Exercise Sheet 8: Short Solutions.

Exercise 1

Firms set the prices such as to maximize profits. Profits are given by $\Pi(P) = PQ(p) - C(Q(P))$. The first order condition is:

$$\frac{\partial \Pi}{\partial P} = Q(P) + P\frac{\partial Q(P)}{\partial P} - \frac{\partial C(Q)}{\partial Q}\frac{\partial Q(P)}{\partial P} = 0$$

Inserting $\frac{\partial C(Q)}{\partial Q} = 1$ and $\frac{\partial Q(P)}{\partial P} = -2S$, as well as the expression for $Q(P)$, the first order condition writes:

$$\frac{S}{n} - 2S(P - \overline{P}) - 2SP + 2S = 0 \tag{1}$$

We now use the fact that all firms do the same thing (since they all solve the same optimization problem). This implies that $P = \overline{P}$. Equation (1) then simplifies to

$$\frac{1}{n} - 2P + 2 = 0 \tag{2}$$

Solving for P yields $\underline{\underline{P = \frac{1}{2n} + 1}}$. This is the price that all firms set.

We now use the condition that firms must make zero profits in equilibrium (this follows from the assumption of free entry into the car market). Profits are given by $\Pi = PQ - 100 - Q$ (this is the same expression as above, of course). We now insert $P = \frac{1}{2n} + 1$ into the expression, as well as $Q = \frac{S}{n}$ (all firms do the same) and set profits to zero. This gives the condition $S - 200n^2 = 0$. Solving this for n yields $n = \sqrt{\frac{S}{200}}$.

Hence the number of firms in country A is given by $n^a = \sqrt{\frac{1800}{200}} = 3$ and in country B by $n^b = 4$. Solving for the prices gives a price of approximately 1.16 in country A and of 1.125 in country B.

b) If the two countries allow for free trade, total market size is $Q^{world} = Q^a + Q^b = 5000$. Since nothing else has changed (firms still solve the same optimization problem, etc.)

we can simply use our formula from a) and solve for n. We get that there are 5 car firms in the unified market. The price of a car decreases to 1.1.

Note that the model does not tell us anything about in which country the car firms are located. In the unified market, cars are cheaper and there is a larger variety of different cars (more car firms). Therefore consumers are better off in the unified market compared to autarky.