The Current Account

Definition

$$CA_t = TB_t + NFP_t$$

$$\begin{aligned} GNP &= GDP + NFP \\ &= C + I + G + TB + NFP \\ &= C + I + G + CA \\ GNP &= C + S + T \end{aligned}$$

$$CA = S - I + \underbrace{T - G}_{\text{budget deficit (surplus)}}$$

Derivation of the CA

Budget constraint of a representative agent

$$Y_t + (1+r)B_{t-1} + (1+r)B^G_{t-1} = C_t + I_t + T_t + B_t + B^G_t$$

Budget constraint of the government

$$G_t + (1+r)B^G_{t-1} = T_t + B^G_t$$

Combining

$$Y_t + (1+r)B_{t-1} = C_t + I_t + G_t + B_t$$

The CA is

$$CA_t \equiv B_t - B_{t-1} = \underbrace{Y_t - C_t - G_t - I_t}_{TB_t} + rB_{t-1} \tag{1}$$

$$= TB_t + NFP \tag{2}$$

The CA in terms of savings and investment

$$CA_t = S_t - I_t + T_t - G_t \tag{3}$$

Equation (1) emphasizes trade while equation (3) savings

The determination of macroeconomic variables: Y, C, I, CA, B, q, ..

- Model 1
  1. Small open economy
  2. Endowments (no production)
  3. A single, perishable good
- Model 2
  1. Small open economy
  2. Production
  3. A single, non-perishable good (investment)
- Model 3
  1. Two (large) economies
  2. Endowments (no production)
  3. A single, perishable good
- Model 4
  1. Two (large) economies
  2. Production
  3. A single, non-perishable good (investment)

- Model 5
  1. Two (large) economies
  2. Endowments (no production)
  3. (Two) Country specific, perishable goods

Assumptions shared across models:

- Abstract from monetary (nominal) considerations
- No uncertainty
- Representative agent
- Two periods

A useful starting point: A closed economy
Preferences

$$u(c_1) + \beta u(c_2)$$

The budget constraints

$$\begin{aligned} Y_1 &= C_1 + B_1 \\ Y_2 + (1+r)B_1 &= C_2 + B_2 \rightarrow \\ Y_1 + \frac{Y_2}{1+r} &= c_1 + \frac{c_2}{1+r} \end{aligned}$$

$B_2 = 0$
Optimal savings-consumption choice

$$u_{c_1} = \beta(1+r)u_{c_2}$$

The equilibrium

$$B_1 = 0$$
$$c_1 = Y_1$$
$$c_2 = Y_2$$

$r : B_1 = 0$

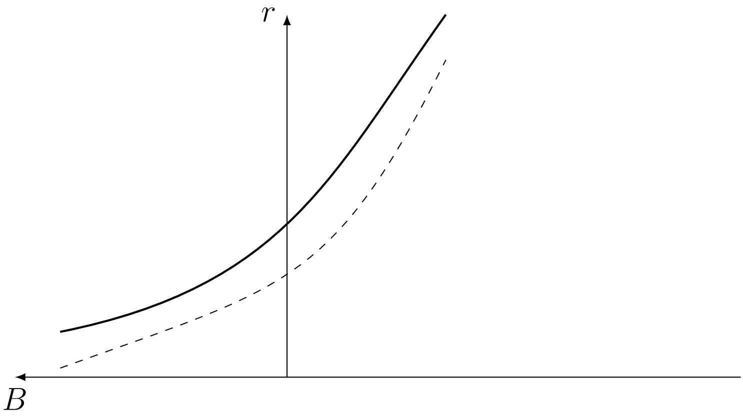
$$u_{c_1}(Y_1) = \beta(1 + r)u_{c_2}(Y_2) \Rightarrow r = \frac{u_{c_1}(Y_1)}{\beta u_{c_2}(Y_2)} - 1$$

The General Equilibrium of the model determines the endogenous variables as a function of the exogenous variables:

$\{Y_1, Y_2\} \quad \Rightarrow \quad \{c_1(Y_1), c_2(Y_2), B_1 = 0, r(Y_1, Y_2)\}$

