Lecture Notes 1

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Definition of the current account and some useful relationships (further details and description of the variables in the supplement to note 1)

The current account (CA) is an important window linking any economy to the rest of the world

$$CA_t = TB_t + NFP$$

$$\begin{aligned} GNP &= GDP + NFP \\ &= C + I + G + TB + NFP \\ &= C + I + G + CA \\ GNP &= C + S + T \end{aligned}$$

$$CA = S - I + \underbrace{T - G}_{\text{budget deficit (surplus)}}$$

Derivation of the CA using the budget constraint of a representative agent

Suppose that the only international asset is a one period bond, B, that pays an interest of r. The budget constraint of an individual is

$$Y_t + (1+r)B_{t-1} + (1+r)B^G_{t-1} = C_t + I_t + T_t + B_t + B^G_t$$

where B^G is the domestic agent's holdings of domestic government bonds, Y is output, T is taxes and $B > 0$ indicates an international lender. The budget constraint of the government is

$$G_t + (1+r)B^G_{t-1} = T_t + B^G_t$$

Combining the two budget constraints gives

$$Y_t + (1+r)B_{t-1} = C_t + I_t + G_t + B_t$$

The CA is

$$CA_t = B_t - B_{t-1} = \underbrace{Y_t - C_t - G_t - I_t}_{TB_t} + rB_{t-1} \tag{1}$$
$$CA_t = B_t - B_{t-1} = TB_t + NFP \tag{2}$$
$$\tag{3}$$

It is useful to express the CA in terms of savings and investment. From the national accounts we have that $Y_t + rB_{t-1} = C_t + T_t + S_t$ where S_t is private savings. Using this definition in equation 1 gives

$$CA_t = S_t - I_t + T_t - G_t \tag{4}$$

Equation 1 emphasizes trade as the key determinant of the CA while equation 4 savings (both private and public, T-G) and investment. Naturally, the two equations are equivalent.

0.1 Lecture Objective: The determination of macroeconomic variables: Y, C, I, CA, B, q, ..

We will work through a succession of models. We will start with the simplest possible macroeconomic environment and then continue adding new features, expanding the number of variables to be determined. We will work with a two period model[1]. We will use these models to determine the properties of the equilibrium aggregate quantities –in particular of the current account– and prices (interest rate and real exchange rate) and then always ask the same question. Namely, how the equilibrium values are affected by changes in the economic environment (shocks to productivity, preferences, economic policy,..).

- Model 1
    1. Small open economy
    2. Endowments (no production)
    3. A single, perishable good
- Model 2
    1. Small open economy
    2. Production
    3. A single, non-perishable good (investment)
- Model 3

[1] Obstfeld and Rogoff and Uribe contain multiperiod versions of our models.

1. Two (large) economies
2. Endowments (no production)
3. A single, perishable good

- Model 4

1. Two (large) economies
2. Production
3. A single, non-perishable good (investment)

- Model 5

1. Two (large) economies
2. Endowments (no production)
3. (Two) Country specific, perishable goods

Assumptions shared across models:

- Abstract from monetary (nominal) considerations
- No uncertainty
- Representative agent
- Two periods

1 MODEL 1: Small open economy, endowment

1.1 A useful starting point: The closed economy (one country) model

Preferences

$$u(c_1) + \beta u(c_2)$$

The budget constraints

$$Y_1 = C_1 + B_1$$
$$Y_2 + (1+r)B_1 = C_2 + B_2$$

where B is savings. The world ends after the second period so $B_2 = 0$

The optimization problem is

$$\max_{c_1,c_2} U$$

subject to

$$\omega = Y_1 + \frac{Y_2}{1+r} = c_1 + \frac{c_2}{1+r}$$

The FOC is

$$u_{c_1} = \beta(1+r)u_{c_2}$$

The assumption of perishability implies that savings cannot take the form of stored (invested) goods but only the form of a "paper" claim.

The representative agent assumption means that there can be no net borrowers or lenders in the economy.

