to the domestic shock even in the absence of any direct response. Hence, to the extent that domestic and foreign cost push shocks are highly, positively correlated, the domestic economy will be able to enjoy -indirectly- a more efficient response to its own cost push shocks through the union central bank than it would have accomplished on its own. Monetary union is thus welfare improving even in the absence of an inflation bias if the correlation of domestic and foreign cost push shocks is sufficiently positive.

As in the traditional optimum currency area literature, a positive correlation of shocks works in favor of monetary union. Note, however, that there is an important difference between our analysis in the absence of an inflation bias and the traditional OCA approach. In the latter, the country joining monetary union can never be made better off as far as macroeconomic stabilization is concerned. At best, it will be indifferent if the shocks are perfectly, positively correlated across countries. In our model, monetary union may bring about positive macroeconomic stability gains if cost push shocks are the main source of macroeconomic volatility and if they are sufficiently positively correlated across countries[13]. This is a novel element that had not been identified before in the literature.

In order to give a more quantitative flavor to these arguments we have considered three specific correlation structures (very positive, very negative and zero correlation of shocks). In order to highlight the forces at work we have also considered each category of shocks separately. The rest of the parameters of the model have been taken from the literature (Woodford, 2003, Galí and Monacelli, 2005) and are given in Table (1). The entries in Table (2) give the inflation equivalent of moving from a float under discretion to monetary union under commitment. By "inflation equivalent" we refer, following Jensen (2002), to the value of the permanent increase in the quarterly inflation rate relative to zero (expressed in percentage points) that would generate a change in the unconditional mean of the loss function equal to $\left|L^{mu} - L^{flex}\right|$, with a positive sign if $L^{flex} \geq L^{mu}$ and a negative sign if $L^{flex} < L^{mu}$. It is computed according to the formula:

$$\Pi \equiv \left\{ \begin{array}{c} 100\sqrt{(1-\beta)\left|L^{mu} - L^{flex}\right|} \text{ if } L^{flex} \geq L^{mu} \\ -100\sqrt{(1-\beta)\left|L^{mu} - L^{flex}\right|} \text{ if } L^{flex} < L^{mu} \end{array} \right\}$$

Table 2 summarizes the welfare comparisons across regimes under the assumption that the inflation bias (the pre-union average inflation difference from the foreign country's rate)

[13] An example of this appears in table 2 below.

Table 1: Parameters

β	κ	ϵ	$\delta = \kappa/\epsilon$	φ	α
0.99	0.024	6	0.004	3	0.4

[a] Source: Woodford (2003), Galí and Monacelli (2005).

is zero. As explained above, currency union is never welfare improving when the only source of variation is productivity shocks (column 3). It can be welfare improving in the presence of cost-push shocks but only when the correlation of these shocks across countries is sufficiently positive (columns 2 and 4, row 1).

Table 2: Welfare comparisons of alternative regimes: The case of a zero inflation bias

	$\sigma_u = 0.001$,$\sigma_a = 0$	$\sigma_u = 0$,$\sigma_a = 0.001$	$\sigma_u = 0.001$,$\sigma_a = 0.001$
corr =0.9	+1.2625	-0.0927	+1.2591
corr =0	-0.7138	-0.2933	-0.7717
corr =-0.9	-1.6165	-0.4042	-1.6663

[a] The numbers represent the inflation equivalent of moving from a float under discretion to monetary union under commitment. A + means a welfare gain and a − a welfare loss. There is NO inflation bias under discretion in the flexible regime. u is the cost push and a the productivity shock. σ represents variance.

The general case

These two special cases (inflation bias with no shocks and shocks without an inflation bias) discussed above bound the more interesting cases which involve simultaneously an inflation bias and macroeconomic stabilization considerations.

Table (3) summarizes the welfare comparisons across regimes under the assumption that the inflation bias (the pre-union average inflation difference from the foreign country's rate) is 3% per annum. This is an arbitrary number but we think it represents a good benchmark case (see below, Table 4). Using higher values (say 6%) would stack the cards too much in favor of credibility and eliminate any meaningful trade off between credibility and stabilization. Lower values (say 1%) would not really constitute an interesting inflation bias.

Before reporting on the comparisons of alternative monetary arrangements let us examine the properties of the optimal equilibrium as well as whether the parametrization that delivers a 3% annual inflation bias is consistent with the sufficiently small distortion assumption that underlies the derivation of the objective functions (13) and (15). Setting the first term of (16) (the quarterly inflation bias) equal to $0.03/4 = 0.0075$, using the parameter values from Table 1 and solving for χ gives a value of $\chi = 0.048$. Plugging this value into equation (14) and solving for τ gives a value of $\tau = -0.65$. That is, in order for the model to generate a 3% inflation bias in the conduct of optimal, discretionary monetary policy, it requires the presence of a 65% *tax*. Why is that so? Recall that in open economy models with monopolistic competition there are typically two distortions that optimal monetary policy would like to address: The standard monopolistic distortion. And a terms of trade externality (see Corsetti and Pesenti, 2001). The former implies that the level of output is low relative to its efficient level. In the absence of an appropriate output subsidy that would bring output to its efficient level, this distortion gives rise to an inflation bias in the conduct of optimal policy. The latter (the externality) implies that lower domestic output would improve the domestic terms of trade and could increase domestic –at the expense of foreign – welfare. In the absence of an appropriate tax that would reduce output and eliminate this incentive, this consideration imparts a deflation bias in the conduct of discretionary monetary policy. Now, in order to get the model to exhibit a positive inflation bias, the former effect must dominate, so that output must be low relative to what is desired by the central bank. But under standard parametrizations of a small open economy, it is the latter effect that tends to dominate. That is, output turns out to be too high relative to the level that maximizes domestic welfare. Consequently, in order to support a positive inflation bias we need to not only introduce a positive tax but also to make it greater than the tax rate that would have led to a zero inflation bias[14]. We simply impose exogenously the tax rate required to deliver the 3% annual inflation bias.

Are the distortions that lead to a 3% inflation bias "small" enough to make the minimization of the quadratic loss function (13) subject to the linearized structural equations deliver a correct linear approximation of the socially optimal monetary policy, as in Woodford (2003, chapter 6) and Galí (2008, chapter 5)? Using the value of $\tau = -0.65$ in the expression for the zero inflation steady state level of output, Y, leads to a value[15] of $Y/Y^e = 0.96$. Hence the discrepancy between the efficient and zero inflation steady states is about 0.04. Given the size of fluctuations in the shocks as well as those in inflation and the output gap, the assumption of a small enough steady state distortion seems justified. Alternatively, one can compute the size of the distortion at the zero inflation steady state –see the Appendix– as $\Phi = 1 - (\epsilon - 1)/(\epsilon * (1 - \tau)(1 - \alpha)) = 0.15$, which seems sufficiently small[16]. Yet another way to address the same issue is to compute the value of the coefficient $-2\delta\chi$ of the linear term in the social loss function. This linear term, which is shown in Footnote 8, reflects the steady state distortion. The value of the coefficient is -0.00038, or -0.038%, which again given the size of fluctuations in the output gap is sufficiently small for the linear term to be considered a second-order term.

[14] Under the parametrization used here the tax rate associated with a zero inflation bias is 39%.

[15] See the Appendix, part A.

[16] In closed economies with monopolistic distortion and no production subsidy, a value of Φ in the range of 0.15 to 0.20 is considered small for the purposes of approximation. See Galí (2008, ch. 5).

Table 3: Welfare comparisons of alternative regimes: The case of a 3% inflation bias

	$\sigma_u = 0.001$,$\sigma_a = 0$	$\sigma_u = 0$,$\sigma_a = 0.001$	$\sigma_u = 0.001$,$\sigma_a = 0.001$
corr =0.9	+1.4645	+0.7364	+1.4616
corr =0	+0.2033	+0.6818	-0.2113
corr =-0.9	-1.4360	+0.6225	-1.4918

[a] The numbers represent the inflation equivalent of moving from a float under discretion to monetary union under commitment. A + means a welfare gain and a − a welfare loss. There is an annual inflation bias of 3% under discretion in the flexible regime. u is the cost push and a the productivity shock. σ represents variance.