Comparative statics

- $\frac{dc_1}{dY_1} > 0 \quad dc_1 = dY_1 \quad dc_2 = dY_2$
- $\frac{dr}{dY_1} = u_{c_1} < 0$
- $\frac{dr}{dY_2} > 0$
- $\frac{dr}{d\beta} < 0$


r
B

The Small Open Economy
Small (and committed to repay) means inability to affect r.

$$Y_t + (1+r)B_{t-1} = C_t + B_t$$
$$CA_t = B_t - B_{t-1} = \underbrace{Y_t - C_t}_{TB} + r_{t-1}B_{t-1}$$

Specialize to two periods

$$B_0(1+r) + Y_1 = C_1 + B_1$$
$$(1+r)B_1 + Y_2 = C_2 + B_2 \rightarrowtail$$
$$B_0(1+r) + Y_1 + \frac{Y_2}{1+r} = C_1 + \frac{C_2}{1+r}$$

$B_2 = 0$

Optimal consumption satisfies the Euler equation

$$u_{c_1} = \beta(1+r)u_{c_2}$$

$c_i = c(B_0, Y_1, Y_2, r)$

$$\begin{aligned} CA_1 = B_1 - B_0 &= Y_1 - C_1 + rB_0 \\ CA_2 = B_2 - B_1 = -B_1 &= Y_2 - C_2 + rB_1 \\ CA_1 + CA_2 = B_1 - B_0 - B_1 &= -B_0 \end{aligned}$$

if $B_0 = 0 \quad \rightarrow \quad CA_1 + CA_2 = 0$
For stationarity in consumption: $\beta(1+r) = 1$
A key property of the model: $u_{c_1} = u_{c_2} \;\; \Rightarrow \;\; c_1 = c_2$
Perfect consumption smoothing over time

Question: Changes in the economic environment and the CA?

A temporary increase in current output $(Y_1 \uparrow, Y_2 \rightarrow)$

$$CA = B_1 - B_0 = Y_1 - C_1 + rB_0 \rightarrow \frac{dCA}{dY_1} = 1 - \frac{dC_1}{dY_1}$$

$$0 < \frac{dC_1}{dY_1} = \frac{(1+r)U_{c_2c_2}}{u_{c_1c_1} + (1+r)u_{c_2c_2}} < 1$$

The CA_1 improves

A temporary increase in future output $(Y_2 \uparrow, Y_1 \rightarrow)$

From the FOC $0 < \frac{dC_1}{dY_2} < 1$. From the definition of the CA,

$CA = Y_1 - C_1 + rB_0 \Rightarrow \frac{dCA_1}{dY_2} = -\frac{dC_1}{dY_2} < 0$

The CA_1 deteriorates

A permanent increase in output

$Y_1 \uparrow \quad Y_2 \uparrow$: $dY_1 = dY_2 = dY$

From the FOC: $u_{c_1c_1} dC_1 = u_{c_2c_2} dC_2 \Rightarrow dC_1 = dC_2 = dc$.

From the budget constraint: $dC = dY$. Hence

$\frac{dC_1}{dY_1} = 1 \qquad \frac{dCA}{dY} = 0$

The big picture: The permanent income hypothesis tells us how changes in income (output) affect consumption-savings decisions and shape the response of the CA.

- $Y_1 \uparrow, Y_2 \rightarrow \Rightarrow B \uparrow CA \uparrow$
- $Y_2 \uparrow, Y_1 \Rightarrow \rightarrow B \downarrow CA \downarrow$
- $Y_1 \uparrow, Y_2 \uparrow dY_1 = dY_2 \Rightarrow B \rightarrow CA \rightarrow$

Exercise: The effects on the CA of output growth:
$Y_1 \uparrow, Y_2 \uparrow dY_2 > dY_1$

Exercise: Redo the analysis in a model which also contains a labor-leisure choice and production (utility is u(c, n) and production is y=f(n) where n is labor).