Consequently, the equilibrium aggregate quantities are

$$B_1 = 0$$
$$c_1 = Y_1$$
$$c_2 = Y_2$$

Even if the quantity of loans in equilibrium is zero, there is a price for loans, r ($r : B = 0$. It is determined by the FOC

$$u_{c_1}(Y_1) = \beta(1+r)u_{c_2}(Y_2)$$

$$r = \frac{u_{c_1}(Y_1)}{\beta u_{c_2}(Y_2)} - 1$$

The General Equilibrium of the model determines the endogenous variables as a function of the exogenous variables:

$\{Y_1, Y_2\} \quad \Rightarrow \quad \{c_1, c_2, B_1, r\}$

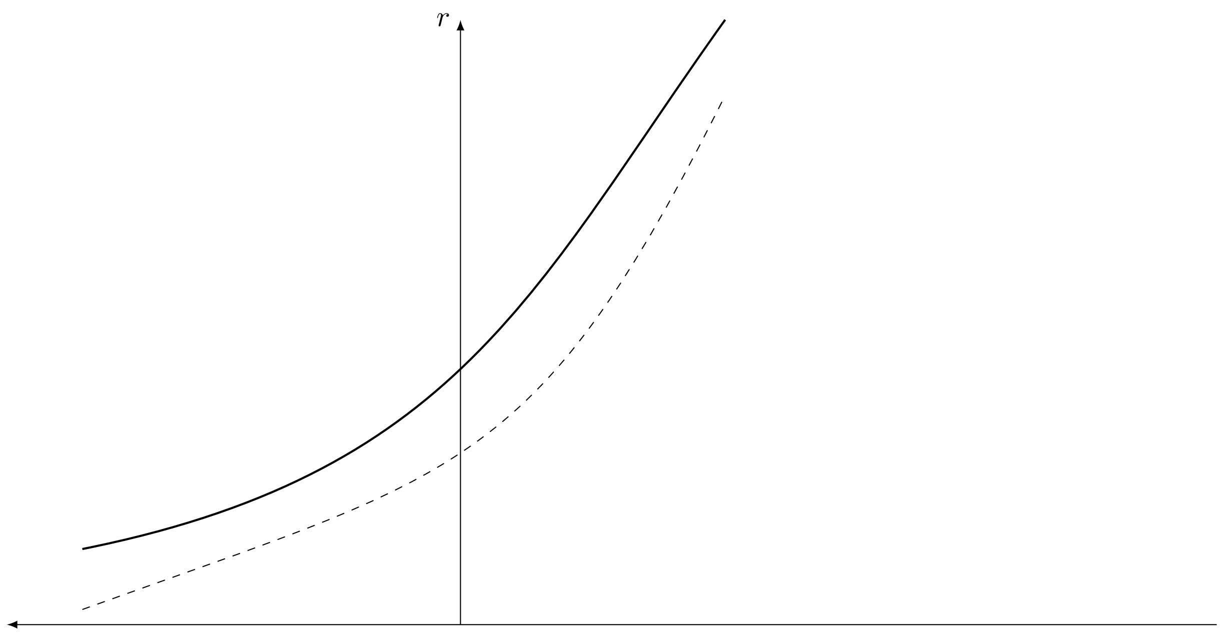
Comparative statics

$Y_1 \uparrow \quad c_1 \uparrow \quad dc_1 = dY_1 \quad dc_2 \rightarrow$

$\frac{dr}{dY_1} = u_{c_1} < 0$

$\frac{dr}{dY_2} > 0$ people want to borrow against high future income. Given a fixed supply of output at present (Y_1, this desire is discouraged by having a higher interest rate.

$\frac{dr}{d\beta} < 0 \qquad \beta \uparrow$ When people become more patient ($\beta \uparrow$), they want to save more. The interest rate must decrease to discourage this desire.



1.2 Small Open Economy

The assumption of small means inability to affect international prices. In our case this means that r is given, that is, the country can borrow as much as its repayment ability permits it without any effect on r.

$$Y_t + (1+r)B_{t-1} = C_t + B_t$$
$$CA_t = B_t - B_{t-1} = \underbrace{Y_t - C_t}_{TB} + r_{t-1}B_{t-1}$$

Specialize this budget constraint to the case of two periods

$$B_0(1+r) + Y_1 = C_1 + B_1$$
$$(1+r)B_1 + Y_2 = C_2 + B_2$$

or combined together,

$$B_0(1+r) + Y_1 + \frac{Y_2}{1+r} = C_1 + \frac{C_2}{1+r}$$

$B_2 = 0$ is the transversality condition.