The properties of the results are consistent with the arguments made above. Comparison of column 2 across Tables 2 and 3 shows that the existence of a positive inflation bias makes participation in a currency union beneficial even when domestic and foreign cost push shocks are not strongly positive correlated (but they should not be too negatively correlated if currency union is to remain superior). As expected, adding the IS shocks to the mix (column 4) works against monetary union and requires a strong positive correlation to make it worthwhile (compare rows 1 and 2 in column 4 of Table 3). Interestingly, under the specification of the model used, the incentive to participate in monetary union that arises from the inflation bias is so strong as to overwhelm the disincentive that comes from the existence of IS shocks, even in the worse scenario case, that is, when these shocks are strongly negatively correlated across countries[17] (column 3).

We do not know much about about the international correlation of cost-push shocks. But if it is positive (for productivity shocks, see Jondeau and Sahuc, 2008) then this would create a presumption that monetary union is likely to represent a welfare improving move even for countries with a modest inflation bias.

How high are inflation biases in practice? Table 4 reports the average CPI inflation differential vis a vis Germany for some EU countries. If these differentials are taken to reflect the average inflation bias in these countries relative to Germany, then our analysis indicates that for countries like Greece and Spain (and perhaps for Italy depending on the period over which the inflation bias is computed) there may have been little trade off involved. But for countries such as France (and even the Netherlands) the decision to participate in EMU may have involved a meaningful trade off between credibility and stabilization.

We have run a large number of experiments involving variation in the parameters of the model: Serial correlation in shocks, asymmetries across countries in the structure of the shocks,

[17] Naturally, the results reported are sensitive to both the specification of the model and the calibration employed. We do not claim that our model economy corresponds closely to any real economy in the world. Nonetheless, the standard calibration adopted means that it has some realistic features and consequently, the results have indicative value.

Table 4: Pre-EMU inflation differences

	FRANCE	ITALY	GREECE	SPAIN	NETHERLANDS
1960-98	2.44	4.91	8.46	5.78	0.99
1970-95	3.15	6.77	12.49	7.21	0.75
1970-90	4.08	7.68	12.27	8.21	1.01
1980-90	4.13	7.89	16.69	7.04	-0.05

[a] The numbers represent the average, CPI inflation difference of the country under consideration vis a vis Germany during the specified period.

smaller and larger inflation biases and so. The basic structure of and intuition for the results reported above remains intact and it is easy to anticipate how changes in the structure affect the relative merits of alternative international monetary arrangements.

Caveats

The analysis has employed a set of special assumptions. In particular, we have assumed unitary elasticities and also that the steady state distortion is small enough to permit accurate welfare results without requiring higher order approximations. Moreover, the model lacks features that are present in country specific DSGE models (such as investment) and that allow such models to reproduce important, open economy stylized facts and thus serve as a more reliable tool for quantitative analysis. Would the key insights of the paper survive the relaxation of these assumptions? The answer is affirmative.

De Paoli (2009) derives the welfare function as well as the characteristics of optimal policy under commitment (and also considers some non-optimal rules) in a model that relaxes both the unitary elasticity and small distortion assumptions. Her main findings are that the welfare function also includes the volatility of the real (CPI based) exchange rate. Consequently, strict GDP deflator targeting is not optimal. And that the degree of optimal exchange rate volatility depends on the elasticity of substitution between domestic and foreign goods. Our view is that these differences from our analysis do not affect the qualitative properties of our results. Our results derive from the following considerations: The existence of an inflation bias under a flexible exchange rate regime. The fact that discretion is associated with efficient response to some shocks and inefficient response to some other shocks. And that the correlation between domestic and foreign shocks determines the degree of indirect stabilization the domestic economy receives through the union central bank. All these factors remain -and continue to play a key role- in more general models. For instance, the existence of an inflation bias under discretion depends on the type and size of distortions present in the steady state, hence it is a general feature. Similarly, the efficiency of the response of monetary policy to a particular shock depends on whether this shock generates a trade off between the various objectives of policy in combination with whether policy is characterized by discretion or not. The existence of additional arguments in the welfare function or additional shocks does not alter this property of the model. And finally the correlation between domestic and foreign shocks that determines the degree of indirect stabilization the domestic economy receives through the union central bank is invariant to the monetary arrangement in place (the shocks are exogenous). Consequently, while the more general specification employed by De Paoli (or a DSGE version) would certainly affect the quantitative properties of the model (for instance, the size of the inflation bias that tilts the scales in favor of monetary union) it does not affect the qualitative properties or the main mechanisms at work in our paper.

Conclusion

We have used the standard NK model to offer a synthesis of two important but distinct branches of the monetary union literature: One emphasizing credibility problems. And another emphasizing issues of macroeconomic stabilization. Our main point is that lack of credibility is not incompatible with some, perhaps imperfect but nevertheless potentially welfare improving stabilization. Monetary union, on the other hand, may not leave much room for stabilization in an individual country, at least in the presence of an asynchronous international business cycle. Under these circumstances, the relevant comparison involves the welfare losses from high average inflation, the benefits of inefficient, national stabilization and the benefits from more efficient, union wide stabilization. Neither monetary arrangement can always be superior and it cannot be determined on theoretical grounds alone which of the two options is likely to be associated with higher welfare.

The main contribution of the paper is to suggest how these conflicting considerations can be combined into a simple welfare criterion that can be then used to judge the desirability of a currency union for a particular country. Using a standard calibration for the parameters of the model we produce examples that shed light on the main factors at work. For instance, we find that participation in a currency union would be economically justified even for low levels of inflation bias as long as business cycles were not too a-synchronized across countries[18].

Our results complement similar results obtained by Cooley and Quadrini (2003) in a limited participation model with flexible prices. It could be due to the fact that macroeconomic stabilization is not of great welfare value in modern macroeconomic models, perhaps because they allow for a great deal of risk sharing. It remains to be seen whether this presumption would be overturned in models with more limited international risk sharing opportunities.

[18] A fruitful avenue for future research would involve working out the optimality of the decision to join a currency union for specific countries in the context of a more general and realistic model.

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Appendix: NOT FOR PUBLICATION

Parts B and C of this Appendix closely follow the computations made by Loisel (2008) for a closed economy.

A Derivation of the welfare loss functions

This Appendix derives a second-order approximation of the representative household's utility function in the domestic economy and the foreign economy. We follow closely Appendix D of Galí and Monacelli (2005). The only two differences are the following. First, while they assume that a constant employment subsidy or tax is in place that makes the steady state efficient and the flexible-price allocation optimal, we assume that a constant employment subsidy or tax is in place that offsets most of, but not all, the steady-state distortion. More precisely, we assume that the steady-state distortion is of the same order of magnitude as fluctuations in the output gap or inflation, *i.e.* of order one. Second, while they derive this second-order approximation in the neighborhood of the flexible-price allocation, we derive it in the neighborhood of the (distorted) zero-inflation steady state, following Galí (2008, chap. 5).[19]

We first derive the second-order approximation of the representative household's utility function in the domestic economy. This utility function is

$$U_t = E_t \left\{ \sum\nolimits_{k=0}^{+\infty} \beta^k \left[\log C_{t+k} - \frac{N_{t+k}^{1+\varphi}}{1+\varphi} \right] \right\},$$

where C_t denotes the composite consumption index and N_t the hours of labour of the representative household at date t, while $\varphi > 0$. Since, as in Galí and Monacelli (2005),

$$c_t = (1-\alpha)\, y_t + \alpha y_t^*,$$

where lower-case letters denote the logs of the corresponding upper-case letters, we have

$$\log C_t = \log C + (1-\alpha)\, \widehat{y}_t + t.i.p.,$$

where letters without time subscript nor superscript e denote the zero-inflation steady-state values of the corresponding variables (*i.e.* C denotes the zero-inflation steady-state value of C_t), letters with a "hat" denote the deviation of the corresponding variables from their zero-inflation steady-state values (*i.e.* $\widehat{y}_t \equiv y_t - y$), and $t.i.p.$ stands for "terms independent of policy" from the point of view of the small open economy considered. Besides, we get

$$\frac{N_t^{1+\varphi}}{1+\varphi} = \frac{N^{1+\varphi}}{1+\varphi} + N^{1+\varphi} \left(\widehat{n}_t + \frac{1+\varphi}{2} \widehat{n}_t^2 \right) + o\left(\Theta^3\right),$$

