Lecture Note 3a

Harris Dellas

1 Sovereign Debt

Case 1: Full commitment to repay

Case 2: Limited commitment to repay

1.1 State-Contingent Contracts

Model: Single good, uncertainty, 2 dates

t=1: Trading Assets

t=2: Consumption

$Y_2 = \overline{Y} + \varepsilon$ with $\varepsilon \in \{\underline{\varepsilon} = \varepsilon_1 < \varepsilon_2 < \ldots < \varepsilon_{N-1} < \varepsilon_N = \overline{\varepsilon}\}$, and prob($\varepsilon_i$)=$\pi(\varepsilon_i)$ with $\sum_{i=1}^{N} \pi(\varepsilon_i) = 1$. The shock ε has a mean of zero, is observable and $\underline{\varepsilon}$ is such that $\overline{Y} + \underline{\varepsilon} > 0$.

Agents can contract with risk neutral competitive foreign insurers.

Contracts deliver $P(\varepsilon)$ on date 2 so that: $C = \bar{Y} + \epsilon - P(\epsilon)$

$P(\varepsilon) < 0$: insurers pay

$P(\varepsilon) > 0$: insurers receive

Risk neutrality + perfect competition imply that profits are

$$\sum_{i=1}^{N} \pi(\varepsilon_i) P(\varepsilon_i) = 0$$

Payment is an issue for the country if $P(\varepsilon) > 0$. This raises the question of *Credibility.*

CASE 1: Full Commitment: An example with two income states for Y_2, namely, Y_{21}, Y_{22}

Output $Y_{21} = \bar{Y} + \epsilon$, $Y_{22} = \bar{Y} - \epsilon$, $Prob(\epsilon > 0) = 0.5$

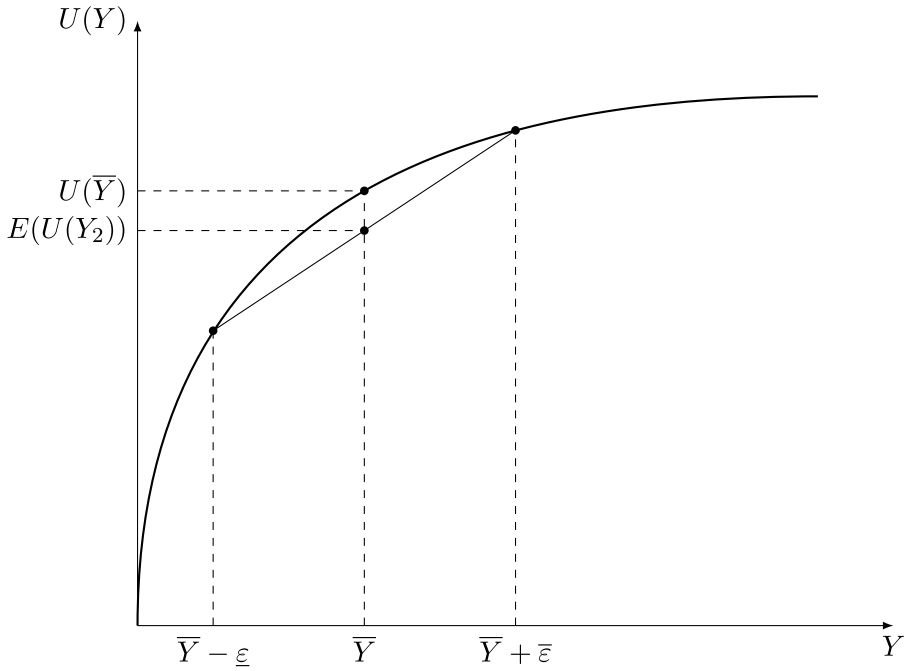
Choose a schedule of payments,P_1, P_2 to maximize the welfare of the country

Zero profit condition and risk neutrality on the part of the insurers means that $P_1 + P_2 = 0 \Rightarrow P_1 = -P_2 = P$.