Let us now solve for the equilibrium. The optimal choice of consumption satisfies the Euler equation

$$u_{c_1} = \beta(1+r)u_{c_2}$$

Solving the FOC for c_2 gives the first equation below. Plugging this equation into the budget constraint in the first period produces an equation for c_1 (the second equation below)

$$c_2 = c(r, c_1, Y_1, Y_2, B_0)$$
$$c_1 = c(r, Y_1, Y_2, B_0)$$

Hence, we can determine the equilibrium quantities of c_1 and c_2 as a function of output, initial assets and the world interest rate.

Using the budget constraints we have the following expressions for the CA

$$CA_1 = B_1 - B_0 = Y_1 - C_1 + rB_0$$
$$CA_2 = B_2 - B_1 = -B_1 = Y_2 - C_2 + rB_1$$
$$CA_1 + CA_2 = B_1 - B_0 - B_1 = -B_0$$

if $B_0 = 0 \quad \rightarrow \quad CA_1 + CA_2 = 0$

In order to have stationarity in consumption we impose the condition[2] $\beta(1+r) = 1$

A key property of the model: The FOC implies $u_{c_1} = u_{c_2} \;\; \Rightarrow \;\; c_1 = c_2$

The small open economy can thus achieve perfect consumption smoothing over time independent of the path of Y. We shall see later that this is no longer the case when this economy cannot fully commit to repaying its international debt.

Question: How do changes in the economic environment affect the CA?

A temporary increase in current output $(Y_1 \uparrow, Y_2 \rightarrow)$

$$CA = B_1 - B_0 = Y_1 - C_1 + rB_0 \rightarrow \frac{dCA}{dY_1} = 1 - \frac{dC_1}{dY_1}$$

Determine $\frac{dC_1}{dY_1}$ by differentiating the FOC and the overall budget constraint (equation 1.2) and combining to get

$$\frac{dc_1}{dY_1} = \frac{(1+r)U_{c_2c_2}}{u_{c_1c_1} + (1+r)u_{c_2c_2}} < 1$$

Consequently, $\frac{dCA}{dY_1} > 0$, that is, the CA_1 improves

A temporary increase in future output $(Y_2 \uparrow, Y_1 \rightarrow)$

Similarly, we can establish $0 < \frac{dC_1}{dY_2} < 1$ and thus from the definition of the CA, $CA = Y_1 - C_1 + rB_0$ we have

[2] Try to figure out what happens in an economy with an infinite time horizon when this condition is violated.

$$\frac{dCA_1}{dY_2} = -\frac{dC_1}{dY_2} < 0$$

That is, the CA_1 deteriorates

A permanent increase in output

$Y_1 \uparrow \quad Y_2 \uparrow$: $dY_1 = dY_2 = dY$

From the FOC we have that: $u_{c_1c_1}dc_1 = u_{c_2c_2}dc_2 \Rightarrow dc_1 = dc_2 = dc$. Differentiating the budget constraint then gives $dc = dY$.

$$\frac{dC_1}{dY_1} = 1 \qquad \frac{dCA}{dY} = 0$$

The big picture: The permanent income hypothesis tells us how changes in income (output) affect consumption-savings decisions and shape the response of the CA.

$Y_1 \uparrow, Y_2 \rightarrow \Rightarrow B \uparrow CA \uparrow$

$Y_2 \uparrow, Y_1 \Rightarrow \rightarrow B \downarrow CA \downarrow$

$Y_1 \uparrow, Y_2 \uparrow dY_1 = dY_2 \Rightarrow \rightarrow B \rightarrow CA \rightarrow$

Exercise: Think of the effects on the CA of a current increase in output that is expected to be followed by an even larger increase in future output (positive output growth)

$Y_1 \uparrow, Y_2 \uparrow dY_2 > dY_1$

Exersise: You may redo the analysis in a model which also contains a labor-leisure choice and production (so that utility is u(c, n) and production is y=f(n) where n is labor).

