HOMEWORK

Harris Dellas

1 THE RBC MODEL WITH INDIVISIBLE LABOR

Solve the standard RBC model using the following utility function

$$U(C_t) = \ln C_t - \psi h_t \tag{1}$$

where C_t and h_t are consumption and employment respectively.

2 A SIMPLE, TWO SECTOR, TWO FACTOR, STOCHASTIC GROWTH MODEL

The economic environment

Two sectors: Sector 1 produces a single good that can be used for consumption and investment purposes. Sector 2 produces human capital. Production uses two inputs (physical and human capital). The subscript it denotes sector i in period t.

Preferences:

$$U(C_t) = \psi \ln C_t \tag{2}$$

Goods production:

$$Y_t = F(K_{1t}, H_{1t}, z_{1t}) = z_{1t}(u_{1t}K_t)^a(N_{1t}H_t)^{1-\alpha} \tag{3}$$

The economy's resource constraint is

$$Y_t = F(K_{1t}, H_{1t}, z_{1t}) = C_t + I_t \tag{4}$$

Physical capital depreciates at the rate of d_k per period (the same in both sectors)

The physical capital stock evolves as follows

$$K_{t+1} = (1 - d_k)K_t + I_t \quad = (1 - d_k)K_t + z_{1t}(u_{1t}K_t)^a(N_{1t}H_t)^{1-\alpha} - C_t \tag{5}$$

Human capital production

$$X_t = G(K_{2t}, H_{2t}, z_{2t}) = z_{2t}(u_{2t}K_t)^w(N_{2t}H_t)^{1-w} \tag{6}$$

The human capital stock evolves as follows

$$H_{t+1} = (1 - d_H)H_t + X_t \ = (1 - d_H)H_t + z_{2t}(u_{2t}K_t)^w(N_{2t}H_t)^{1-w} \tag{7}$$

Note that

$$N_{1t} + N_{2t} = 1 \tag{8}$$

$$u_{1t} + u_{2t} = 1 \tag{9}$$

The technological shock in sector i evolves according to

$$\log(z_{it+1}) = (1 - \rho_i)\log(z_i) + \rho_i \log(z_{it}) + v_{it+1}, \quad v_{it+1} \ \ iid \tag{10}$$

Information: v_{it} becomes known in period t

The value function is

$$V(K_t, H_t, z_t) = \max\{u(C_t, N_{1t}) + bE_tV(K_{t+1}, H_{t+1}, z_t) \tag{11}$$

where E_t is conditional expectation taken in period t and K_t, H_t and z_t are the state variables.

The objective is to maximize (11) subject to (5),(7),(8), (9), and (10) by selecting the optimal sequence $\{C_t, K_{t+1}, H_{t+1}, N_{1t}, N_{2t}, u_{1t}, u_{2t}\}_{t=0}^{\infty}$. Use (8), (9) in (6) and (7) to express u_{2t} and N_{2t} in terms of u_{1t} and N_{1t} respectively. Let Λ_{1t},and Λ_{2t} be the Langrange multipliers associated with (5) and (7). The FOC are

$$U_{Ct} = \Lambda_{1t} \tag{12}$$

$$bE_t\{(1 - d_k + F_{kt+1})\Lambda_{1t+1} + G_{kt+1}\Lambda_{2t+1} = \Lambda_{1t} \tag{13}$$

$$bE_t\{F_{Ht+1}\Lambda_{1t+1} + (1 - d_H + G_{Ht+1})\Lambda_{2t+1} = \Lambda_{2t} \tag{14}$$

$$\Lambda_{1t}F_{N1t} + \Lambda_{2t}G_{N1t} = 0 \tag{15}$$

$$\Lambda_{1t}F_{u1t} + \Lambda_{2t}G_{u1t} = 0 \tag{16}$$

Equations(12)-(16) together with (5), (7) and (10) form a system of equations that fully characterizes the equilibrium of this economy. Because the economy experiences sustained -endogenous-growth, the levels of the variables K, C and H are non-stationary. We can get rid of non-stationarity by expressing variables in terms of human capital. The corresponding system is

$$g1(c_t, \lambda_{1t}) = 0 \tag{17}$$

$$E_t g2(k_{t+1}, \lambda_{1t+1}, \lambda_{2t+1}, u_{1t+1}, N_{1t+1}, z_{1t+1}, z_{2t+1}) = 0 \tag{18}$$

$$E_t g3(k_{t+1}, \lambda_{1t+1}, \lambda_{2t+1}, u_{1t+1}, N_{1t+1}, z_{1t+1}, z_{2t+1}) = 0 \tag{19}$$

$$g4(k_t, \lambda_{1t}, \lambda_{2t}, u_{1t}, N_{1t}, z_{1t}, z_{2t}) = 0 \tag{20}$$

$$g5(k_t, \lambda_{1t}, \lambda_{2t}, u_{1t}, N_{1t}, z_{1t}, z_{2t}) = 0 \tag{21}$$

$$g6(k_{t+1}, k_t, u_{1t}, N_{1t}, z_{1t}, z_{2t}) = 0 \tag{22}$$

$$g7(z_{1t+1}, z_{1t}, v_{1t+1}) = 0 \tag{23}$$

$$g8(z_{2t+1}, z_{2t}, v_{2t+1}) = 0 \tag{24}$$

where $k_t = K_t/H_t,\ c_t = C_t/H_t$ and $\lambda_{it} = \Lambda_{it} H_t$. We now proceed as in the case of the one factor growth model. We first calculate the steady state solution $\{z_1, z_2, k, c, u_1, N_1, \lambda_1, \lambda_2\}$. Let $\{\rho_1, \rho_2, \alpha, w, \psi, b, d_K, d_H\} = \{0.95, 0.95, 0.35, 0.3, 1, 0.99, 0.02, 0.01\}$. The steady state vector is then $\{z_1, z_2, k, c, u_1, N_1, \lambda_1, \lambda_2\} = \{1, 0.0123, 33.7072, 1.3247, 0.6533, 0.6, 0.7549, 73.2089\}$ and the growth rate of the economy is $\mu = 1.0035$.