$\max Eu(c) = 0.5u(\bar{Y} + \epsilon - P) + 0.5u(\bar{Y} - \epsilon + P)$

Concavity of the utility function implies that $P = \epsilon$ so that $C_{12} = C_{22} = \bar{Y}$. Consumption is independent of the state of nature. Perfect consumption smoothing

Figure 1: Concavity and Uncertainty $U(\overline{Y}) > E(U(Y_2))$



The more general case

$$\max U = \sum_{i=1}^{N} \pi(\varepsilon_i) U(C_i)$$
$$\text{s.t. } C_i = \overline{Y} + \varepsilon_i - P(\varepsilon_i)$$
$$\sum_{i=1}^{N} \pi(\varepsilon_i) P(\varepsilon_i) = 0$$

Associate a Lagrange multiplier μ to the zero profit condition and write the Lagrangean

$$\mathcal{L} = \sum_{i=1}^{N} \pi(\varepsilon_i) \big(U(\overline{Y} + \varepsilon_i - P(\varepsilon_i)) + \mu P(\varepsilon_i) \big)$$

The first order condition is

$$\pi(\varepsilon_i) \big(-U'(C_i) + \mu \big) = 0 \Longleftrightarrow U'(C_i) = \mu \forall i = 1, \ldots, N$$

This implies

$$C_i = \overline{Y} \text{ and } P(\varepsilon_i) = \varepsilon_i \ \forall i = 1, \ldots, N$$

There is full insurance.

CASE 2: Imperfect commitment to pay back: If the borrower lacks commitment to pay and if international insurers are competitive and the cost of not paying is **zero** then there will be no int'l asset trade.

$$P(\varepsilon_i) = 0 \ \forall i = 1, \ldots, N$$
$$C_i = \overline{Y} + \varepsilon_i$$

Zero consumption smoothing/insurance: $C_{2i} = Y_{2i}$ Suboptimal due to concavity of utility curve.

In order to support international international asset trade (debt) we need to introduce a cost of not paying the contracted amount (of default), L. Let it be a function of output: $L = \eta Y_2$ with $\eta \in (0, 1)$.

Incentive compatibility constraint (pay only when the payment is less than the sanction):

$$P(\varepsilon_i) \leqslant \eta Y_2 = \eta(\overline{Y} + \varepsilon_i)$$

An example with two states Assume that η is such that $\epsilon > \eta(\bar{Y} + \epsilon)$. In other words, η is sufficiently small as to make the commitment equilibrium with $P = \epsilon$ infeasible.

The maximum payment that the sovereign will make in the good state 1 for fear of sanctions is $P = \eta(\bar{Y} + \epsilon) < \epsilon$. Let $\epsilon - \eta(\bar{Y} + \epsilon) = m > 0$. Consequently, $C_{21} = \bar{Y} + \epsilon - P = \bar{Y} + \epsilon - \eta(\bar{Y} + \epsilon) = \bar{Y} + m$ and $C_{22} = \bar{Y} - m$ which leads to lower welfare than in the case of commitment as the idiosyncratic risk cannot be completely eliminated (consumption cannot be perfectly smoothed).

The more general case The optimization problem is

$$\max U = \sum_{i=1}^{N} \pi(\varepsilon_i) U(C_i)$$
$$\text{s.t. } C_i = \overline{Y} + \varepsilon_i - P(\varepsilon_i)$$
$$\sum_{i=1}^{N} \pi(\varepsilon_i) P_i = 0$$
$$P(\varepsilon_i) \leqslant \eta(\overline{Y} + \varepsilon_i)$$

The Lagrangian is then given by

$$\mathcal{L} = \sum_{i=1}^{N} \pi(\varepsilon_i) \left(U(\overline{Y} + \varepsilon_i - P(\varepsilon_i)) + \mu P(\varepsilon_i) \right) + \lambda(\varepsilon_i) \left(\eta(\overline{Y} + \varepsilon_i) - P(\varepsilon_i) \right)$$

The first order condition wrt $P(\varepsilon_i)$ is then

$$-\pi(\varepsilon_i) U'(C_i) + \mu \pi(\varepsilon_i) - \lambda(\varepsilon_i) = 0$$

and the slackness condition is

$$\lambda(\varepsilon_i) \left(\eta(\overline{Y} + \varepsilon_i) - P(\varepsilon_i) \right) = 0$$

There are then 2 possibilities. Either the incentive compatibility constraint is binding (satisfied with equality) and $\lambda(\varepsilon_i) > 0$, or it is not in which case $\lambda(\varepsilon_i) = 0$.