[19] The (distorted) zero-inflation steady state corresponds to the (distorted) steady state under commitment, at which the output level is lower than the output level at the (distorted) steady state under discretion, itself lower than the output level at the fictitious undistorted or efficient steady state.

where $o(\Theta^m)$ represents terms that are of order equal to or higher than m in the bound on the amplitude of the relevant shocks and in the size of the zero-inflation steady-state distortion. Since

$$N_t = \frac{Y_t}{A_t} \int_0^1 \left[\frac{P_{H,t}(i)}{P_{H,t}} \right]^{-\epsilon} di$$

with $A = 1$, where $P_{H,t}(i)$ denotes the price set by firm i in the domestic economy at date t and

$$P_{H,t} \equiv \left[\int_0^1 P_{H,t}(i)^{1-\epsilon} di \right]^{\frac{1}{1-\epsilon}},$$

we have

$$\widehat{n}_t = \widehat{y}_t - a_t + z_t,$$

where

$$z_t \equiv \log \int_0^1 \left[\frac{P_{H,t}(i)}{P_{H,t}} \right]^{-\epsilon} di.$$

Now, the zero-inflation steady state coincides with the fictitious flexible-price steady state determined by Galí and Monacelli (2005), so that

$$\frac{\epsilon - 1}{\epsilon} = (1 - \tau) N^{1+\varphi},$$

where τ denotes the constant employment subsidy (when positive) or tax (when negative). Moreover, the fictitious efficient steady state, also determined by Galí and Monacelli (2005), is characterized by

$$(N^e)^{1+\varphi} = 1 - \alpha,$$

where letters without time subscript and with superscript e denote the values of the corresponding variables at the fictitious efficient steady state (*i.e.* N^e denotes the value of N_t at the fictitious efficient steady state). Our assumption that the steady-state distortion is of order one then implies

$$\chi \equiv \frac{\epsilon (1 - \tau)(1 - \alpha) - (\epsilon - 1)}{(\epsilon - 1)(1 + \varphi)} = \frac{\epsilon (1 - \tau)}{(\epsilon - 1)(1 + \varphi)} \left[(N^e)^{1+\varphi} - N^{1+\varphi} \right] = o(\Theta).$$

Therefore

$$\begin{aligned}
\log C_t - \frac{N_t^{1+\varphi}}{1+\varphi} &= \log C - \frac{N^{1+\varphi}}{1+\varphi} + (1-\alpha)\,\widehat{y}_t - N^{1+\varphi}\left(\widehat{n}_t + \frac{1+\varphi}{2}\widehat{n}_t^2\right) \\
&\quad + o\left(\Theta^3\right) + t.i.p. \\
&= \log C - \frac{N^{1+\varphi}}{1+\varphi} + (1-\alpha)\,\widehat{y}_t - \left[1-\alpha-\frac{(\epsilon-1)(1+\varphi)}{\epsilon(1-\tau)}\chi\right]\widehat{n}_t \\
&\quad -\frac{1+\varphi}{2}(1-\alpha)\,\widehat{n}_t^2 + o\left(\Theta^3\right) + t.i.p. \\
&= \log C - \frac{N^{1+\varphi}}{1+\varphi} - (1-\alpha)\,z_t + \frac{(\epsilon-1)(1+\varphi)}{\epsilon(1-\tau)}\chi\left(\widehat{y}_t + z_t\right) \\
&\quad -\frac{1+\varphi}{2}(1-\alpha)\left(\widehat{y}_t - a_t + z_t\right)^2 + o\left(\Theta^3\right) + t.i.p. \\
&= \log C - \frac{N^{1+\varphi}}{1+\varphi} - (1-\alpha)\,z_t + (1-\alpha)(1+\varphi)\,\chi\left(\widehat{y}_t + z_t\right) \\
&\quad -\frac{1+\varphi}{2}(1-\alpha)\left(\widehat{y}_t - a_t + z_t\right)^2 + o\left(\Theta^3\right) + t.i.p. \\
&= \log C - \frac{N^{1+\varphi}}{1+\varphi} - (1-\alpha)\,z_t + (1-\alpha)(1+\varphi)\,\chi\widehat{y}_t \\
&\quad -\frac{1+\varphi}{2}(1-\alpha)\left(\widehat{y}_t - a_t\right)^2 + o\left(\Theta^3\right) + t.i.p. \\
&= \log C - \frac{N^{1+\varphi}}{1+\varphi} - (1-\alpha)\left[z_t + \frac{1+\varphi}{2}\left(\widehat{y}_t - a_t - \chi\right)^2\right] \\
&\quad + o\left(\Theta^3\right) + t.i.p.
\end{aligned}$$

where the last but one equality follows from

$$z_t = \frac{\epsilon}{2} var_i\left\{p_{H,t}(i)\right\} + o\left(\Theta^3\right) = o\left(\Theta^2\right),$$

as shown by Galí and Monacelli (2005). Since

$$\sum\nolimits_{k=0}^{+\infty} \beta^k var_i\left\{p_{H,t+k}(i)\right\} = \frac{1}{\lambda}\sum\nolimits_{k=0}^{+\infty}\beta^k\left(\Delta p_{H,t+k}\right)^2,$$

where $\lambda > 0$, as they also show, we get

$$\begin{aligned}
U_t &= U - \frac{(1-\alpha)\,\epsilon}{2\lambda}E_t\left\{\sum\nolimits_{k=0}^{+\infty}\beta^k\left[\left(\Delta p_{H,t+k}\right)^2 + \frac{\lambda(1+\varphi)}{\epsilon}\left(\widehat{y}_t - a_t - \chi\right)^2\right]\right\} \\
&\quad + o\left(\Theta^3\right) + t.i.p.
\end{aligned}$$

Now, as shown again by Galí and Monacelli (2005),

$$Y_t^e = A_t(1-\alpha)^{\frac{1}{1+\varphi}},$$

where letters with time subscript and superscript e denote the values of the corresponding variables that would be chosen by a social planner maximizing households' utility in the domestic economy subject to the production function, the international risk-sharing condition and the goods market clearing condition and taking foreign variables as given (*i.e.* Y_t^e denotes the fictitious efficient value of Y_t from the viewpoint of the domestic economy). Defining the welfare-relevant output gap as

$$x_t \equiv y_t - y_t^e,$$

we therefore have

$$\widehat{x}_t = (y_t - y_t^e) - (y - y^e) = (y_t - y) - (y_t^e - y^e) = \widehat{y}_t - a_t,$$

so that we finally get

$$\begin{aligned} U_t &= U - \frac{(1-\alpha)\,\epsilon}{2\lambda} E_t \left\{ \sum_{k=0}^{+\infty} \beta^k \left[(\Delta p_{H,t+k})^2 + \frac{\lambda(1+\varphi)}{\epsilon} (\widehat{x}_t - \chi)^2 \right] \right\} \\ &\quad + o\left(\Theta^3\right) + t.i.p. \end{aligned}$$

Hence the choice of the welfare loss function

$$L_t = E_t \left\{ \sum_{k=0}^{+\infty} \beta^k \left[(\Delta p_{H,t+k})^2 + \delta\,(\widehat{x}_t - \chi)^2 \right] \right\}$$

in the main text, where $\delta \equiv \frac{\lambda(1+\varphi)}{\epsilon}$.

