Exercise Sheet 3: Short Solutions.

Note: Most calculations are only approximately true. If you have slightly different results because you rounded differently, it does not matter. The important thing is that you understand the formulas and apply them correctly. You can solve the exercise with or without using the log-approximations. In the solutions we usually use the log-approximations because it is more convenient and faster.

Exercise 1

a) A first order Taylor approximation of $f(i) = ln(1+i)$ around $i = 0$ is given by:

$$f(i) \approx f(0) + f'(0)(i - 0)$$

Since $f'(0) = \frac{1}{1}$ and $f(0) = 0$, we get that $f(i) \approx i$, which is what we needed to show.

b) The exact version of UIP is: $\dfrac{1 + i_{CH}}{1 + i_{EUR}} = \dfrac{E^e}{E}$. Take logs on both sides of the equation: $ln\left(\dfrac{1 + i_{CH}}{1 + i_{EUR}}\right) = ln\left(\dfrac{E^e}{E}\right)$.

Rewriting this gives $ln(1 + i_{CH}) - ln(1 + i_{EUR}) = ln(E^e) - ln(E)$.[1] Using the two approximations this is approximately the same condition as $i_{CH} - i_{EUR} = \dfrac{E^e - E}{E}$, which is the approximate version of UIP that is usually used in calculations.

c) Covered interest parity says that investing one Franc in CHF must give the same return as investing the Franc in Euro and then selling the return forward to CHF.

With one Franc you can buy $\dfrac{1}{E}$ Euros today. You then invest them in Euro and get

[1] We assume that you know how to calculate with logs. If not, you should review the relevant material from your introductory math classes.

$\frac{1}{E}(1 + i_{EUR})$ Euros in one year. You sell this return forward to CHF at the current forward rate F, which means that you *know* that you will get $\frac{F}{E}(1 + i_{EUR})$ CHF in one year. Covered interest parity says that this return must be equal to the return on an investment in CHF. The exact version of covered interest parity is:

$$(1 + i_{CH}) = \frac{F}{E}(1 + i_{EUR})$$

This is exactly the same as uncovered interest parity, just with the forward rate instead of the future expected exchange rate. In order to derive the approximate version of covered interest parity, take logs on both sides of the equation and apply the two approximations.

d) We know that the percentage change in the real exchange rate, $\frac{E_{real,t+1} - E_{real,t}}{E_{real,t}}$, is approximately equal to the log-difference $log(E_{real,t+1}) - log(E_{real,t})$. Inserting the definition of the real exchange rate, we get that the percentage change in the real exchange is given by: $log\left(\frac{E_{t+1}P_{t+1}^{US}}{P_{t+1}^{CH}}\right) - ln\left(\frac{E_t P_t^{US}}{P_t^{CH}}\right)$. Rewriting gives:

$$\underbrace{ln(E_{t+1}) - ln(E_t)}_{\text{percentage nominal depreciation of CHF}} + \underbrace{ln(P_{t+1}^{US}) - ln(P_t^{US})}_{\text{Inflation in US}} - \underbrace{(ln(P_{t+1}^{CH}) - ln(P_t^{CH}))}_{\text{Inflation in CH}}$$

Exercise 2

a) From the graph we see that there was an appreciation of the CHF of about 15 %. Inflation in CH was about 2.5% lower in CH than in USA. If relative PPP holds, the two should have been rougly equal, so inflation in CH should have been about 15% lower than in the USA. Since this was not true, we conclude that relative PPP did not hold over that time period.

b) Define E as the exchange rate given in CHF per USD. E_1 is the exchange rate in September 2011, E_2 in September 2012. Relative PPP says that

$$ln(E_2) - ln(E_1) = inflation_{CH} - inflation_{US}$$

Inserting $E_1 = 1.23$, $inflation_{CH} = -0.0041$ and $inflation_{US} = 0.0199$ gives $ln(E_2) = 0.18$. Hence $E_2 = e^{0.18} = \underline{\underline{1.2}}$.

c) There was a real appreciation of the CHF. Using the formula derived in 1c), we get that the percentage change in the real exchange rate is: $ln(E_{real,2}) - ln(E_{real,1}) = ln(1.04) - ln(1.23) + 0.0199 + 0.0041 = -0.14$. So there was a real appreciation of the CHF of about $\underline{\underline{14\%}}$

d) If relative PPP does not hold, absolute PPP cannot hold either.