1. If $\lambda(\varepsilon_i) = 0$, then $P(\varepsilon_i) < \eta(\overline{Y} + \varepsilon_i)$ and

   $$u'(C_i) = \mu \ \forall i = 1, \dots, N$$

2. If $P(\varepsilon_i) = \eta(\overline{Y} + \varepsilon_i) \Longrightarrow \lambda(\varepsilon_i) > 0$

   $$U'(C_i) = \mu - \frac{\lambda(\varepsilon_i)}{\pi(\varepsilon_i)} \neq \mu$$

   Imperfect consumption insurance. Consumption is not constant across states of nature. It depends on ε_i

How much consumption smoothing can a sovereign have?

Guess: The ICC will not bind for low values of ε (because the country receives rather than pays) then

$$\lambda(\varepsilon_i) = 0 \Longrightarrow U'(C_2(\varepsilon_i)) = \mu$$

For low values of ε, period 2 consumption, C_2, is constant. Hence

$$C_2 = \overline{Y} + \varepsilon_i - P(\varepsilon_i) = \text{constant} \Longleftrightarrow P(\varepsilon_i) = \underbrace{\overline{Y} - \text{constant}}_{P_0} + \varepsilon_i$$

Hence

$$P(\varepsilon_i) = P_0 + \varepsilon_i$$

Let $\widetilde{\varepsilon}$ be such that the country is indifferent between paying or not paying and therefore suffering the sanction.

Default	Pay
$(1-\eta)(\overline{Y}+\widetilde{\varepsilon})$	$\overline{Y}+\varepsilon_i - P(\varepsilon_i) = \overline{Y}+\widetilde{\varepsilon} - P_0 - \widetilde{\varepsilon} = \overline{Y} - P_0$

Indifference implies that

$$(1-\eta)(\overline{Y}+\widetilde{\varepsilon}) = \overline{Y} - P_0 \tag{1}$$

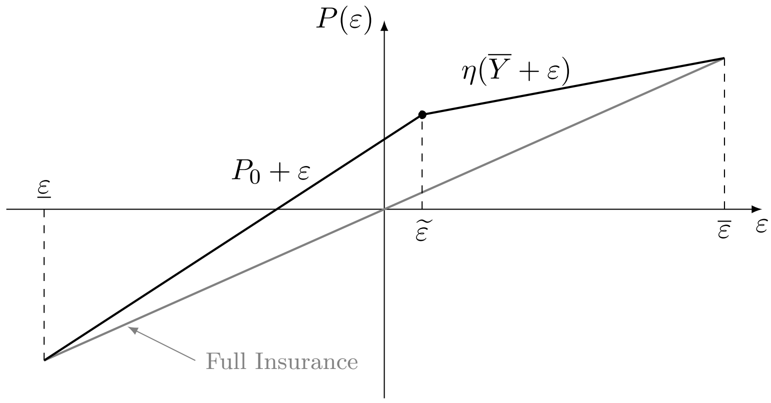
For $\varepsilon > \widetilde{\varepsilon}$ the country will never pay more than the sanction, $\eta(\overline{Y}+\varepsilon_i)$. Hence

$$P(\varepsilon) = \begin{cases} P_0 + \varepsilon & \text{if } \varepsilon \leqslant \widetilde{\varepsilon} \\ \eta(\overline{Y}+\varepsilon) & \text{if } \varepsilon > \widetilde{\varepsilon} \end{cases}$$

$$\int_{\underline{\varepsilon}}^{\widetilde{\varepsilon}} (P_0 + \varepsilon_i) \mathrm{d}f(\varepsilon_i) + \int_{\widetilde{\varepsilon}}^{\overline{\varepsilon}} \eta(\overline{Y}+\varepsilon_i) \mathrm{d}f(\varepsilon_i) = 0 \tag{2}$$

Equations (1)-(2) are two equations in the two unknown, P_0 and $\widetilde{\varepsilon}$.

