

# Stylized facts of real aggregate activity (1)

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## 1 Introduction

We begin this course by summarizing important regularities in macroeconomic time series data for economies such as the U.S. or Switzerland since WW II. Section 2 describes how the cyclical fluctuations of a macroeconomic time series is separated from its long-run growth component. In Section 3, our focus is the business cycle. In Section 4, important facts of economic growth are summarized.

## 2 Isolating the cyclical component

Figure 1 shows the U.S. price level from 1869-1996 on a proportionate scale. Over these almost 130 years, the price index has increased considerably. Also evident in Figure 1 are prolonged periods of increases and declines. We say, the

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index fluctuates around a long run growth trend. Such a behavior is typical for macroeconomic time series like real GDP, aggregate consumption, and aggregate investment.

FIGURE 1: U.S. PRICE LEVEL FROM 1869-1996  
(TAKEN FROM BARRO [1], FIG. 1.4)

If we want to study the business cycle properties of a macroeconomic time series, we have to separate the *cyclical fluctuations of the series* (considered to be a stationary stochastic process) from its *long-run growth component*. There are several alternative ways to do so:

- If the long run growth component in the macro time series is posited to be a linear time trend, then a natural way to eliminate this trend component is to regress the log of the series against time and to work with the resulting residual. - This procedure is statistically valid only if the long run growth component is *trend-stationary*. In most cases considered here this assumption is rather questionable.
- If the macro time series is thought to be *difference-stationary*, then one way to eliminate its trend is to first difference the series which, when the series is in logarithms, transforms the series into quarterly growth rates.
- Trend extraction can also be done with the help of the so called *Hodrik-Prescott (HP) filter*. The HP filter involves defining the cyclical component of a variable,  $x_t^c$ , as the current value of the variable,  $x_t$ , less a measure of trend of the variable,  $x_t^g$ , with trend output being a weighted average of past, current and future observations

$$x_t^c = x_t - x_t^g = x_t - \sum_{j=-J}^J a_j x_{t-j}$$

where the weights satisfy  $\sum_{j=-J}^J a_j = 1$ .

- Another way to extract the trend of macro time series is to apply a *Band-Pass filter*. Band-Pass filters split the series up into moderate frequency fluctuations associated with the business cycle, low frequency fluctuations associated with trend growth, and high frequency fluctuations associated with noise in the data (e.g. measurement errors).

Those who are interested in the subject find more info about linear filtering in Stock and Watson [4].

### 3 Business cycle facts

After having extracted the business cycle components of the series of aggregate activity of interest (e.g., GDP, consumption, investment, wage, labor productivity) we would like to statistically summarize these cyclical components. To this end we consider three distinct sets of empirical evidence.

- A first set of evidence is simply a *plot* of the cyclical component of each series along with the cyclical component of real GDP.
- Next, the comovements in these plots are quantified with the means of the *cross-correlation* of the cyclical component of each series with the cyclical component of real GDP. Specifically, this is the correlation between  $x_t$  and  $y_{t+k}$ , where  $x_t$  is the filtered series of interest and  $y_{t+k}$  is the  $k$ -quarter lead of the filtered logarithm of real GDP. A large positive correlation at  $k = 0$  indicates *procyclical* behavior of the series of interest (positive contemporaneous correlation with output); a large negative correlation at  $k = 0$  indicates *countercyclical* behavior; and a maximum correlation at, for example,  $k = -1$  indicates that the cyclical component of the series tends to lag the aggregate business cycle by one quarter. A variable that has little relation to real GDP is *acyclical*.
- Finally, we consider the *persistence* of the cyclical components of each series. With persistence we mean the long term effect of a shock on a particular variable. It is usually summarized by means of an estimate of the parameter  $\rho$  of the process

$$y_t = \rho y_{t-1} + \varepsilon_t$$

where  $y_t$  is the stationary component under consideration and  $\varepsilon_t$  is a residual.

The key facts can be summarized as follows:

#### 3.1 Volatility

- Consumption of non-durables is less volatile than output; Consumption of durables is more volatile than output.
- Investment is three times more volatile than output.
- Government purchases are less volatile than output.
- Total hours worked has about the same volatility as output; Hours per worker is much less volatile than output.