Government Spending and the Current Account: Twin Deficits

$$Y_1 = C_1 + T_1 + B$$
$$Y_2 + (1+r)B = C_2 + T_2$$

Combining these two budget constraints, we get

$$C_1 + \frac{C_2}{1+r} + T_1 + \frac{T_2}{1+r} = Y_1 + \frac{Y_2}{1+r}$$

In an analogous manner, the government's budget constraint is

$$T_1 + B_1^G = G_1$$
$$T_2 = (1+r)B_1^G + G_2$$

or, combining

$$T_1 + \frac{T_2}{1+r} = G_1 + \frac{G_2}{1+r}$$

Combining the private and public budget constraints produces

$$Y_1 + \frac{Y_2}{1+r} = C_1 + \frac{C_2}{1+r} + G_1 + \frac{G_2}{1+r}$$

Note that it does not make any difference whether we assume a balanced budget $T_1 = G_1$, $T_2 = G_2$ or debt financed deficits, $G_1 = T_1 + B^g$, $G_2 + (1+r)B^g = T_2, B^g > 0$. This is due to Ricardian equivalence.

The social planning problem is

$$\max_{C_1} U(C_1) + \beta U(C_2) = U(C_1) + \beta U((1+r)Y_1 + Y_2 - (1+r)C_1 - (1+r)G_1 - G_2)$$

which leads to the first order Euler condition

$$U_{C_1} = \beta(1+r)U_{C_2}$$

We now compute $\frac{\mathrm{d}(CA)}{\mathrm{d}G_1}$. Again assume that $\beta(1+r)$. We have $CA = B - B_0 = Y_1 - C_1 - G_1$ so that $\frac{dCA}{dG_1} = -\frac{dC_1}{dG_1} - 1$. We need to determine $\frac{dC_1}{dG_1}$. Total differentiation of the Euler equation with regard to C_1 and G_1 leads to

$$\frac{\mathrm{d}C_1}{\mathrm{d}G_1} = -\frac{(1+r)U_{C_2C_2}}{U_{C_1C_1} + (1+r)U_{C_2C_2}} < 0 \text{ and } \left|\frac{\mathrm{d}C_1}{\mathrm{d}G_1}\right| < 1$$

Hence, $\frac{dCA_1}{dG_1} < 0$. In particular,

$CA_1 \downarrow (-), CA_2 \uparrow (+), CA_1 + CA_2 = 0$

What about the effect of an expected future increase in government spending? Differentiating the Euler equation with regard to G_2 we can see that

$$\frac{\mathrm{d}C_1}{\mathrm{d}G_2} < 0$$

Differentiation of the resource constraint ($Y_1 = C_1 + G_1 + B_1$) gives

$$0 = \frac{\mathrm{d}C_1}{\mathrm{d}G_2} + 0 + \frac{\mathrm{d}B}{\mathrm{d}G_2} \Longrightarrow \frac{\mathrm{d}B}{\mathrm{d}G_2} > 0 \quad \begin{matrix} CA_1+ \\ CA_2- \end{matrix}$$

Again, the permanent income hypothesis can be used to understand the effects of changes in the level of government spending on savings and the current account. Recall that government spending is completely useless (from the point of view of the consumers) in the economy. For the households, an increase in government spending is equivalent a reduction in their output (income). Hence, with temporary increase in G savings decreases and the CA deteriorates.

Exercise: (a)The effects on the CA of a permanent increase in G. (b) The effects of a change in G that satisfies: $\mathrm{d}G_1 + \frac{\mathrm{d}G_2}{(1+r)} = 0$.

2 MODEL 2: Small open economy, Investment

In the previous section, a country could transfer resources over time only by borrowing from or lending to the rest of the world. In this section we study the current account in a model where there is an additional means of saving, namely physical capital. For simplicity, we assume that all domestic capital is owned by domestic residents[3]

Specification of the model

- Production technology: $Y = F(K)$
- Capital Accumulation: $K_{t+1} = I_t + K_t$, depreciation, $\delta = 0$
- Government Budget Constraint: $T_t = G_t$
- Resource (Budget) Constraint:

$$F(K_t) + (1+r)B_t + K_t = C_t + T_t + B_{t+1} + K_{t+1}$$

Combining these equations:

$$Y_t + (1+r)B_t = C_t + G_t + I_t + B_{t+1}$$

The current account is:

$$CA_t = B_{t+1} - B_t = Y_t + rB_t - \underbrace{(C_t + I_t + G_t)}_{\text{absorption}}$$