Figure 2: Debt Contract



1.2 Non-contingent contracts (one period debt)

Sanction: Exclusion from credit markets in case of default in addition to the standard cost of default (output loss)

2 periods with some outstanding debt in the first period. Consumption in the first and second period respectively is

$C_1 = Y_1 - \aleph b_1 - (1-\aleph)kY_1 + \aleph q b_2$

$C_2 = Y_2 - \aleph b_2$

where $\aleph$ is an indicator variable such that $\aleph = 1$ indicates full repayment and $\aleph < 1$ default (with $\aleph = 0$ representing full default). b_1 is debt overhang in the first period. b_2 is debt issued in period 1 at price q (a discount bond: the borrower obtains qb_1 in period 1 and repays b_1 in period 2). k is the share of output lost in case of default. Y_2 is assumed to be known in the first period. Without loss of generality, we will assume that $k = 1$ in the second period and since Y_2 is known this implies that $b_2 \leq Y_2$ and the sovereign always pays back in period 2.

Let β and δ be the discount factor of the risk neutral lender and the borrower respectively with $\beta > \delta$. The lender will demand $q = \beta$ on risk free loans. If $\aleph < 1$ there is default and the sovereign cannot borrow anything in period 1. Since this punishment applies equally to partial or total default, the sovereign will opt for full default if he defaults at all (will choose $\aleph = 0$).

The utility levels associated with default and no-default in the first period are respectively:

$D : u(Y_1 - kY_1) + \delta u(Y_2)$

$ND : u(Y_1 - b_1 + qb_2) + \delta u(Y_2 - b_2)$

Let the borrower be risk neutral. The utility level associated with default and no-default in the first period respectively is:

$D : Y_1 - kY_1 + \delta Y_2$

$ND : Y_1 - b_1 + qb_2 + \delta(Y_2 - b_2) = Y_1 - b_1 + qb_2$

Note that debt in this case is $b_2 = Y_2$ due to the linearity of utility and the fact that $\beta > \delta$.

Consequently, default is selected[1] when $b_1 > kY_1 + (\beta - \delta)Y_2$. That is, when the current level of income is low, the future level of income is expected to be low (which limits the amount of fresh funds that can be obtained in period 1 in case the country did not default) and when the outstanding level of debt in the beginning of period 1 is high.

More general treatment See Eaton and Gersovitz, 1981, Arellano, 2008, Uribe, 2013 ch 8.

1.3 Investment

Suppose that the country receives an endowment in the first period, Y_1 but she needs to invest in capital in order to have any output in period 2. Assuming $K_1 = 0$,$Y_2 = F(K_2)$ with $F' > 0$,$F" < 0$.

[1] In the more general case where the cost of default in the second period is $k < 1$, default is selected in period 1 if $b_1 > kY_1 + (\beta - \delta)Y_2 - \beta kY_2$.

The budget constraint in the first period is $Y_1 + D - C_1 - K_2 = 0$ and in the 2nd period $F(K_2) + K_2 - C_2 - \aleph(1+r)D - (1-\aleph)k(F(K_2) + K_2) = 0$, where D is debt issued in the first period. Suppose the country cannot commit to repay its debt. Let us consider two cases: a) After borrowing the country enjoys discretion over the level of investment. b) The country commits to a particular level of investment.

No commitment regarding the level of investment We will solve the problem in two steps. In the first step we will assume that the lenders have made a loan, D, and the country then decides on the optimal amount of investment as well as whether it will default on its debt or not in period 2. In general, given a level of debt, D, and an investment decision, K_2, the debtor defaults if $(1+r)D < k(F(K_2) + K_2)$.

