Harris Dellas, Fall 1998

The empirics of general equilibrium macroeconomic models

I. Solution of the model

Formulate a fully specified model

Select convenient functional forms for its "primitives" (preferences, production, technology, exogenous driving forces)

Solve the model: Find decision rules for the endogenous variables in terms of the exogenous and predetermined variables (the states) and the parameters

If closed form solutions do not exist then find an approximate solution

II. Evaluation of the model

a) Standard econometric approach

Find the parameters of the decisions rule that best fit the data by either ML (Maximum likelihood) or GMM (Generalized methods of moments). Test the validity of the model by testing the restrictions implied by the model, by goodness of fit tests and so on.

An example: The stochastic, two factor, endogenous growth model

Let s_t be the vector of the state variables $s_t = \{k_t; z_{1t}; z_{2t}\}$ and w be the vector of the control variables $w_t = \{c_t; i_t; y_t; u_{1t}; N_{1t}\}$ (I have dropped the $'$ notation but the variables are still expressed in percentage deviations from steady state values). Suppose that I have not yet specified values for the parameters of the model. As it was shown earlier, the solution for the states takes the form

$$s_{t+1} = As_t + Be_{t+1} \tag{1}$$

where

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ 0 & \rho_1 & 0 \\ 0 & 0 & \rho_2 \end{pmatrix}; \quad B = \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{pmatrix}; \quad e_{t+1} = \begin{pmatrix} \varepsilon_{1t+1} \\ \varepsilon_{2t+1} \end{pmatrix}$$

and the solution for the controls is

$$w_t = Cs_t \tag{2}$$

The elements of A and C are functions of the unknown parameters of the model $(\alpha; \psi; \rho; \ldots)$ Combining (1) and (2) leads to

$$\begin{pmatrix} s_{t+1} \\ w_{t+1} \end{pmatrix} = \begin{pmatrix} A & 0 \\ CA & 0 \end{pmatrix} \begin{pmatrix} s_t \\ w_t \end{pmatrix} + \begin{pmatrix} B \\ CB \end{pmatrix} e_{t+1} \tag{3}$$

or more compactly

$$x_{t+1} = Dx_{t+1} + Ee_{t+1} \tag{4}$$

The system (4) is a restricted VAR. For instance, it assumes that lagged values of w_t do not enter. Such restrictions can be used to test the validity of the specification.

Another test of the model could utilize the fact that the elements of A and C are functions of the parameters of the model, which gives rise to cross equation restrictions involving these parameters. As these restrictions are non linear, the estimation-testing procedure can be quite complicated.

Finally, if parameter values are somehow assigned then (4) involves a set of simple linear restrictions which can also be used to evaluate the model.

b) The model as an economic laboratory: Calibration

Assign values to the parameters of the model (calibrate) using one of the available methods. Then assess its validity to fit the data by comparing quantitative and qualitative features of the model to those of the data (typically using some informal distance criterion).

The testing of the model is usually based on the examination of various moments (variances, covariances, autocorrelations, cross correlations at various lags and leads, impulse functions, forecast error decompositions and so on).

STEP 1

Model calibration

Assign values to the parameters of the model by

a) Using information from prior studies
b) Matching (some) sample and model moments. Usually long run averages
c) Estimating the parameters by GMM or Maximum Likelihood

b) Matching moments

One sector, one input, stochastic growth model
Output is given by $y_t = f(k_t) = z_t k_t^{\alpha}$
The steady state value of y_t; y; is then $y = k^{\alpha}$
The steady state level of capital, k, is

$$k = \left\{\frac{\beta^{-1} + \delta - 1}{\alpha}\right\}^{\frac{1}{\alpha - 1}}$$

Hence the steady state capital/output ratio is simply $k/y = \alpha/(\beta^{-1} - 1 + \delta_K)$.

We calculate $k/y = \frac{1}{n}\sum_{t=0}^{n}(k_t/y_t)$ from sample -actual- data on k_t and y_t where n is our sample size. Let $k/y = 10$. Using the standard values for β and α (0.99 and 0.35 respectively) we can assign a value to the rate of capital depreciation. In this case, $\delta_k = 0.034$.