The second-order approximation of the representative household's utility function in the foreign economy is derived in a similar way. The corresponding computations are the same as for the domestic economy in the limit case where $\alpha = 0$. We therefore obtain

$$L_t^* = E_t \left\{ \sum_{k=0}^{+\infty} \beta^k \left[\left(\Delta p_{t+k}^*\right)^2 + \delta\,(\widehat{x}_t^* - \chi^*)^2 \right] \right\}$$

where $\chi^* \equiv \frac{\epsilon(1-\tau^*)-(\epsilon-1)}{(\epsilon-1)(1+\varphi)} = o\,(\Theta)$.

B Determination of the flexible exchange rate equilibrium

At each date t the central bank chooses r_t so as to minimize (13) subject to (1), (4) and (5) or, equivalently, at each date t the central bank chooses $\Delta p_{H,t}$ and $\widehat{x}_t$ so as to minimize (13) subject to (1). Since $\Delta p_{H,t+k}$ and $\widehat{x}_{t+k}$ for $k \geq 1$ will be chosen in the future and since today's choice of $\Delta p_{H,t}$ and $\widehat{x}_t$ will not influence tomorrow's choice of $\Delta p_{H,t+k}$ and $\widehat{x}_{t+k}$ (as the model is purely forward-looking), the private agents' expectations $E_t\{\Delta p_{H,t+k}\}$ and $E_t\{\widehat{x}_{t+k}\}$ do not depend on the choice of $\Delta p_{H,t}$ and $\widehat{x}_t$, so that the central bank considers these expectations as given when minimizing (13) subject to (1).

The first-order condition of the minimization programme at date t is $\kappa \Delta p_{H,t} + \delta \widehat{x}_t = \delta\chi$, from which we derive $E_t\left\{\Delta p_{H,t+1}\right\} = \frac{\kappa^2+\delta}{\beta\delta}\Delta p_{H,t} - \frac{\kappa\chi}{\beta} - \frac{u_t}{\beta}$ with the help of the Phillips curve taken at date t. Similarly, for $k \geq 1$, the first-order condition of the minimization programme at date $t+k$ taken in expectations $E_t\left\{.\right\}$ is $\kappa E_t\left\{\Delta p_{H,t+k}\right\} + \delta E_t\left\{\widehat{x}_{t+k}\right\} = \delta\chi$, from which we derive the recurrence equation $E_t\left\{\Delta p_{H,t+k+1}\right\} = \frac{\kappa^2+\delta}{\beta\delta} E_t\left\{\Delta p_{H,t+k}\right\} - \frac{\kappa\chi}{\beta} - \frac{\rho_u^k u_t}{\beta}$ with the help of the Phillips curve taken in expectations $E_t\{.\}$ at date $t+k$. These two equations lead in turn to

$$\begin{aligned} E_t\{\Delta p_{H,t+k}\} &= \left(\frac{\kappa^2+\delta}{\beta\delta}\right)^k \left[\Delta p_{H,t} - \frac{\kappa\delta\chi}{\kappa^2+\delta(1-\beta)} - \frac{\delta u_t}{\kappa^2+\delta(1-\beta\rho_u)}\right] \\ &\quad + \frac{\kappa\delta\chi}{\kappa^2+\delta(1-\beta)} + \frac{\delta\rho_u^k u_t}{\kappa^2+\delta(1-\beta\rho_u)} \end{aligned}$$

for $k \geq 1$. The solution to the optimization programme satisfies therefore

$$\Delta p_{H,t} = \frac{\kappa\delta\chi}{\kappa^2+\delta(1-\beta)} + \frac{\delta u_t}{\kappa^2+\delta(1-\beta\rho_u)},$$

since L_t would take an infinite value otherwise. The condition $\kappa\Delta p_{H,t} + \delta\widehat{x}_t = \delta\chi$ then leads to

$$\widehat{x}_t = \frac{\delta(1-\beta)\chi}{\kappa^2+\delta(1-\beta)} - \frac{\kappa u_t}{\kappa^2+\delta(1-\beta\rho_u)}.$$

C Determination of the foreign economy equilibrium

The timeless-perspective equilibrium corresponds to the limit case of the t_0-commitment equilibrium when $t_0 \longrightarrow -\infty$. At the t_0-commitment equilibrium, the central bank chooses at date t_0 the state-contingent path of r_t^* for all dates $t \geq t_0$ so as to minimize (15) subject to (8), (11) and (12) taken all dates $t \geq t_0$ or, equivalently, it chooses at date t_0 the state-contingent path of Δp_t^* and $\widehat{x}_t^*$ for all dates $t \geq t_0$ so as to minimize (15) at date t_0 subject to (8) taken all dates $t \geq t_0$. We follow the undetermined coefficients method to compute the t_0-commitment equilibrium, rewriting the variables in the following way prior to the minimization of $L_{t_0}^*$:

$$\Delta p_{t_0+k}^* \equiv \sum\nolimits_{j=0}^{k-1} a_{j,k}\varepsilon_{t_0+k-j}^{u^*} + g_k \text{ and } \widehat{x}_{t_0+k}^* \equiv \sum\nolimits_{j=0}^{k-1} b_{j,k}\varepsilon_{t_0+k-j}^{u^*} + h_k$$

for $k \geq 0$. We look for the coefficients $a_{j,k}$, $b_{j,k}$, g_k and h_k for $k \geq 0$ and $0 \leq j \leq k-1$ which minimize $L_{t_0}^*$ subject to (8) considered at all dates, *i.e.* which minimize the following Lagrangian:

$$\begin{aligned} &E_{t_0}\left\{\sum\nolimits_{k=0}^{+\infty} \beta^k \left[\left(\Delta p_{t_0+k}^*\right)^2 + \delta\left(\widehat{x}_{t_0+k}^* - \chi^*\right)^2\right]\right\} \\ &\qquad - \sum\nolimits_{k=0}^{+\infty} \mu_k \left(\Delta p_{t_0+k}^* - \beta E_{t_0+k}\left\{\Delta p_{t_0+k+1}^*\right\} - \kappa\widehat{x}_{t_0+k}^* - u_{t_0+k}^*\right). \end{aligned}$$

The first-order conditions of the Lagrangian's minimization with respect to $a_{0,k}$ for $k \geq 1$, $a_{j,k}$ for $k \geq 2$ and $j \in \{1,...,k-1\}$, $b_{j,k}$ for $k \geq 1$ and $j \in \{0,...,k-1\}$, g_0, g_k for $k \geq 1$, h_k for

$k \geq 0$ can be respectively written in the following way:

$$\begin{aligned}
2\beta^k V_{u^*} a_{0,k} - \mu_k \varepsilon_{t_0+k}^{u^*} &= 0 \text{ for } k \geq 1, \\
2\beta^k V_{u^*} a_{j,k} - \mu_k \varepsilon_{t_0+k-j}^{u^*} + \beta \mu_{k-1} \varepsilon_{t_0+k-j}^{u^*} &= 0 \text{ for } k \geq 2 \text{ and } j \in \{1, ..., k-1\}, \\
2\beta^k \delta V_{u^*} b_{j,k} + \kappa \mu_k \varepsilon_{t_0+k-j}^{u^*} &= 0 \text{ for } k \geq 1 \text{ and } j \in \{0, ..., k-1\}, \\
2g_0 - \mu_0 &= 0, \\
2\beta^k g_k - \mu_k + \beta \mu_{k-1} &= 0 \text{ for } k \geq 1, \\
2\beta^k \delta (h_k - \chi^*) + \kappa \mu_k &= 0 \text{ for } k \geq 0,
\end{aligned}$$

and the Phillips curve considered at all dates provides the following two additional equations:

$$\begin{aligned}
\beta a_{j+1,k+1} - a_{j,k} + \kappa b_{j,k} &= -\rho_{u^*}^j \text{ for } k \geq 1 \text{ and } j \in \{0, ..., k-1\}, \\
\beta g_{k+1} - g_k + \kappa h_k &= -\rho_{u^*}^k u_{t_0}^* \text{ for } k \geq 0.
\end{aligned}$$