Government Spending and the Current Account: Twin Deficits

$$Y_1 = C_1 + T_1 + B$$
$$Y_2 + (1+r)B = C_2 + T_2 \mapsto$$
$$C_1 + \frac{C_2}{1+r} + T_1 + \frac{T_2}{1+r} = Y_1 + \frac{Y_2}{1+r}$$

The government's budget constraint

$$T_1 + B_1^G = G_1$$
$$T_2 = (1+r)B_1^G + G_2 \mapsto$$
$$T_1 + \frac{T_2}{1+r} = G_1 + \frac{G_2}{1+r}$$

Private plus public budget constraints

$$Y_1 + \frac{Y_2}{1+r} = C_1 + \frac{C_2}{1+r} + G_1 + \frac{G_2}{1+r}$$

Ricardian equivalence: Timing of taxes does not matter.

$$U(C_1)+\beta U(C_2) = U(C_1)+\beta U((1+r)Y_1+Y_2-(1+r)C_1-(1+r)G_1-G_2)$$

$$U_{C_1} = \beta(1+r)U_{C_2}$$

Assume again $\beta(1+r)$. The effect of a change in G_1 on the CA_1 is
$\frac{\mathrm{d}(CA_1)}{\mathrm{d}G_1} = -\frac{dC_1}{dG_1} - 1.$

$$\frac{\mathrm{d}C_1}{\mathrm{d}G_1} = -\frac{(1+r)U_{C_2C_2}}{U_{C_1C_1}+(1+r)U_{C_2C_2}} < 0 \text{ and } \left|\frac{\mathrm{d}C_1}{\mathrm{d}G_1}\right| < 1$$

$\frac{dCA_1}{dG_1} < 0$. $CA_1 \downarrow (-), CA_2 \uparrow (+)$, $CA_1 + CA_2 = 0$
The effect of an expected increase in G_2 on the CA. $\frac{\mathrm{d}C_1}{\mathrm{d}G_2} < 0$

$$0 = \frac{\mathrm{d}C_1}{\mathrm{d}G_2} + 0 + \frac{\mathrm{d}B}{\mathrm{d}G_2} \Longrightarrow \frac{\mathrm{d}CA_1}{\mathrm{d}G_2} = \frac{\mathrm{d}B}{\mathrm{d}G_2} = -\frac{\mathrm{d}C_1}{\mathrm{d}G_2} > 0$$

- The permanent income hypothesis can be used to understand the effects of changes in the level of government spending on savings and the current account.
- Government spending is completely useless (from the point of view of the consumers) in the economy.
- For the households, an increase in government spending is equivalent a reduction in their output (income). Hence, temporary increases in G make savings and the CA change.

<u>Exercise:</u> (a)Determine the effects on the CA of a permanent increase in G.

Small open economy with investment

An additional means of saving, physical capital

- Production technology: $Y = F(K)$
- Capital Accumulation: $K_{t+1} = I_t + K_t$, $\delta = 0$
- Balanced Government Budget: $T_t = G_t$
- Budget Constraint:

$$F(K_t) + K_t + (1+r)B_t = C_t + T_t + B_{t+1} + K_{t+1} \Rightarrow \quad (4)$$

$$Y_t + (1+r)B_t = C_t + G_t + I_t + B_{t+1} \quad (5)$$

The current account is:

$$CA_t \equiv B_{t+1} - B_t = Y_t + rB_t - \underbrace{(C_t + I_t + G_t)}_{\text{absorption}}$$

Private savings is $S_t = Y_t + rB_t - C_t - T_t$. Combined with the NIA definition,$GNP_t = Y_t + rB_t = C_t + G_t + I_t + CA_t$ it gives the twin deficit expression $CA_t = S_t - I_t + T_t - G_t$.

$$F(K_1) + K_1 = C_1 + K_2 + G_1 + B$$
$$F(K_2) + K_2 + (1+r)B = C_2 + K_3 + G_2 \mapsto$$
$$C_1 + I_1 + G_1 + \frac{C_2 + I_2 + G_2}{1+r} = Y_1 + \frac{Y_2}{1+r} =$$
$$F(K_1) + \frac{F(I_1 + K_1)}{1+r}$$