The optimal investment decision is determined by solving the following problem.

$$max_{K_2} u(Y_1 + D - K_2) + \delta u(F(K_2) + K_2 - min\{(1+r)D, k(F(K_2) + K_2)\}) \tag{3}$$

The country solves the problem in the case of default and in the case of no default. Let K_2^d and K_2^{nd} be the corresponding optimal choices of investment. If $U(D, K_2^d(D)) > U(D, K_2^{nd}(D))$ then the country defaults. Otherwise, it pays back. The lenders choose a level of debt, $\bar{D}$, such that the country does not default: That is, $\bar{D} : U(\bar{D}, K_2^d(\bar{D})) = U(\bar{D}, K_2^{nd}(\bar{D}))$. Figure 6.3 in Obstfeld and Rogoff shows how the maximal level of debt as well as the corresponding optimal level of investment are determined. In drawing the graph, the assumption has been made that if the level of debt is not too high then there is a level of investment that makes repayment optimal. An additional assumption is also needed regarding the relationship between the interest rate, the cost of default and the marginal product of capital to make sure that repayment is not always optimal independent of the level of debt. Note that the kink, where $(1+r)D = k(F(K_2) + K_2)$ is not on the optimal frontier!

Formally, the optimization problem takes the form

$$\Lambda = u(Y_1 + D - K_2) + \delta u(F(K_2) + K_2 - (1+r)D) - \lambda(D - \bar{D}) \tag{4}$$

The FOCs are

$$u'(C_1) = (1+r)\delta u'(C_2) + \lambda \tag{5}$$

$$u'(C_1) = (1 + F'(K_2))\delta u'(C_2) \tag{6}$$

$$0 = \lambda(\bar{D} - D) \tag{7}$$

When the borrowing constraint binds ($\lambda > 0$) consumption is tilted towards the future (C_1 is too low). But at the same time, C_2 is also below the level it would attained in the absence of default risk.

Commitment to the level of investment If the country can commit to a level of investment $K_2(D)$ when it receives the loan then the lenders will be willing to lend all the way up to the point where $(1+r)D = k(F(K_2) + K_2)$ as they know they will always get repaid. The optimization problem is then

$$\Lambda = u(Y_1 + D - K_2) + \delta u(F(K_2) + K_2 - (1+r)D) - \lambda((1+r)D - k(F(K_2) + K_2)) \qquad (8)$$

The FOCs are

$$u'(C_1) = (1+r)(\delta u'(C_2) + \lambda) \qquad (9)$$
$$u'(C_1) = (1+F'(K_2))(\delta u'(C_2) + k\lambda) \qquad (10)$$
$$0 = \lambda((1+r)D - k(F(K_2) + K_2)) \qquad (11)$$

When the borrowing constraint binds ($\lambda > 0$) consumption is tilted towards the future (C_1 is too low). At the same time, the country invests too little relative to the zero risk case ($F' > r$) but more than in the case of no investment commitment. Thus it can receive more funds relative to that case. The ability to tie one's hands helps.

1.4 Dellas-Niepelt, Differential sanctions across creditor groups

- A model with official and private creditors
- Probability of sovereign default depends on both the level and the composition of debt
- Higher exposure to official lenders improves incentives to repay but also carries extra costs such as reduced ex post flexibility.

The model can account for important features of sovereign debt crises:

- official lending to sovereigns takes place only in times of debt distress
- carries a favorable rate

- tends to *displace* private funding
- in the presence of debt overhang the availability of official funding increases the probability of default on outstanding debt

Justification for the key assumption. Club members

Description of model

$$
\begin{aligned}
G_1 &= \max_{\aleph_1\in[0,1],\ (b_2,b_2^e)} u(Y_1 - \aleph b_1 - (1-\aleph)L_1 + d_1) + \delta E_1\left[G_2\right)] \\
G_2 &= \max_{\aleph_2\in[0,1]} u(Y_2 - \aleph\tilde{b}_2 - (1-\aleph)(L_2 + \mathcal{L}(b_2^e))).
\end{aligned}
$$

$$
d_1(s_1,\pi_1) \equiv b_2 q_1(s_1,\pi_1) - b_2^e \Delta_1(s_1,\pi_1) \tag{12}
$$

$\Delta_1(s_1,\pi_1) \equiv q_1(s_1,\pi_1) - p_1(s_1,\pi_1)$, $\tilde{b}_2 = b_{02} + b_2 + b_2^e$