Let us note $u \equiv k - j$, $v \equiv j$, $A_{u,v} \equiv a_{j,k}$ and $B_{u,v} \equiv b_{j,k}$, so that $A_{u,v}$ and $B_{u,v}$ characterize respectively the responses of $\Delta p_{t_0+u+v}^*$ and $\widehat{x}_{u+v}^*$ to $\varepsilon_u^{u^*}$. Our eight equations are then equivalent to the following systems of equations:

$$\begin{cases}
\kappa g_0 + \delta h_0 = \delta \chi^* & \\
\kappa g_{k+1} + \delta h_{k+1} - \delta h_k = 0 & \text{for } k \geq 0 \\
\beta g_{k+1} - g_k + \kappa h_k = -\rho_{u^*}^k u_{t_0}^* & \text{for } k \geq 0
\end{cases} \tag{23}$$

$$\text{and} \begin{cases}
\kappa A_{u,0} + \delta B_{u,0} = 0 & \text{for } u \geq 1 \\
\kappa A_{u,v+1} + \delta B_{u,v+1} - \delta B_{u,v} = 0 & \text{for } u \geq 1 \text{ and } v \geq 0 \\
\beta A_{u,v+1} - A_{u,v} + \kappa B_{u,v} = -\rho_{u^*}^v & \text{for } u \geq 1 \text{ and } v \geq 0
\end{cases}. \tag{24}$$

System (23) implies that the coefficients g_k satisfy the following equations:

$$\begin{aligned}
\beta \delta g_1 - (\kappa^2 + \delta) g_0 &= -\kappa \delta \chi^* - \delta u_{t_0}^*, \\
\beta \delta g_{k+2} - (\beta \delta + \kappa^2 + \delta) g_{k+1} + \delta g_k &= \delta (1 - \rho_{u^*}) \rho_{u^*}^k u_{t_0} \text{ for } k \geq 0.
\end{aligned}$$

The latter equation corresponds to a recurrence equation on the g_k for $k \geq 0$. The corresponding characteristic polynomial has three positive real roots ρ, ω and ω' with:

$$\begin{aligned}
\omega &\equiv \frac{(\beta \delta + \kappa^2 + \delta) - \sqrt{(\beta \delta + \kappa^2 + \delta)^2 - 4\beta \delta^2}}{2\beta \delta} < 1, \\
\omega' &\equiv \frac{(\beta \delta + \kappa^2 + \delta) + \sqrt{(\beta \delta + \kappa^2 + \delta)^2 - 4\beta \delta^2}}{2\beta \delta} > 1.
\end{aligned}$$

The general form of the solution to the recurrence equation is therefore $g_k = p_1 \rho^k + p_2 \omega^k + p_3 \omega'^k$ for $k \geq 0$, where $(p_1, p_2, p_3) \in \mathbb{R}^3$. Three equations are then needed to determine (p_1, p_2, p_3). Two are provided by the initial conditions $\beta \delta g_1 - (\kappa^2 + \delta) g_0 = -\kappa \delta \chi^* - \delta u_{t_0}^*$ and $\beta \delta g_2 -$

$\left(\beta\delta+\kappa^2+\delta\right)g_1+\delta g_0=\delta\left(1-\rho_{u^*}\right)u^*_{t_0}$. The third one is simply $p_3=0$ and comes from the fact that $\beta\omega'^2\geq 1$, as can be readily checked, so that no solution with $p_3\neq 0$ would fit the bill as $L^*_{t_0}$ would then be infinite. We thus eventually obtain the following solution for system (23):

$$\begin{aligned} g_k &= \frac{\delta\left(1-\omega\right)\omega^k\chi^*}{\kappa}+\frac{\omega\left[\left(1-\rho_{u^*}\right)\rho^k_{u^*}-\left(1-\omega\right)\omega^k\right]u^*_{t_0}}{\left(1-\beta\rho_{u^*}\omega\right)\left(\omega-\rho_{u^*}\right)} \text{ for } k\geq 0,\\ h_k &= \omega^{k+1}\chi^*+\frac{\kappa\omega\left(\rho^{k+1}_{u^*}-\omega^{k+1}\right)u^*_{t_0}}{\delta\left(1-\beta\rho_{u^*}\omega\right)\left(\omega-\rho_{u^*}\right)} \text{ for } k\geq 0. \end{aligned}$$

The similarity between systems (23) and (24) enables us to derive the solution of system (24) from the solution of system (23) in a straightforward way:

$$\begin{aligned} A_{u,v} &= \frac{\omega\left[\left(1-\rho_{u^*}\right)\rho^v_{u^*}-\left(1-\omega\right)\omega^v\ \right]}{\left(1-\beta\rho_{u^*}\omega\right)\left(\omega-\rho_{u^*}\right)} \text{ for } u\geq 1 \text{ and } v\geq 0,\\ B_{u,v} &= \frac{\kappa\omega\left(\rho^{v+1}_{u^*}-\omega^{v+1}\right)}{\delta\left(1-\beta\rho_{u^*}\omega\right)\left(\omega-\rho_{u^*}\right)} \text{ for } u\geq 1 \text{ and } v\geq 0, \end{aligned}$$

so that we obtain the following results for $k\geq 0$:

$$\begin{aligned} \Delta p^*_{t_0+k} &= \frac{\delta\left(1-\omega\right)\omega^k\chi^*}{\kappa}+\frac{\omega\left[\left(1-\rho_{u^*}\right)u^*_{t_0+k}-\left(1-\omega\right)\xi^*_{t_0+k}\right]}{\left(1-\beta\rho_{u^*}\omega\right)\left(\omega-\rho_{u^*}\right)},\\ \widehat{x}^*_{t_0+k} &= \omega^{k+1}\chi^*+\frac{\kappa\omega\left(\rho_{u^*}u^*_{t_0+k}-\omega\xi^*_{t_0+k}\right)}{\delta\left(1-\beta\rho_{u^*}\omega\right)\left(\omega-\rho_{u^*}\right)}, \end{aligned}$$

where $\xi^*_{t_0+k}\equiv\sum_{j=0}^{k}\omega^j\varepsilon^{u^*}_{t_0+k-j}+\omega^k\rho_{u^*}u^*_{t_0-1}$. Making t_0 tend towards $-\infty$ while keeping $t\equiv t_0+k$ finite, we eventually obtain

$$\Delta p^*_t = \frac{\omega\left[\left(1-\rho_{u^*}\right)u^*_t-\left(1-\omega\right)\zeta^*_t\right]}{\left(1-\beta\rho_{u^*}\omega\right)\left(\omega-\rho_{u^*}\right)}, \tag{25}$$

$$\widehat{x}^*_t = \frac{\kappa\omega\left(\rho_{u^*}u^*_t-\omega\zeta^*_t\right)}{\delta\left(1-\beta\rho_{u^*}\omega\right)\left(\omega-\rho_{u^*}\right)}, \tag{26}$$

where $\zeta^*_t\equiv\sum_{j=0}^{+\infty}\omega^j\varepsilon^{u^*}_{t-j}$.

D Determination of the monetary union equilibrium

The equations for the efficient domestic and foreign output are

$$y^e_t=\Omega_\alpha+a_t, \tag{27}$$

$$y^{e*}_t=a^*_t, \tag{28}$$

where a_t and a^*_t denote exogenous productivity shocks occurring in period t, and Ω_α takes the value zero when $\alpha=0$.