Private savings, S is equal to the change in –total– wealth:

$$S_t = B_{t+1} + K_{t+1} - (B_t + K_t) = Y_t + rB_t - C_t - G_t = CA_t + I_t$$

(and also $S_t = Y_t + rB_t - C_t - T_t$)

The two period version of the model is

$$F(K_1) + K_1 = C_1 + K_2 + G_1 + B$$
$$F(K_2) + K_2 + (1+r)B = C_2 + K_3 + G_2$$

Combining these equations, we get

$$C_1 + I_1 + G_1 + \frac{C_2 + I_2 + G_2}{1+r} = Y_1 + \frac{Y_2}{1+r} = F(K_1) + \frac{F(I_1 + K_1)}{1+r}$$

[3] If there is foreign ownership we need to take this into account in defining the NFIP of the country and hence the current account.

where $I_1 = K_2 - K_1$ and $I_2 = K_3 - K_2 = -K_2$. The maximization problem is then

$$\max U(C_1)+\beta U(C_2) = U(C_1)+\beta U\left((1+r)(F(K_1) - C_1 - G_1 - I_1) + F(I_1 + K_1) - G_2 + \underbrace{I_1 + K_1}_{K_2}\right)$$

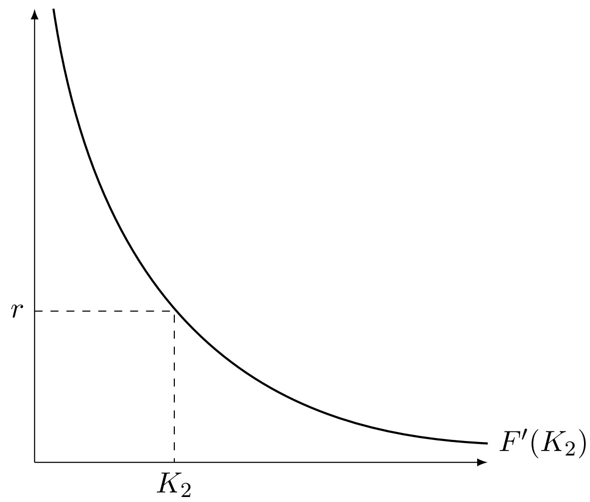
which leads to the first order conditions

$$\begin{aligned} /C_1 :&\ U_{C_1} = \beta(1+r)U_{C_2} \\ /I_1 :&\ \beta U_{C_2}\left(-(1+r) + F_{K_2} + 1\right) = 0 \Longrightarrow F_{K_2} = r \end{aligned}$$

where $F_{K_2} = F'(K_2)$.

Note that differentiating the 2nd FOC gives $F'' dK_2 = dr$ so that $dK_2/dr = 1/F'' < 0$.

Figure 1: Capital choice



$$\left.\begin{matrix} I_1 \\ K_2 \end{matrix}\right\} \Longleftarrow r$$

Main points:

Because r is exogenous (given by the rest of the world) the desired investment and capital stock are independent of demand conditions (preferences).

Consequently, there is complete separation of savings from investment decisions. But note that this requires the following conditions

1. Small Open Economy
2. Perfect capital mobility
3. (No borrowing limits)

The Feldstein–Horioka puzzle: A strong positive correlation between national savings and investment rates. If capital is very mobile across countries, then the correlation between savings and investment should be close to zero, as the preceding analysis shows.

Exercise: Write the production function as Y=AF(K). Examine the effects of a current temporary, expected future and a permanent positive productivity shock ($A \uparrow$). What is the response of S, I and the CA?

3 MODEL 3: Two -large- country model, endowments

Now we will g

$$Y_1 = C_1 + B \tag{5}$$
$$Y_2 + (1+r)B = C_2 \tag{6}$$
$$Y_1^\star = C_1^\star + B^\star \tag{7}$$
$$Y_2^\star + (1+r)B^\star = C_2^\star \tag{8}$$
$$B + B^\star = 0 \tag{9}$$
$$\tag{10}$$

Combining (5)–(9), we obtain the market clearing conditions for the two outputs

$$Y_1 + Y_1^\star = C_1 + C_1^\star \tag{11}$$
$$Y_2 + Y_2^\star = C_2 + C_2^\star \tag{12}$$