The Choice of Repayment Rate in the Second Period

$$
r_2(s_2) = \begin{cases} 1 & \text{if } L_2 \geq \tilde{b}_2 - \mathcal{L}(b_2^e) \\ 0 & \text{if } L_2 < \tilde{b}_2 - \mathcal{L}(b_2^e) \end{cases}. \tag{13}
$$

The Choice of Debt Issued to Private Lenders Elasticity of debt offer curve

The Choice of Debt Issued to Official Lenders Description of equilibria

Corner solutions

Examples:

$u'(c) = 1$, $\mathcal{L}'(b_2^e) = \mathcal{L}'$ with $0 \leq \mathcal{L}' < 1$, and $F_2'(L_2) = f_2$

Exogenous Price Discount, No Long-Term Debt Overhang ($b_{02}\xi_1 = 0$) Private creditors

$$
b_2^{\mathrm{PR}} = \frac{1}{f_2}\frac{\beta-\delta}{2\beta-\delta},\quad b_2^{e\,\mathrm{PR}} = 0,\quad G_1^{\mathrm{PR}} = \frac{1}{2f_2}\frac{(\beta-\delta)^2}{2\beta-\delta}.
$$

Official creditors only

$$
b_2^{\mathrm{OF}} = \frac{1}{f_2}\frac{\beta\kappa-\delta}{2\beta\kappa-\delta(1-\mathcal{L}')}\frac{1}{1-\mathcal{L}'},\quad b_2^{e\,\mathrm{OF}} = b_2^{\mathrm{OF}},\quad G_1^{\mathrm{OF}} = \frac{1}{2f_2}\frac{(\beta\kappa-\delta)^2}{2\beta\kappa-\delta(1-\mathcal{L}')}\frac{1}{1-\mathcal{L}'}.
$$

Official is favored when financing needs are high (low δ), enforcement is strong (high $\mathcal{L}'$) and official charges are low (high κ), see (Figure 3

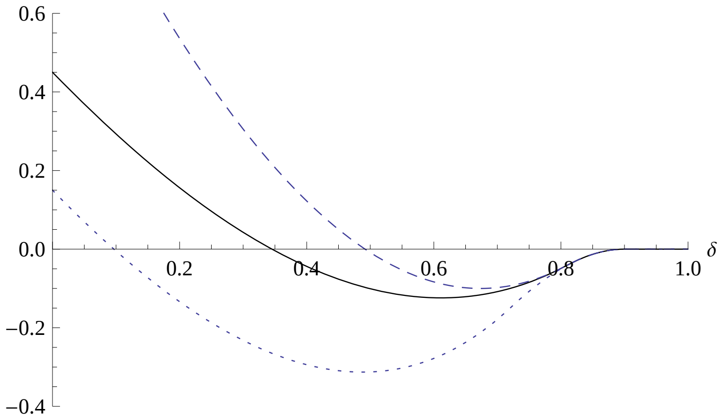


Figure 3: $G_1^{\text{OF}} - G_1^{\text{PR}}$ as function of δ. Higher $\mathcal{L}'$ shifts the curve up (dashed line), lower κ shifts the curve down (dotted line).

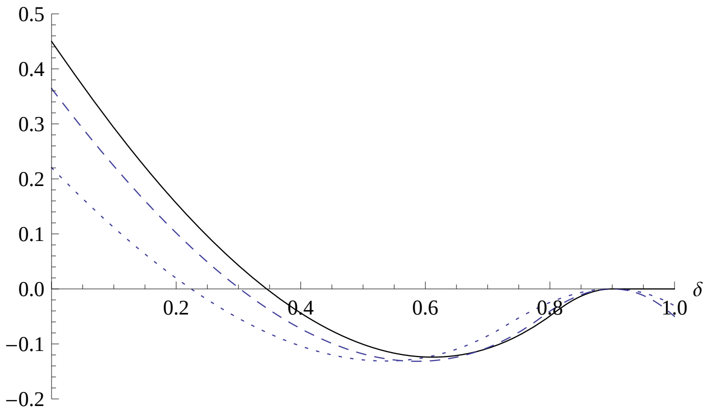


Figure 4: $G_1^{\text{OF}} - G_1^{\text{PR}}$ as function of δ. Higher debt overhang $b_{02}\xi_1$ reduces $\delta^\star$.