Using (1), (2), (3), (6), (9), (10), (19), (27) and (28) leads to the following second-order equation in p_H:

$$\beta E_t \{p_{H,t+1}\} - (1+\beta+\kappa)\, p_{H,t} + p_{H,t-1} + \kappa\,(e + x^* - x - \Omega_\alpha) - \kappa a_t + u_t + \kappa a_t^* + \kappa \widehat{x}_t^* + \kappa p_t^* = 0. \tag{29}$$

This equation admits a unique stationary solution since the roots

$$\begin{aligned} \theta &\equiv \frac{1+\beta+\kappa-\sqrt{(1+\beta+\kappa)^2-4\beta}}{2\beta} \\ \text{and } \phi &\equiv \frac{1+\beta+\kappa+\sqrt{(1+\beta+\kappa)^2-4\beta}}{2\beta} \end{aligned}$$

of the corresponding second-order characteristic polynomial $P(X) \equiv \beta X^2 - (1+\beta+\kappa)\, X + 1$ are such that $0 < \theta < 1$ and $\phi > 1$ (as can be readily checked). We follow the undetermined coefficients method to find this solution, writing it in the form

$$p_{H,t} = p_H + \sum_{k=0}^{+\infty} \left(\psi_k^a \varepsilon_{t-k}^a + \psi_k^u \varepsilon_{t-k}^u + \psi_k^{a^*} \varepsilon_{t-k}^{a^*} + \psi_k^{u^*} \varepsilon_{t-k}^{u^*} \right).$$

Using (26), (29) and the equation

$$\begin{aligned} p_t^* &= p^* + \sum_{k=0}^{+\infty} \Delta p_{t-k}^* \\ &= p^* + \frac{\omega}{(1-\beta\rho_{u^*}\omega)(\omega-\rho_{u^*})} \sum_{k=0}^{+\infty} \left(\omega^{k+1} - \rho_{u^*}^{k+1} \right) \varepsilon_{t-k}^{u^*}, \end{aligned} \tag{30}$$

straightforwardly derived from (25), we obtain:

$$\begin{gathered} p_H = e + p^* + x^* - x - \Omega_\alpha \\ \beta\psi_1^a - (1+\beta+\kappa)\,\psi_0^a - \kappa = 0, \\ \beta\psi_{k+2}^a - (1+\beta+\kappa)\,\psi_{k+1}^a + \psi_k^a - \kappa\rho_a^{k+1} = 0 \text{ for } k \geq 0, \\ \beta\psi_1^u - (1+\beta+\kappa)\,\psi_0^u + 1 = 0, \\ \beta\psi_{k+2}^u - (1+\beta+\kappa)\,\psi_{k+1}^u + \psi_k^u + \rho_u^{k+1} = 0 \text{ for } k \geq 0, \\ \beta\psi_1^{a^*} - (1+\beta+\kappa)\,\psi_0^{a^*} + \kappa = 0, \\ \beta\psi_{k+2}^{a^*} - (1+\beta+\kappa)\,\psi_{k+1}^{a^*} + \psi_k^{a^*} + \kappa\rho_{a^*}^{k+1} = 0 \text{ for } k \geq 0, \\ \beta\psi_1^{u^*} - (1+\beta+\kappa)\,\psi_0^{u^*} - \frac{\omega\kappa\,(\kappa-\delta)}{(1-\beta\rho_{u^*}\omega)\,\delta} = 0, \\ \beta\psi_{k+2}^{u^*} - (1+\beta+\kappa)\,\psi_{k+1}^{u^*} + \psi_k^{u^*} - \frac{\omega\kappa\,(\kappa-\delta)\left(\omega^{k+2} - \rho_{u^*}^{k+2}\right)}{(1-\beta\rho_{u^*}\omega)\,\delta\,(\omega-\rho_{u^*})} = 0 \text{ for } k \geq 0, \end{gathered}$$

from which we get, for $k \geq 0$,

$$\begin{aligned}
\psi_k^a &= \frac{-\kappa}{P(\rho_a)}\left(\theta^{k+1} - \rho_a^{k+1}\right), \\
\psi_k^u &= \frac{1}{P(\rho_u)}\left(\theta^{k+1} - \rho_u^{k+1}\right), \\
\psi_k^{a^*} &= \frac{\kappa}{P(\rho_{a^*})}\left(\theta^{k+1} - \rho_{a^*}^{k+1}\right), \\
\psi_k^{u^*} &= \frac{\omega\kappa(\kappa-\delta)}{(1-\beta\rho_{u^*}\omega)\,\delta\,(\omega-\rho_{u^*})}\left[\frac{\rho_{u^*}}{P(\rho_{u^*})}\left(\theta^{k+1} - \rho_{u^*}^{k+1}\right) - \frac{\omega}{P(\omega)}\left(\theta^{k+1} - \omega^{k+1}\right)\right]
\end{aligned}$$

and therefore

$$\begin{aligned}
\Delta p_{H,t} &= \frac{-\kappa}{P(\rho_a)}\sum\nolimits_{k=0}^{+\infty}\left[(1-\rho_a)\,\rho_a^k - (1-\theta)\,\theta^k\right]\varepsilon_{t-k}^a \\
&\quad + \frac{1}{P(\rho_u)}\sum\nolimits_{k=0}^{+\infty}\left[(1-\rho_u)\,\rho_u^k - (1-\theta)\,\theta^k\right]\varepsilon_{t-k}^u \\
&\quad + \frac{\kappa}{P(\rho_{a^*})}\sum\nolimits_{k=0}^{+\infty}\left[(1-\rho_{a^*})\,\rho_{a^*}^k - (1-\theta)\,\theta^k\right]\varepsilon_{t-k}^{a^*} \\
&\quad + \frac{\omega\kappa(\kappa-\delta)}{(1-\beta\rho_{u^*}\omega)\,\delta\,(\omega-\rho_{u^*})}\sum\nolimits_{k=0}^{+\infty}\left[\frac{\rho_{u^*}}{P(\rho_{u^*})}\left[(1-\rho_{u^*})\,\rho_{u^*}^k - (1-\theta)\,\theta^k\right]\right. \\
&\quad \left. - \frac{\omega}{P(\omega)}\left[(1-\omega)\,\omega^k - (1-\theta)\,\theta^k\right]\right]\varepsilon_{t-k}^{u^*} \qquad (31)
\end{aligned}$$

with the convention $0^0 = 1$. Then, using (1) and (31), we eventually obtain

$$\begin{aligned}
\widehat{x}_t &= -\sum\nolimits_{k=0}^{+\infty}\left[\frac{\kappa}{P(\rho_a)}\left(\rho_a^{k+1} - \theta^{k+1}\right) + \rho_a^k\right]\varepsilon_{t-k}^a \\
&\quad + \frac{1}{P(\rho_u)}\sum\nolimits_{k=0}^{+\infty}\left[\rho_u^{k+1} - \theta^{k+1}\right]\varepsilon_{t-k}^u \\
&\quad + \sum\nolimits_{k=0}^{+\infty}\left[\frac{\kappa}{P(\rho_{a^*})}\left(\rho_{a^*}^{k+1} - \theta^{k+1}\right) + \rho_{a^*}^k\right]\varepsilon_{t-k}^{a^*} \\
&\quad + \frac{\omega(\kappa-\delta)}{(1-\beta\rho_{u^*}\omega)\,\delta\,(\omega-\rho_{u^*})}\sum\nolimits_{k=0}^{+\infty}\left[\frac{\kappa\rho_{u^*}}{P(\rho_{u^*})}\left(\rho_{u^*}^{k+1} - \theta^{k+1}\right)\right. \\
&\quad \left. - \frac{\kappa\omega}{P(\omega)}\left(\omega^{k+1} - \theta^{k+1}\right) + \left(\rho_{u^*}^{k+1} - \omega^{k+1}\right)\right]\varepsilon_{t-k}^{u^*}. \qquad (32)
\end{aligned}$$