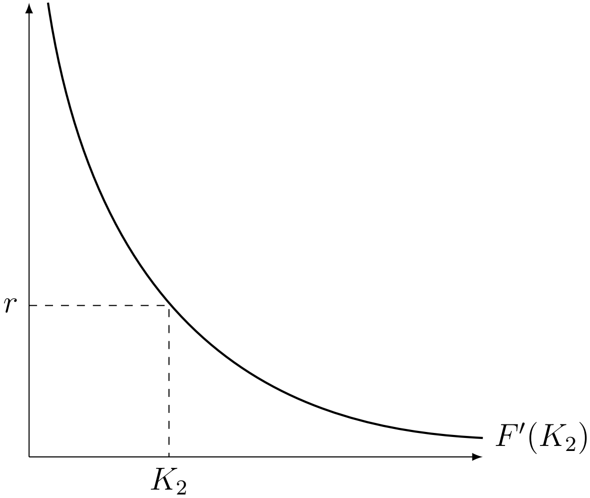
$I_1 = K_2 - K_1,\ I_2 = K_3 - K_2 = -K_2.$

$$U(C_1)+\beta U(C_2) = U(C_1)+\beta U\left((1+r)(F(K_1) - C_1 - G_1 - I_1) + F(I_1 + K_1) - G_2 + \underbrace{I_1 + K_1}_{K_2}\right)$$

$$/C_1:\ U_{C_1} = \beta(1+r)U_{C_2}$$
$$/I_1:\ \beta U_{C_2}\left(-(1+r) + F_{K_2} + 1\right) = 0 \Longrightarrow F_{K_2} = r$$

where $F_{K_2} = F'(K_2)$. $F'' dK_2 = dr$ so $dK_2/dr = 1/F'' < 0$.

Figure : Capital choice



$$\left.\begin{array}{l} I_1 \\ K_2 \end{array}\right\} \Longleftarrow r$$

Main points:
Because r is exogenous the desired investment and capital stock are independent of demand conditions (preferences). There is complete separation of savings from investment decisions. This requires the following

1. Small open economy
2. Perfect capital mobility

The Feldstein–Horioka puzzle: A strong positive correlation between national savings and investment rates. If capital is very mobile across countries, then the correlation between savings and investment should be close to zero, as the preceding analysis shows.

Two -large- country model, endowments

$$Y_1 = C_1 + B \tag{6}$$

$$Y_2 + (1+r)B = C_2 \tag{7}$$

$$Y_1^\star = C_1^\star + B^\star \tag{8}$$

$$Y_2^\star + (1+r)B^\star = C_2^\star \tag{9}$$

$$B + B^\star = 0 \tag{10}$$

Combining (6)–(10), we obtain the market clearing conditions for the two outputs

$$Y_1 + Y_1^\star = C_1 + C_1^\star \tag{11}$$

$$Y_2 + Y_2^\star = C_2 + C_2^\star \tag{12}$$

Combining (6)–(7) and (8)–(9)gives the intertemporal budget constraint for each country

$$Y_1 + Y_2/(1+r) = C_1 + C_2/(1+r) \tag{13}$$

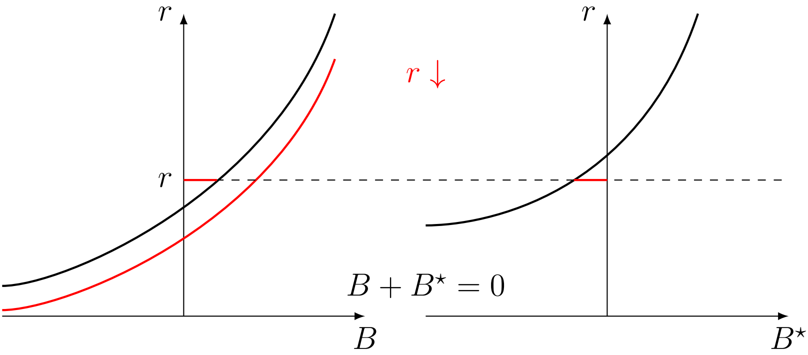
$$Y_1^\star + Y_2^\star/(1+r) = C_1^\star + C_2^\star/(1+r) \tag{14}$$