Combining (5)–(8)gives the intertemporal budget constraint for each country

$$Y_1 + Y_2/(1+r) = C_1 + C_2/(1+r) \tag{13}$$
$$Y_1^\star + Y_2^\star/(1+r) = C_1^\star + C_2^\star/(1+r) \tag{14}$$

Finally the FOCs for the optimal choice of consumption in each country are

$$U_{C_1} = \beta(1+r)U_{C_2} \Longrightarrow C_2 = \mathcal{C}_2(C_1, r) \tag{15}$$
$$U_{C_1}^\star = \beta(1+r)U_{C_2}^\star \Longrightarrow C_2^\star = \mathcal{C}_2^\star(C_1^\star, r) \tag{16}$$

Equations 11-16 are 6 equations in 5 unknowns, $\{C_1, C_2, C_1^\star, C_2^\star, r\}$.

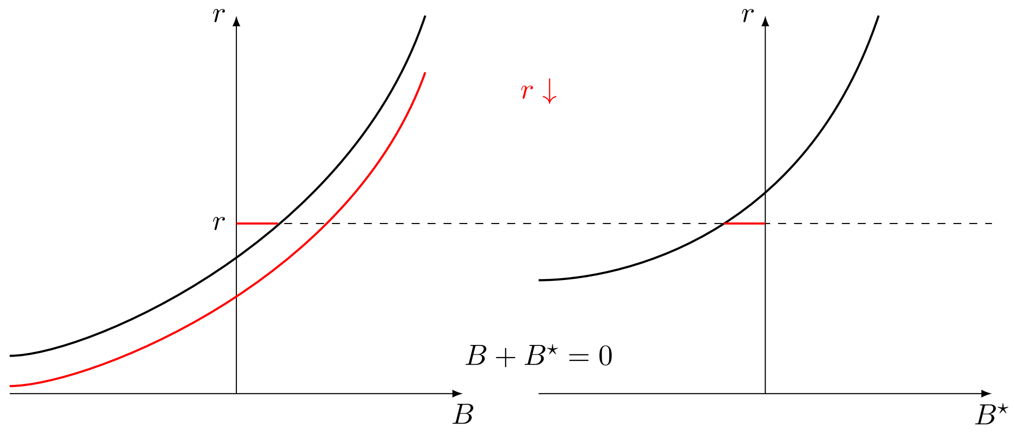
To solve for the equilibrium of the model we use Walras law: If there are N markets we only need consider the equilibrium ($D = S$) in the N-1 markets. The N-th market will clear automatically when the N-1 markets clear.

Exercise: Assume $U(C) = \log(C)$, Solve the model. Determine $\{C_1, C_2, C_1^\star, C_2^\star, r\}$ for arbitrary $\{Y_1, Y_2, Y_1^\star, Y_2^\star\}$. That is, find

$$C_t = \mathcal{C}_t(Y_1, Y_2, Y_1^\star, Y_2^\star) \text{ for } t = 1, 2$$
$$r = \mathcal{R}_t(Y_1, Y_2, Y_1^\star, Y_2^\star)$$
$$B = \mathcal{B}(Y_1, Y_2, Y_1^\star, Y_2^\star)$$

Compute $\frac{\mathrm{d}C_t}{\mathrm{d}Y}, \frac{\mathrm{d}r}{\mathrm{d}Y}, \frac{\mathrm{d}B}{\mathrm{d}Y}$.

Figure 2: Variation in Y_1, endowment world economy



- What is the main difference between a small and a large economy? In the former case r is exogenous. In the large economy: Smaller variations in the current account as a result of economic disturbances than in the small open economy due to the "dampening effect" of the induced change in r. *Small open economies exhibit greater macroeconomic volatility.*

- Growth and welfare. Is a country hurt by an increase in trading partners' growth rates? In Fig. 2 an increase in Y_2/Y_1 shifts the Home saving curve upwards, increasing world interest rates and making the foreign country (borrower) worse off.

- Empirical implication: War and the CA. War waging countries run a current account ("borrow from abroad to finance the war"). Japan's CA experience during the Russian War, 1904-05 (no disruption of world financial markets, relative certainty about the winner)

4 MODEL 4: Two country model, investment

- 2 country model with investment: $S = B + I$

- 2 periods.