Criticism: Which moments to match? The same parameter may appear in several moments. The selection of a particular moment may lead to poor performance along other dimensions.

c) Estimation

I. GMM

One sector, two input, exogenous growth model. Let

$$u(C_t, 1 - H_t) = \frac{1}{\sigma}[C_t^{1-\gamma}(1 - H_t)^{\gamma}]^{\sigma} \quad \sigma \neq 1 \tag{5}$$

Suppose we want to estimate the preference parameter γ. One of the first order conditions associated with the optimization problem is

$$E(u_{Ht}/u_{Ct} - F_{Ht}) = 0 \tag{6}$$

where $F_{Ht} = (1 - \alpha)K_t^{\alpha}H_t^{-\alpha} = w$ (the real wage). Hence, (6) takes the form

$$E_t\{\gamma(1 - H_t)w - (1 - \gamma)C_t\} = 0 \tag{7}$$

However, once could have used a different FOC, namely

$$\beta E_t u_{Ct+1}(f_{Kt+1} + 1 - \delta) = u_{Ct}$$

which, after some manipulation can be written as

$$E_t\{\beta(C_{t+1}/C_t)^{\sigma-1}(1 - \delta_K + r_{t+1})(w_{t+1}/w_t)^{(1-\gamma)\sigma}\} = 0 \tag{8}$$

One can exploit the moment condition $E_t(Q \otimes m) = 0$ to estimate the three unknown parameters, σ, γ and δ_k (Q is the term inside the expectations operator in (8) and m is the vector of instruments; $m \geq 3$).

Problems: (7) and (8) will in general give you different answers. Despite the fact that (8) imposes more structure (it requires forward looking individuals) its parameter estimation tends to be more imprecise (because of poor instruments). Small sample problem.

II Maximum likelihood

Problems with

a) Unobservable variables. If # observables > # state variables then a transformation is possible to allow estimation to proceed.

b) # of error terms (exogenous shocks) < # of variables in the system. The covariance matrix of the variables is singular and the log-likelihood cannot be evaluated

Solutions:

1. Reduce the dimension of the system to match the dimension of the vector of errors (shocks)
2. Add enough errors in variables to remove the singularity.

STEP II

Use the calibrated model as a laboratory to generate artificial data (time series) on the variables of interest.

a) Generate random numbers from the probability distribution of the exogenous driving forces (ε_{it}). We usually assume that the shocks have a normal pdf. The variance parameter is calibrated like any other parameter in the model (note: if there are several non-independent shocks one may still draw random numbers from independent distributions and construct the shocks).

b) Then feed these random numbers into equation (1) to obtain time series for the state variables. The simulated series for the states can in turn be fed into (2) to get simulated series for the control variables, say, $\hat{w}'_t = \{\hat{c}'_t; \hat{\imath}'_t; \hat{y}'_t; ...\}_{t=1}^T$. These time series constitute one artificial sample. The length of the artificial series (sample), T, should match that of the actual data.

c) Recall that the variables in (1) are measured in percentage deviations from the steady state. The artificially generated series can be transformed into their non-stationary counterparts, say $\{\hat{c}_t; \hat{\imath}_t; \hat{y}_t; ...\}_{t=1}^T$ by using the definition $\hat{w}_t = (1 + \hat{w}'_t)w$ (w is the steady state value of w_t).

d) Detrend the artificial series. The Prescott-Hodrick (HP) method is a popular detrending method, despite the fact that most models postulate a trend that is inconsistent with HP detrending (the method is described in the appendix). Let the HP detrended series be represented by w_t^*.

e) Repeat (a)-(d) many times and then calculate the moments of interest as the average across the artificial samples that have been generated. For instance, suppose that one is interested in the correlation between consumption and output. Let $cor_i = \text{Correlation}(c_t^*; y_t^*)_i$ in sample i; $i = 1; ...N$. The statistics of interest is simply constructed as $cor(c^*; y^*) = (1/N)\sum_{i=1}^N cor_i$ (we also typically calculate and report the variance of $cor(c; y)$).

f) Compare the moments of the artificial data to the corresponding moments of the -appropriately detrended- real data.

The impulse response functions and variance decompositions that are associated with (1) can be used to explicitly test the model. Even of one does not use them to test the model, it is common to report them too in order to present the information that is contained in the data.

Appendix

The Hodrick-Prescott filter

This is a method for decomposing a series into a trend and a stationary (cyclical) component. Suppose that you have a times series of observations on $\{x_t\}_{t=1}^T$ and you want to decompose this series into its trend $\{\tau_t\}$ and stationary $\{x_t - \tau_t\}$ elements. This can be done by selecting a series $\{\tau_t\}$ that minimizes the following sum

$$\frac{1}{T}\sum_{t=1}^{T}(x_t - \tau_t)^2 + (\lambda/T)\sum_{t=2}^{T-1}\left[\left(\tau_{t+1} - \tau_t\right) - \left(\tau_t - \tau_{t-1}\right)\right]^2$$

where λ is an arbitrary constant reflecting how much one wants to penalize the incorporation of fluctuations into the trend ($\lambda \to \infty$ gives a linear time trend; $\lambda = 0$ gives $\tau_t = y_t$). Hodrick-Prescott have a suggested a value of λ equal to 1600 (for quarterly data). The appeal of HP is that it can extract the same trend from a set of variables.