Exercise 3

Denote E_{11} as the exchange rate, given in CHF per USD, in September 2011. Analogously E_{12} is the exchange rate in September 12 and $E\{E_{13}\}$ is the expected exchange rate in September 2013.

a) According to uncovered interest parity (UIP): $i_{CH} - i_{US} = ln(E\{E_{13}\}) - ln(E_{12})$. Inserting $E_{12} = 1.04$, $E\{E_{13}\} = 0.99$ and $i_{CH} = 0.0105$, we get $i_{US} \approx 6\%$.

b) Here we can used covered interest rate parity: $i_{CH} - i_{US} = ln(F) - ln(E_{11})$. Inserting $i_{CH} = 0.008$, $E_{11} = 1.23$ and $F = 1.18$, we get $i_{US} \approx 4.9\%$

c) The effective return of a US bond for a Swiss investor is given by: $i_{US} + ln(E_{12}) - ln(E_{11})$. Inserting $i_{US} = 0.049$, $E_{12} = 1.04$ and $E_{11} = 1.23$ yields an effective return of

-0.119 or of -11.9%, meaning that the Swiss investor made a loss. The effective return on an investment in CHF would have been 0.8%.

d) The fact that the ex-post effective return on USD was lower than on CHF does not say anything about uncovered interest parity (UIP). UIP says that the *expected* effective return should be the same accross countries. But in this case, it might just have been that expectations were wrong.

<u>*Exercise 4*</u>

a) There was a depreciation of the CHF vis-à-vis the Yen by approximately 2.6%. ($ln(113) - ln(116) = -0.026$)

b) Apply the formula for the cross rates:

$$\frac{YEN}{USD} = \underbrace{\frac{YEN}{CHF}}_{116} * \underbrace{\frac{CHF}{USD}}_{0.9} = \underline{\underline{104.4}} \tag{1}$$

c) **The solution to subquestion c) presented in the exercise session was wrong. Sorry!**

Let E_1=exchange rate in March and E_2=exchange rate in September. Suppose you are a Swiss investor (this is just for illustration, it does not matter whether you are a Swiss or a Japanese investor). The ex-post return of an investment in CHF is $1 + i_{CHF}$. The ex-post return of an investment in Yen is given by $\frac{E_1(1+i_{Yen})}{E_2}$. Taking logs, the difference in the ex-post return is given by $\underbrace{i_{CHF} - i_{Yen}}_{?} + \underbrace{ln(E_2) - ln(E_1)}_{-0.026}$.

From covered interest parity, we know that $i_{CHF} - i_{Yen} = ln(E) - ln(F) = -0.03$. Since $-0.03 - 0.026 < 0$, the ex post return for an investment in CHF was lower than for an investment in JPY.

d) Relative PPP says that the difference in inflation between Switzerland and Japan ($\pi_{CH} - \pi_{JP}$) should be equal to the percentage-depreciation of the CHF vis-$\grave{a}$-vis the JPY. We know that the CHF depreciated by 2.6% vis-$\grave{a}$-vis the JPY. So if relative PPP holds, it must be true that $\pi_{CH} - \pi_{JP} = 2.6\%$. Inserting $\pi_{CH} = -0.5\%$, we get that π_{JP} should have been at $\pi_{JP} = -3.1\%$, that is, a deflation in Japan of 3.1%.

e) There was a real depreciation of the CHF vis-$\grave{a}$-vis the JPY. (Inflation in Japan was higher than what "it should have been" according to relative PPP, meaning that there was a real appreciation of the JPY vis-$\grave{a}$-vis the CHF).

Exercise 5

a) Let E denote the spot exchange rate, given in USD per CHF (!), F^{3M} denote the three months forward rate and i^{3M} denote the 3-months interest rate. Then, for bonds paying out in 3 months, uncovered interest parity implies that $i_{US}^{3M} - i_{CH}^{3M} = ln(F^{3M}) - ln(E)$. Note that interest rates are usually given in annual terms. So if a CHF bond is said to pay an interest rate of 1% the 3-months interest is $\frac{1\%}{4} = 0.25\%$. Hence we have $i_{CH}^{3M} = 0.0025$, $E = 1.077$ and $F^{3M} = 1.081$. Inserting this we get $i_{US}^{3M} \approx 0.0062 = 0.62\%$. Since we express interest rates in annualized terms, a 3-months US bond has an interest rate of $4 * 0.62\% = 2.48\%$.

Doing the same thing for the 6-months interest rate, we get $i_{US}^{6M} \approx 3.2\%$.

Doing the same thing for the 12-months interest rate, we get $i_{US}^{12M} \approx 3.84\%$.

b) If covered interest parity did not hold, investors could make riskless arbitrage profits buy borrowing in USD and lending out in CHF. Example: An investor buys a Swiss bond for 1000 CHF, maturing in 12 months. That costs him 1.077*1000 = 1077 USD. He borrows the USD at a rate of 3.5%. He then needs to pay back 1.035*1077 $\approx$ 1115 USD in 12 months. From the bond he knows that he gets 1000*1.02 = 1020 CHF in 12 months. He sells the SFR forward and gets a secure income of 1.108*1020 $\approx$ 1130 USD in 12 months. His arbitrage profit is 1130 USD - 1115 USD = 15 USD.

c) CHF has a flat term structure, USD has an increasing term structure.

d) The statement is true.