- Domestic technology: $Y = AF(K)$
- Foreign technology: $Y^\star = A^\star F(K^\star)$

- Period $t = 1$: $K_1 + A_1F(K_1) = C_1 + K_2 + B$ and $K_2 = I_1 + K_1$ imply: $Y_1 = C_1 + I_1 + B$
- Period $t = 2$: $K_2 + A_2F(K_2) + (1 + r)B = C_2$

where r is the interest rate on the one period international bonds.

Note also that K_1 is predetermined and that we have assumed a zero rate of capital depreciation.

The representative individual maximizes:

$$\max_{C_1,I_1} U(C_1) + \beta U\left((1 + r)(A_1F(K_1) - C_1 - I_1) + A_2F(K_1 + I_1) + K_1 + I_1\right)$$

The first order conditions are then given by

$$/I_1 \ : \beta U'(C_2)\big(-(1 + r) + A_2F'(K_2) + 1\big) = 0 \tag{17}$$

$$/C_1 \ : U'(C_1) = \beta(1 + r)U'(C_2) \tag{18}$$

The first order condition for I_1 reduces to

$$r = A_2F'(K_2)$$

This equation represents an arbitrage condition between the two possible investments (I_1, B):

$$r = A_2F'(K_1 + I_1) \Longleftrightarrow I_1 = \mathcal{I}(\underset{-}{r}, \underset{+}{A_2}, \underset{-}{K_1})$$

Solving the model

The second first order condition above expresses the intertemporal allocation of consumption, from which we get

$$C_2 = \mathcal{C}_2(\underset{+}{r}, \underset{+}{C_1})$$

The intertemporal budget constraint of the agent writes

$$K_1 + A_1F(K_1) - C_1 - (I_1 + K_1) = \frac{C_2 - K_1 - I_1 - A_2F(K_1 + I_1)}{1 + r}$$

which can be used to solve for period 1 consumption as

$$C_1 = \mathcal{C}_1(r, K_1, A_1, A_2)$$

From the first period resource constraint, $A_1F(K_1) = C_1 + I_1 + B$, one can get B as

$$B = \mathcal{B}(r, K_1, A_1, A_2) = CA$$

Plaguing all these equation in the budget constraint allows to solve for the equilibrium r, that is, r satisfies $A_1F(K_1) = C_1 + I_1 + B$

If $B > 0$, $S > I$, $CA > 0$, home lends its excess savings to the rest of the world.

Comparative exercises

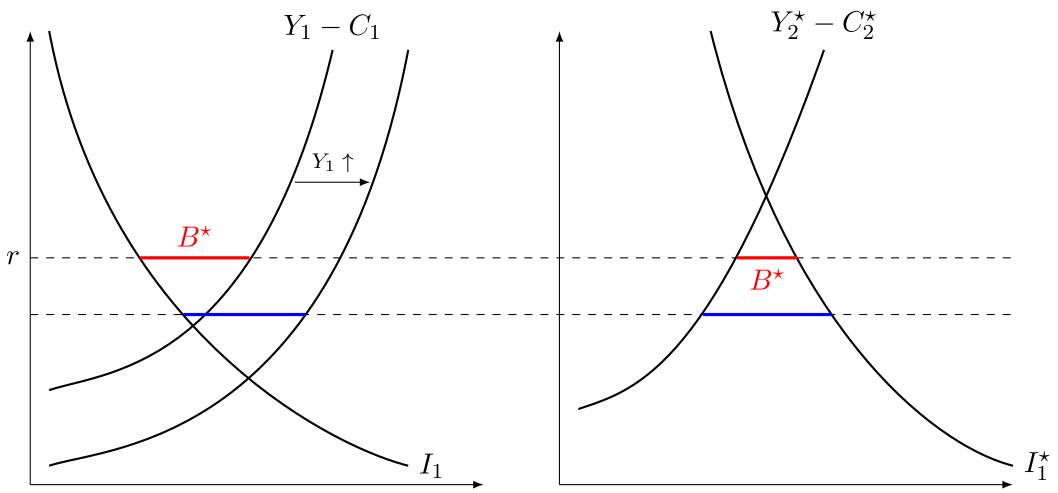
An increase in current domestic productivity (3)

$A_1 \uparrow$, $A_2 \rightarrow$, and $\{A_1^\star, A_2^\star\} \rightarrow$:

- I_1 curve does not shift ($I_1 \rightarrow$)
- $Y_1 - C_1$: $A_1 \uparrow \Longrightarrow Y_1 = A_1F(K_1) \uparrow$. By the permanent income hypothesis, we have $\Delta C_1 < \Delta Y_1$, hence $Y_1 - C_1$ shifts to the right.