The unconditional mean of the social welfare loss function is then

$$L^{mu} = \frac{\delta\chi^2}{1-\beta} + M_1 V_a + M_2 V_{a^*} + M_3 \mu_a + M_4 V_u + M_5 V_{u^*} + M_6 \mu_u$$

where

$$\begin{aligned}
M_1 &\equiv \frac{1}{(1-\beta)\,P(\rho_a)^2}\left\{\frac{1-\rho_a}{1+\rho_a}\left[\kappa^2 + \delta(1-\beta\rho_a)^2\right]\right. \\
&\quad \left. + \frac{\kappa^2}{1-\theta^2}\left[(1-\theta)^2 + \delta\theta^2\right] - \frac{2\kappa(1-\rho_a)}{1-\rho_a\theta}\left[\kappa(1-\theta) + \delta(1-\beta\rho_a)\,\theta\right]\right\},
\end{aligned}$$

$$
\begin{aligned}
M_2 \quad \equiv \quad & \frac{1}{(1-\beta)\, P\left(\rho_{a^*}\right)^2} \left\{ \frac{1-\rho_{a^*}}{1+\rho_{a^*}} \left[\kappa^2 + \delta \left(1-\beta\rho_{a^*}\right)^2\right] \right. \\
& \left. + \frac{\kappa^2}{1-\theta^2} \left[(1-\theta)^2 + \delta\theta^2\right] - \frac{2\kappa \left(1-\rho_{a^*}\right)}{1-\rho_{a^*}\theta} \left[\kappa (1-\theta) + \delta \left(1-\beta\rho_{a^*}\right)\theta\right] \right\},
\end{aligned}
$$

$$
\begin{aligned}
M_3 \quad \equiv \quad & \frac{-2}{(1-\beta)\, P(\rho_a)\, P\left(\rho_{a^*}\right)} \left\{ \kappa^2 \left[\frac{(1-\rho_a)\left(1-\rho_{a^*}\right)}{1-\rho_a\rho_{a^*}} + \frac{1-\theta}{1+\theta} - \frac{(1-\rho_a)(1-\theta)}{1-\rho_a\theta} \right. \right. \\
& \left. - \frac{\left(1-\rho_{a^*}\right)(1-\theta)}{1-\rho_{a^*}\theta} \right] + \delta \left[\frac{(1-\rho_a)(1-\beta\rho_a)\left(1-\rho_{a^*}\right)\left(1-\beta\rho_{a^*}\right)}{1-\rho_a\rho_{a^*}} \right. \\
& \left. \left. + \frac{\kappa^2\theta^2}{1-\theta^2} - \frac{(1-\rho_a)(1-\beta\rho_a)\kappa\theta}{1-\rho_a\theta} - \frac{\left(1-\rho_{a^*}\right)\left(1-\beta\rho_{a^*}\right)\kappa\theta}{1-\rho_{a^*}\theta} \right] \right\},
\end{aligned}
$$

$$
\begin{aligned}
M_4 \quad \equiv \quad & \frac{1}{(1-\beta)\, P(\rho_u)^2} \left\{ \left[\frac{1-\rho_u}{1+\rho_u} + \frac{1-\theta}{1+\theta} - \frac{2(1-\rho_u)(1-\theta)}{1-\rho_u\theta} \right] \right. \\
& \left. + \delta \left[\frac{\rho_u^2}{1-\rho_u^2} + \frac{\theta^2}{1-\theta^2} - \frac{2\rho_u\theta}{1-\rho_u\theta} \right] \right\},
\end{aligned}
$$

$$
\begin{aligned}
M_5 \quad \equiv \quad & \frac{(\kappa-\delta)^2 \omega^2}{(1-\beta)\,\delta^2 \left(1-\beta\rho_{u^*}\omega\right)^2 \left(\omega-\rho_{u^*}\right)^2} \left\{ \kappa^2 \left[\frac{\rho_{u^*}^2 \left(1-\rho_{u^*}\right)}{P\left(\rho_{u^*}\right)^2 \left(1+\rho_{u^*}\right)} \right. \right. \\
& + \frac{\omega^2 (1-\omega)}{P(\omega)^2 (1+\omega)} + \frac{1-\theta}{1+\theta} \left[\frac{\omega}{P(\omega)} - \frac{\rho_{u^*}}{P\left(\rho_{u^*}\right)} \right]^2 \\
& + \frac{2\rho_{u^*}\left(1-\rho_{u^*}\right)(1-\theta)}{P\left(\rho_{u^*}\right)\left(1-\rho_{u^*}\theta\right)} \left[\frac{\omega}{P(\omega)} - \frac{\rho_{u^*}}{P\left(\rho_{u^*}\right)} \right] \\
& \left. - \frac{2\omega(1-\omega)(1-\theta)}{P(\omega)(1-\omega\theta)} \left[\frac{\omega}{P(\omega)} - \frac{\rho_{u^*}}{P\left(\rho_{u^*}\right)} \right] - \frac{2\rho_{u^*}\left(1-\rho_{u^*}\right)\omega(1-\omega)}{P\left(\rho_{u^*}\right)P(\omega)\left(1-\rho_{u^*}\omega\right)} \right] \\
& + \delta \left[\frac{\rho_{u^*}^2\left(1-\rho_{u^*}\right)\left(1-\beta\rho_{u^*}\right)^2}{P\left(\rho_{u^*}\right)^2\left(1+\rho_{u^*}\right)} + \frac{\omega^2(1-\omega)(1-\beta\omega)^2}{P(\omega)^2(1+\omega)} \right. \\
& + \frac{\kappa^2\theta^2}{1-\theta^2} \left[\frac{\omega}{P(\omega)} - \frac{\rho_{u^*}}{P\left(\rho_{u^*}\right)} \right]^2 + \frac{2\kappa\left(1-\rho_{u^*}\right)\left(1-\beta\rho_{u^*}\right)\rho_{u^*}\theta}{P\left(\rho_{u^*}\right)\left(1-\rho_{u^*}\theta\right)} \\
& \left[\frac{\omega}{P(\omega)} - \frac{\rho_{u^*}}{P\left(\rho_{u^*}\right)} \right] - \frac{2\kappa(1-\omega)(1-\beta\omega)\omega\theta}{P(\omega)(1-\omega\theta)} \left[\frac{\omega}{P(\omega)} - \frac{\rho_{u^*}}{P\left(\rho_{u^*}\right)} \right] \\
& \left. \left. - \frac{2\left(1-\rho_{u^*}\right)\left(1-\beta\rho_{u^*}\right)(1-\omega)(1-\beta\omega)\rho_{u^*}\omega}{P\left(\rho_{u^*}\right)P(\omega)\left(1-\rho_{u^*}\omega\right)} \right] \right\},
\end{aligned}
$$

$$
\begin{aligned}
M_6 \quad \equiv \quad & \frac{2(\kappa-\delta)\omega}{(1-\beta)\delta P(\rho_u)(1-\beta\rho_{u^*}\omega)(\omega-\rho_{u^*})}\left\{\kappa\left[\frac{(1-\rho_u)\rho_{u^*}(1-\rho_{u^*})}{P(\rho_{u^*})(1-\rho_u\rho_{u^*})}\right.\right. \\
& -\frac{(1-\rho_u)\omega(1-\omega)}{P(\omega)(1-\rho_u\omega)}+\frac{(1-\rho_u)(1-\theta)}{1-\rho_u\theta}\left[\frac{\omega}{P(\omega)}-\frac{\rho_{u^*}}{P(\rho_{u^*})}\right] \\
& \left.-\frac{\rho_{u^*}(1-\rho_{u^*})(1-\theta)}{P(\rho_{u^*})(1-\rho_{u^*}\theta)}+\frac{\omega(1-\omega)(1-\theta)}{P(\omega)(1-\omega\theta)}-\frac{1-\theta}{1+\theta}\left[\frac{\omega}{P(\omega)}-\frac{\rho_{u^*}}{P(\rho_{u^*})}\right]\right] \\
& +\delta\left[\frac{(1-\rho_{u^*})(1-\beta\rho_{u^*})\rho_u\rho_{u^*}}{P(\rho_{u^*})(1-\rho_u\rho_{u^*})}-\frac{(1-\omega)(1-\beta\omega)\rho_u\omega}{P(\omega)(1-\rho_u\omega)}\right. \\
& +\frac{\kappa\rho_u\theta}{1-\rho_u\theta}\left[\frac{\omega}{P(\omega)}-\frac{\rho_{u^*}}{P(\rho_{u^*})}\right]-\frac{(1-\rho_{u^*})(1-\beta\rho_{u^*})\rho_{u^*}\theta}{P(\rho_{u^*})(1-\rho_{u^*}\theta)} \\
& \left.\left.+\frac{(1-\omega)(1-\beta\omega)\omega\theta}{P(\omega)(1-\omega\theta)}-\frac{\kappa\theta^2}{1-\theta^2}\left[\frac{\omega}{P(\omega)}-\frac{\rho_{u^*}}{P(\rho_{u^*})}\right]\right]\right\}.
\end{aligned}
$$

E Proof that log-linearization around the efficient steady state does not affect the results

This Appendix derives the first-order approximation of the structural equations and the second-order approximation of households' utility function in the neighbourhood of the fictitious efficient steady state, instead of the (distorted) zero-inflation steady state, and shows that, as could be expected, this change does not affect the results obtained in the paper.