- Capital is much less volatile than output; Capital utilization is more volatile than output.
- Employment is as volatile as output.
- Labor productivity (output per man-hour) is less volatile than output; The real wage is much less volatile than output.
- The expected real interest rate is weakly countercyclical.<sup>1</sup>

## 3.2 Comovement

- Most series are procyclical; particularly striking is the high degree of comovement between total hours worked and aggregate output.
- Weakly countercyclical is the expected real interest rate; strongly countercyclical is unemployment.
- Three series are essentially acyclical: real wage, government purchases and capital stock.

## 3.3 Persistence

- All macroeconomic aggregates display substantial persistence; the first-order autocorrelation for most detrended quarterly variables is on the order of 0.9.

FIGURE 2: VOLATILITY AND PERSISTENCE FOR THE U.S. ECONOMY  
(TAKEN FROM KING AND REBELO [3], TABLE 3)

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<sup>1</sup>According to Barro [1], economists have employed at least three methods to measure expectations of a variable like inflation or the real interest rate (Barro, p. 253-6): (i) Ask a sample of people about their beliefs. (ii) Use the hypothesis of rational expectations, then use statistical techniques to figure out these optimal predictions. (iii) Use market data, such as interest rates or prices of financial contracts, to infer what people believe. Every of these methods has its own advantages and disadvantages. Barro uses the second method to create a measure of the expected real interest rate; he takes the *three-month* U.S. Treasury bill rate less an estimate of expected inflation for the corresponding three-month interval. A plot of the detrended value of the expected real interest rate against detrended real GDP (see Barro, Figure 9.10) reveals that the expected real interest rate is weakly procyclical: the correlation with detrended real GDP is 0.26 (i.e., weakly *procyclical*).

## 4 Facts of economic growth and development

The focus so far has been on fluctuations over business cycle frequencies. There are, however, some important relations among macroeconomic variables that hold over long horizons, although their relationship might be less transparent over short horizons. They are called *Kaldor facts of growth* since they were presented by Nicholas Kaldor at a 1958 conference on capital accumulation.<sup>2</sup>

1. The real rate of return to capital,  $r$ , shows no trend upward or downward.
2. The shares of income devoted to capital,  $\frac{rK}{Y}$ , and labor,  $\frac{wN}{Y}$  (where  $w$  denotes real wage in units of commodities), show no trend, i.e., they appear to fluctuate around constant means. Notice that fact 1 and 2 imply that the capital-output ratio,  $\frac{K}{Y}$ , is roughly constant.
3. Capital per worker,  $\frac{K}{N}$ , shows steady growth.
4. The average growth rate of output per worker,  $\frac{Y}{N}$ , has been positive and relatively constant over time.
5. There are wide differences in the growth rates across countries.

Jones [2] mentions two observations related to point 5. (i) There is enormous variation in per capita income across economies. (ii) Growth rates are not necessarily constant over time. As a corollary, a country's relative position in the world distribution of per capita incomes is not immutable [unveränderlich]. Countries can move from being 'poor' to being 'rich', and vice-versa.

## References

- [1] Barro, Robert J. (1997), *Macroeconomics*, 5th edition, MIT Press
- [2] Jones, Charles I. (1998), *Introduction to economic growth*, W.W. Norton & Company Ltd.
- [3] King, Robert G. and Sergio T. Rebelo (1999), Resuscitating real business cycles, in *Handbook of Macroeconomics*, Volume 1, edited by J.B. Taylor and M. Woodford, Elsevier Science B.V.
- [4] Stock, James H. and Mark W. Watson (1999), Business cycle fluctuations in U.S. macroeconomic time series, in *Handbook of Macroeconomics*, Volume 1, edited by J.B. Taylor and M. Woodford, Elsevier Science B.V.

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<sup>2</sup>Kaldor, Nicholas (1961), Capital accumulation and economic growth, in *The theory of capital*, ed. F.A. Lutz and D.C. Hague. New York: St. Martins