Answers

Question 1. Consider the model of section 2 (small open economy with investment, p.9 in class note 13.1). Assume now that utility is given by $u(c)-m*v(n)$ and production by $Y=AK^a n^{1-a}$, where n is work and m is a preference shock. Derive formally the effect on the current account of period 1 of an expected temporary increase in m in period 2.

Answer 1. I will make a number of simplifying assumptions in order to have the simplest possible answer to this question. I will assume that $\beta(1+r)=1$, $A=1$, $\delta=1$, that there is no production in the first period and that utility is given by:

$$ln(C_1)+\beta(ln(C_2)-m*ln(n))$$

and the budget constraints are

$$\begin{aligned} Y_1 &= C_1+K+B \quad (1)\\ C_2 &= K^a n^{1-a}+B(1+r) \quad (2) \end{aligned}$$

The FOCs are

$$\begin{aligned} C_1 &= C_2 \quad (3)\\ a(\frac{K}{n})^{a-1} &= 1+r \quad (4)\\ (1-1)(\frac{k}{n})^a &= m\frac{C_1}{n}(1+r) \quad (5) \end{aligned}$$

Equation (4) implies that the capital-labor ratio, k/n, is constant, so investment (capital) is proportional to labor. Hence, equation (5) implies that $m\frac{C_1}{n}$ is constant too. An increase in m thus implies a decrease in the ratio $\frac{C_1}{n}$. Consequently, if an increase in the marginal utility of leisure decreases labor supply both investment and current consumption must decrease. The current account $CA=Y_1-C_1-K$ improves.

Question 2. Consider the two period model with non-state contingent debt (section 1.2.1 in the class notes 13.3). a) Assume that the fraction of income that can be captured by the creditors in the case of default in the second period is $k_2<1$. Assume again risk neutrality on the part of both the creditor and the borrower and determine the optimal default decision in the first and second period as well as the maximum amount of debt that can be issued in period 1 (b_2) and its price (q) as a function also of k_2. How does an increase in k_2 affect the default decision as well as the maximum amount of debt that can be issued?

Answer 2a. The maximum amount that can be borrowed cannot exceed what can be captured by the creditors in the case of default. That is $b^{max}\leq k*Y_2$. Because there is no uncertainty about the level of future output (repaying ability), the creditors will only provide loans that are repaid, so these loans are risk free and their price equals the price of risk free debt, $q=\beta$. A lower k_2 thus means a lower b^{max} but the price is always β.

The optimal default decision is determined by the comparison of utility under default and no default:

If default: $Y_1 - k_1 Y_1 + \delta Y_2$

If no default: $Y_1 - b_1 + \underbrace{\beta k_2 Y_2}_{\text{new loans=qb}} + \delta(1 - k_2) Y_2$

So default is preferred if $b_1 > k_1 Y_1 + (\beta - \delta) k_2 Y_2$. The right hand side of this inequality is increasing in k_2, which implies that the incentive to default on current debt is decreasing in k_2. The intuition for this result is that the higher k_2, the larger the amount that can be borrowed now and hence the greater the benefit from participation in the credit markets (recall that default now means exclusion from the credit markets).

b) Suppose now that the utility of the borrower is concave, for instance, logarithmic, $u = log(c)$. How does the existence of risk aversion affect the likelihood of default in period 1 as well as the maximum amount of debt that can be issued? You do not need to do the math in this case, a intuitive economic argument suffices for full credit.

Answer 2b. Curvature in utility has two implications. First, it means that the gain from default (the extra consumption) is not as valuable as in the case of risk neutrality because of diminishing marginal utility. This lowers the incentive to default. Second, it means that the maximum desired loan is lower (because a larger loan now means a lower marginal utility in the second period), which lowers the value of participation in the credit market. This makes the cost of default lower. The net effect is determined as follows:

Incentive to default, $ID(k_2) = U(D) - U(ND) = u((1 - k_1)Y_1) + \delta u(Y_2) - u(Y_1 - b_1 + \beta k_2 Y_2) - \delta u((1 - k_2)Y_2)$

$\frac{dID}{dk_2} = Y_2(\delta u_2 - \beta u_1)$

The term inside the parenthesis is negative[1] so the incentive to default in the first period declines with more severe sanctions in the second period.

[1] To see this solve the problem under commitment. The FOC is $\beta u_1 - \delta u_2 = 0$. In the absence of commitment, the country cannot do optimal consumption smoothing, that is, it cannot borrow and consume enough in the first period. Hence $\beta u_1 > \delta u_2$.