Exogenous Price Discount, Long-Term Debt Overhang The sovereign's incentive to choose private debt for refinancing increases with the stock of outstanding long-term debt (Figure 4)

Default is more likely when official debt is available for fresh funds. Example: for low values of δ (less than 0.62), intermediate realizations of L_1 (for instance, $L_1 = 1.3$ for $\delta = 0$) induce the sovereign to default if refinancing is provided by official but not if it is provided from private sources ((Figure 5)

Interaction of the default decision and debt ownership structure. Variation with δ and L_1 (Figure 6).

When the source of funds can be chosen along side the default decision, default occurs more often (with lower realizations of L_1) relative to the case where the government can only tap

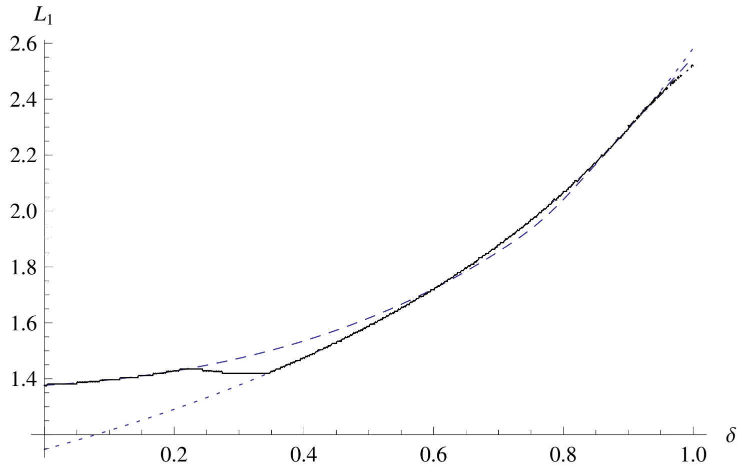


Figure 5: $\hat{L}_1^{\mathrm{PR}}$ (dotted), $\hat{L}_1^{\mathrm{OF}}$ (dashed), $\hat{L}_1$ (solid) as functions of δ.

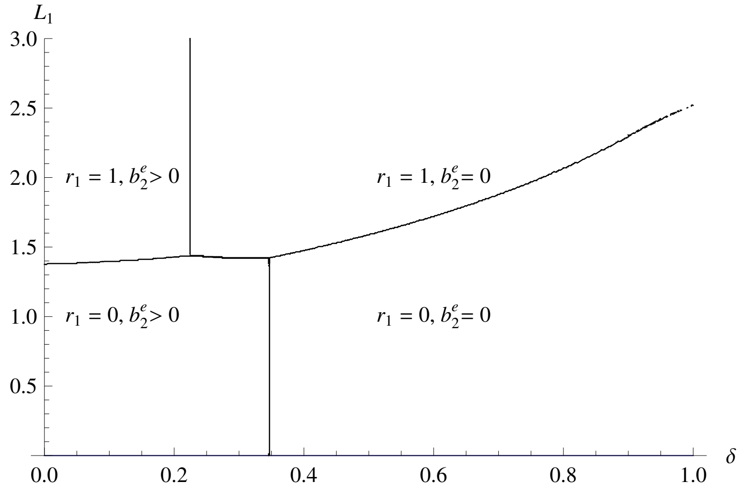


Figure 6: Default and official lending regions.

official creditors but less often than in the case where the government is forced to use private funds. Intuitively, low realizations of L_1 and the ensuing default lead the government to seek official funds because default wipes out long-term debt overhang and given this, the credibility benefits render official funding attractive. But conditional on high realizations of L_1 and the ensuing non-default the presence of honored long-term debt overhang renders official refinancing unattractive.

Conclusion: The availability of official funding may increase default risk on outstanding debt when refinancing needs are high. Interestingly, it may not only be the borrowing country that favors default in these circumstances, but also the official creditors. For they may profit from the debt they buy as long as $\kappa < 1$ and, as a consequence, from a default because it increases the demand for official funds.