First, this leads to the same log-linearized structural equations. Indeed, we get the following IS equations:

$$
\begin{aligned}
x_t &= E_t\{x_{t+1}\} - (r_t - E_t\{\Delta p_{H,t+1}\} - rr_t), \\
x_t^* &= E_t\{x_{t+1}^*\} - (r_t^* - E_t\{\Delta p_{t+1}^*\} - rr_t^*),
\end{aligned}
$$

which can straightforwardly be rewritten as in the paper:

$$
\begin{aligned}
\widehat{x}_t &= E_t\{\widehat{x}_{t+1}\} - (r_t - E_t\{\Delta p_{H,t+1}\} - rr_t), \\
\widehat{x}_t^* &= E_t\{\widehat{x}_{t+1}^*\} - (r_t^* - E_t\{\Delta p_{t+1}^*\} - rr_t^*).
\end{aligned}
$$

Moreover, given that the fictitious flexible-price steady state coincides with the zero-inflation steady state, we get the same Phillips curves as in the paper. Finally, since the other log-linearized structural equations do not involve the output gap, they are left unchanged too.

Second, this leads to the same second-order approximation of households' utility function. To see that, let us derive this second-order approximation along the lines of Appendix A of the paper. We start with the representative household's utility function in the domestic economy. This utility function is

$$
U_t = E_t\left\{\sum_{k=0}^{+\infty}\beta^k\left[\log C_{t+k} - \frac{N_{t+k}^{1+\varphi}}{1+\varphi}\right]\right\}.
$$

Since

$$
c_t = (1-\alpha)y_t + \alpha y_t^*,
$$

we have

$$\log C_t = \log C^e + (1-\alpha)(y_t - y^e) + t.i.p.$$

Besides, we get

$$\frac{N_t^{1+\varphi}}{1+\varphi} = \frac{(N^e)^{1+\varphi}}{1+\varphi} + (N^e)^{1+\varphi}\left[(n_t - n^e) + \frac{1+\varphi}{2}(n_t - n^e)^2\right] + o\left(\Theta^3\right).$$

Since

$$N_t = \frac{Y_t}{A_t}\int_0^1 \left[\frac{P_{H,t}(i)}{P_{H,t}}\right]^{-\varepsilon} di,$$

with $A = 1$ and

$$P_{H,t} \equiv \left[\int_0^1 P_{H,t}(i)^{1-\varepsilon} di\right]^{\frac{1}{1-\varepsilon}},$$

we have

$$n_t - n^e = (y_t - y^e) - a_t + z_t,$$

where

$$z_t \equiv \log \int_0^1 \left[\frac{P_{H,t}(i)}{P_{H,t}}\right]^{-\varepsilon} di.$$

Now,

$$(N^e)^{1+\varphi} = 1-\alpha,$$

so that

$$\begin{aligned}
\log C_t - \frac{N_t^{1+\varphi}}{1+\varphi} &= \log C^e - \frac{(N^e)^{1+\varphi}}{1+\varphi} + (1-\alpha)(y_t - y^e) \\
&\quad - (N^e)^{1+\varphi}\left[(n_t - n^e) + \frac{1+\varphi}{2}(n_t - n^e)^2\right] \\
&\quad + o\left(\Theta^3\right) + t.i.p. \\
&= \log C^e - \frac{(N^e)^{1+\varphi}}{1+\varphi} + (1-\alpha)(y_t - y^e) \\
&\quad - (1-\alpha)(n_t - n^e) - (1-\alpha)\frac{1+\varphi}{2}(n_t - n^e)^2 \\
&\quad + o\left(\Theta^3\right) + t.i.p. \\
&= \log C^e - \frac{(N^e)^{1+\varphi}}{1+\varphi} - (1-\alpha) z_t \\
&\quad - (1-\alpha)\frac{1+\varphi}{2}\left[(y_t - y^e) - a_t + z_t\right]^2 + o\left(\Theta^3\right) + t.i.p. \\
&= \log C^e - \frac{(N^e)^{1+\varphi}}{1+\varphi} - (1-\alpha) z_t \\
&\quad - (1-\alpha)\frac{1+\varphi}{2}\left[(y_t - y^e) - a_t\right]^2 + o\left(\Theta^3\right) + t.i.p. \\
&= \log C^e - \frac{(N^e)^{1+\varphi}}{1+\varphi} - (1-\alpha)\left[z_t + \frac{1+\varphi}{2}x_t^2\right] \\
&\quad + o\left(\Theta^3\right) + t.i.p.
\end{aligned}$$

where the last two equalities follow from

$$z_t = \frac{\varepsilon}{2} var_i \{p_{H,t}(i)\} + o\left(\Theta^3\right) = o\left(\Theta^2\right)$$

and

$$(y_t - y^e) - a_t = (y_t - y_t^e) + (y_t^e - y^e) - a_t = x_t + a_t - a_t = x_t.$$

Since

$$\sum\nolimits_{k=0}^{+\infty} \beta^k var_i \{p_{H,t+k}(i)\} = \frac{1}{\lambda} \sum\nolimits_{k=0}^{+\infty} \beta^k \left(\Delta p_{H,t+k}\right)^2,$$

we get

$$\begin{aligned} U_t &= U^e - \frac{(1-\alpha)\varepsilon}{2\lambda} E_t \left\{ \sum\nolimits_{k=0}^{+\infty} \beta^k \left[\left(\Delta p_{H,t+k}\right)^2 + \frac{\lambda(1+\varphi)}{\varepsilon} x_t^2 \right] \right\} \\ &\quad + o\left(\Theta^3\right) + t.i.p. \end{aligned}$$

Now,

$$\begin{aligned} \chi &= \frac{\varepsilon(1-\tau)}{(\varepsilon-1)} \left[\frac{(N^e)^{1+\varphi} - N^{1+\varphi}}{1+\varphi} \right] \\ &= \frac{\varepsilon(1-\tau)}{(\varepsilon-1)} \left[-(N^e)^{1+\varphi}(n-n^e) \right] + o\left(\Theta^2\right) \\ &= \frac{-1}{1-\alpha} (N^e)^{1+\varphi}(n-n^e) + o\left(\Theta^2\right) \\ &= -(n-n^e) + o\left(\Theta^2\right) \\ &= -(y-y^e) + o\left(\Theta^2\right) \\ &= -x + o\left(\Theta^2\right), \end{aligned}$$

where the third equality follows from $\chi = o(\Theta)$, so that we finally get

$$\begin{aligned} U_t &= U^e - \frac{(1-\alpha)\varepsilon}{2\lambda} E_t \left\{ \sum\nolimits_{k=0}^{+\infty} \beta^k \left[\left(\Delta p_{H,t+k}\right)^2 + \frac{\lambda(1+\varphi)}{\varepsilon} \left(\widehat{x}_t - \chi\right)^2 \right] \right\} \\ &\quad + o\left(\Theta^3\right) + t.i.p. \end{aligned}$$

Hence the welfare loss function for the domestic economy

$$L_t = E_t \left\{ \sum\nolimits_{k=0}^{+\infty} \beta^k \left[\left(\Delta p_{H,t+k}\right)^2 + \delta \left(\widehat{x}_t - \chi\right)^2 \right] \right\}$$

is the same as in the paper. Similarly, the welfare loss function for the foreign economy is identical to that in the paper. Hence, log-linearizing around the zero inflation steady state rather than around the efficient one is of no consequence for our results.