The FOCs

$$U_{C_1} = \beta(1+r)U_{C_2} \Longrightarrow C_2 = \mathcal{C}_2(C_1, r) \qquad (15)$$
$$U^{\star}_{C_1} = \beta(1+r)U^{\star}_{C_2} \Longrightarrow C^{\star}_2 = \mathcal{C}^{\star}_2(C^{\star}_1, r) \qquad (16)$$

Equations (11)-(16) are 6 equations in 5 unknowns, $\{C_1, C_2, C^{\star}_1, C^{\star}_2, r\}$.
To solve for the equilibrium of the model we use Walras law: If there are N markets we only need consider the equilibrium ($D = S$) in the N-1 markets. The N-th market will clear automatically when the N-1 markets clear.

FIGURE : Variation in Y_1, endowment world economy



- What is the main difference between a small and a large economy? In the former case r is exogenous. In the large economy: Smaller variations in the current account as a result of economic disturbances than in the small open economy due to the "dampening effect" of the induced change in r. *Small open economies exhibit greater macroeconomic volatility*.
- Growth and welfare. Is a country hurt by an increase in trading partners' growth rates? In Fig. 2 an increase in Y_2/Y_1 shifts the Home saving curve upwards, increasing world interest rates and making the foreign country (borrower) worse off.

Empirical project: Examine the empirical relationship between country size and volatility.

2 country model with investment

- Domestic technology: $Y = AF(K)$
- Foreign technology: $Y^\star = A^\star F(K^\star)$
- $S = B + I$

- Period $t = 1$: $K_1 + A_1 F(K_1) = C_1 + K_2 + B$ and $K_2 = I_1 + K_1$ imply: $Y_1 = C_1 + I_1 + B$
- Period $t = 2$: $K_2 + A_2 F(K_2) + (1 + r)B = C_2$

$$K_1 + A_1 F(K_1) - C_1 - (I_1 + K_1) = \frac{C_2 - K_1 - I_1 - A_2 F(K_1 + I_1)}{1 + r}$$

The maximization problem: Select C_1, I_1

$$U(C_1)+\beta U\left((1+r)(A_1F(K_1)-C_1-I_1)+A_2F(K_1+I_1)+K_1+I_1\right)$$

FOCs

$$/I_1 \;:\; \beta U'(C_2)\left(-(1+r)+A_2F'(K_2)+1\right)=0 \tag{17}$$

$$/C_1 \;:\; U'(C_1)=\beta(1+r)U'(C_2) \tag{18}$$

$$r = A_2F'(K_2)$$

Arbitrage condition between the two possible investments (I_1, B):

$$r = A_2F'(K_1+I_1) \Longleftrightarrow I_1 = \mathcal{I}(\underset{-}{r}, \underset{+}{A_2}, \underset{-}{K_1})$$

Comparative exercises

An increase in current domestic productivity

$A_1 \uparrow$, $A_2 \rightarrow$, and $\{A_1^\star, A_2^\star\} \rightarrow$:

- I_1 curve does not shift
- $Y_1 - C_1$: $A_1 \uparrow \Longrightarrow Y_1 = A_1 F(K_1) \uparrow$. By the permanent income hypothesis, we have $\Delta C_1 < \Delta Y_1$, hence $Y_1 - C_1$ shifts to the right.

The world interest rate decreases. The foreign CA worsens further.

An increase in expected future domestic productivity can be analyzed similarly. Starting from a zero initial CA, the effect on the CA is negative.