The world interest rate decreases. The foreign CA worsens further.

Figure 3: An increase in A_1



An increase in expected future domestic productivity can be analyzed similarly (4). Starting from a zero initial CA, the effect on the CA is negative (consider the foreign CA).

The non-separation of investment from savings

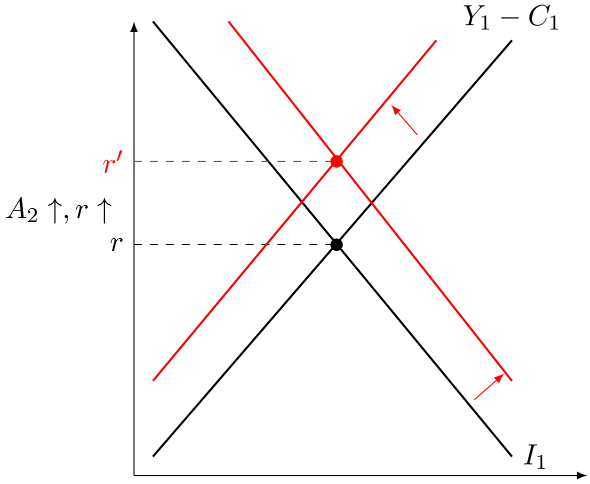
Can the model account for the Felstein-Horioka finding? How?

(Hint: Consider a combination of current and future productivity shocks)

Question: Can the model account for movements in real world interest rates? For instance, why rates were high in 80s? How?

An increase in the expected profitability of investment. It increases r and may increase or decrease I. Under log utility, world I goes down

Figure 4: An increase in A_2



5 Multiperiod version

We now consider a model with many periods, ending in period $T > 0$. The agents maximizes its intertemporal utility

$$\sum_{t=0}^{T} \beta^t U(C_t)$$

subject to

$$(1+r)B_{t-1} + Y_t = C_t + B_t$$

The intertemporal budget constraint then writes

$$(1+r)B_{-1} + \sum_{t=0}^{T} \left(\frac{1}{1+r}\right)^t Y_t = \sum_{t=0}^{T} \left(\frac{1}{1+r}\right)^t C_t + \frac{B_T}{(1+r)^T}$$

The infinite horizon version of the budget constraint is given by

$$(1+r)B_{-1} + \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t Y_t = \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t C_t + \lim_{T\to\infty} \frac{B_T}{(1+r)^T}$$

which simplifies to

$$(1+r)B_{-1} + \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t Y_t = \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t C_t$$

because $\lim_{T\to\infty} \frac{B_T}{(1+r)^T} = 0$.

We can now address an important question about CA deficit sustainability:

Can a country run perpetual current account deficits when $B_{-1} < 0$?

$$C_t + B_t = (1+r)B_{t-1} + Y_t$$

denoting the trade balance by $TB_t = Y_t - C_t$, the last equation rewrites

$$B_t = (1+r)B_{t-1} + TB_t$$

Let $TB = -\alpha r B_{t-1}$ with $\alpha < 1$. Then plugging this in the last equation yields

$$B_t = (1 + r - \alpha r)B_{t-1}$$

Then starting from $B_{-1} < 0$ implies that $B_t < 0\ \forall t$

$$B_t = (1 + r(1-\alpha))^t B_{-1}$$

then

$$CA_t = B_t - B_{t-1} = r(1-\alpha)B_{t-1} < 0$$

Is this feasible? Under what conditions?

$$\lim_{t\to\infty} \frac{B_t}{(1+r)^t} = \frac{(1+r-\alpha r)^t}{(1+r)^t} B_{-1} = 0 \Longleftrightarrow \left(\frac{1+r(1-\alpha)}{1+r}\right) < 1$$

TB grows at rate $r(1-\alpha)$, Y must grow at least $r(1-\alpha)$ to prevent $C < 0$