FIGURE : An increase in A_1

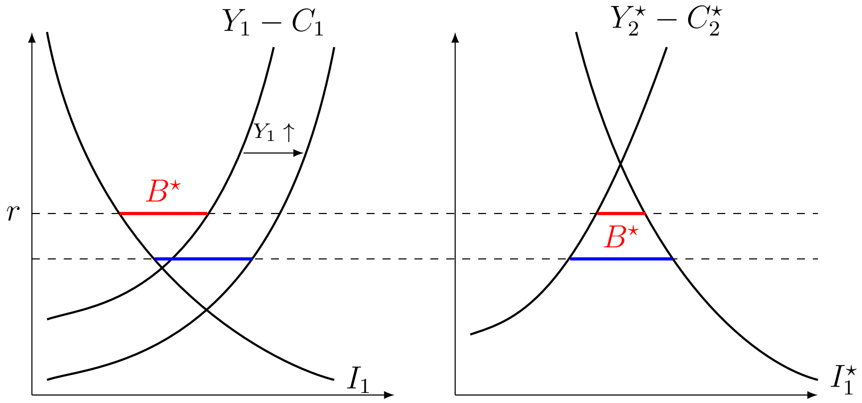
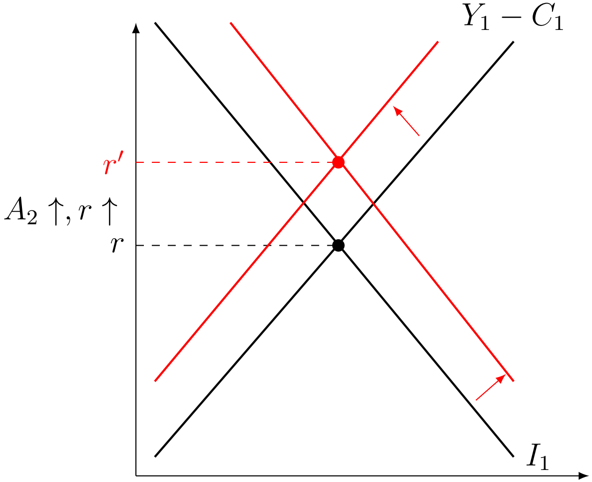


Figure : An increase in A_2



The non-separation of investment from savings
Can the model account for the Felstein-Horioka finding? How?
(Hint: Consider a combination of current and future productivity shocks)
Question: Can the model account for movements in real world interest rates? For instance, why rates were high in 80s? How?
An increase in the expected profitability of investment. It increases r and may increase or decrease I. Under log utility, world I goes down

A multiperiod version

$$\sum_{t=0}^{T} \beta^t U(C_t)$$

subject to

$$(1+r)B_{t-1} + Y_t = C_t + B_t$$

The intertemporal budget constraint

$$(1+r)B_{-1} + \sum_{t=0}^{T} \left(\frac{1}{1+r}\right)^t Y_t = \sum_{t=0}^{T} \left(\frac{1}{1+r}\right)^t C_t + \frac{B_T}{(1+r)^T}$$

Its infinite horizon version

$$(1+r)B_{-1} + \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t Y_t = \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t C_t + \lim_{T\to\infty} \frac{B_T}{(1+r)^T}$$

$$(1+r)B_{-1} + \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t Y_t = \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t C_t$$

because $\lim_{T\to\infty} \frac{B_T}{(1+r)^T} = 0$.

CA deficit sustainability: **Can a country run perpetual current account deficits when $B_{-1} < 0$?**

$$C_t + B_t = (1+r)B_{t-1} + Y_t \Rightarrow B_t = (1+r)B_{t-1} + TB_t$$

Let $TB = -\alpha r B_{t-1}$ with $\alpha < 1$.

$$B_t = (1 + r - \alpha r)B_{t-1}$$

If $B_{-1} < 0$ then $B_t < 0 \ \forall t$

$$B_t = (1 + r(1-\alpha))^t B_{-1}$$

then

$$CA_t = B_t - B_{t-1} = r(1-\alpha)B_{t-1} < 0$$

Is this feasible? Under what conditions?

$$\lim_{t\to\infty} \frac{B_t}{(1+r)^t} = \frac{(1+r-\alpha r)^t}{(1+r)^t} B_{-1} = 0 \Longleftrightarrow \left(\frac{1 + r(1-\alpha)}{1+r}\right) < 1$$

TB grows at rate $r(1-\alpha)$, Y must grow at least at $r(1-\alpha)$ to prevent $C < 0$