The real exchange rate

A. The terms of trade

A static version: 1 period, 2 countries, 2 goods,
Y_1, Y_2
Home: $u(c_{11}, c_{12})$

$$p_1 c_{11} + p_2 c_{12} = p_1 Y_1 \Rightarrow c_{11} + q c_{12} = Y_1 \tag{1}$$

Foreign: $u(c_{21}, c_{22}) \qquad c_{21} + q c_{22} = q Y_2$
Given $\{Y_1, Y_2\}$ we want to determine $\{c_1, c_2, q\}$
$c_{11}: \qquad u_{c_{11}} = \lambda_1$
$c_{12}: \qquad u_{c_{12}} = \lambda_1 q$

$$\frac{u_{c_{12}}}{u_{c_{11}}} = q = \frac{u_{c_{22}}}{u_{c_{21}}}$$

- In general q depends on both supply (the levels of Y_1, Y_2) and demand factors (preferences, the distribution of the endowments ownership across countries, for instance, home residents may own part of the foreign output, etc).
- In a dynamic model it may be very hard to keep track of the distribution of world wealth when trying to determine prices and quantities.
- To avoid this difficulty we can make assumptions such as:
    1. complete asset markets
    2. incomplete markets together with particular specifications of utility (+ production)

$$u_1 = \ln c_{11} + \theta \ln c_{12}, u_2 = \ln c_{21} + \theta \ln c_{22}$$

$$\frac{\theta c_{11}}{c_{12}} = q = \frac{\theta c_{21}}{c_{22}}$$

$$\theta c_{11} = q c_{12} \qquad + \qquad \theta c_{21} = q c_{22}$$

$$\theta \underbrace{(c_{11} + c_{21})}_{Y_1} = q \underbrace{(c_{12} + c_{22})}_{Y_2}$$

$q = \theta \frac{Y_1}{Y_2}$ $\qquad \frac{dq}{d\theta} > 0$

Supply shocks: $Y_1 \uparrow, q \uparrow$ Demand shocks: $\theta \uparrow, q \uparrow$

Key implication: Faster growth (an increase in supply) deteriorates a country's real exchange rate (leads to a real depreciation).

Empirical project: 1. Using the Blanchard-Quah decomposition in order to identify demand and supply shocks examine the effect of supply shocks on the terms of trade

The distribution of consumption across countries

Combining $\theta c_{11} = q c_{12}$ with $c_{11} + q c_{12} = Y_1$ gives $c_{11} = \frac{1}{1+\theta} Y_1$. Using the market clearing condition $c_{11} + c_{21} = Y_1$ gives $c_{21} = \frac{\theta}{1+\theta} Y_1$

$$c_{12} = \frac{1}{1+\theta} Y_2 \qquad c_{22} = \frac{\theta}{1+\theta} Y_2$$
$$c_{11} = \frac{1}{1+\theta} Y_1 \qquad c_{21} = \frac{\theta}{1+\theta} Y_1$$

If $\theta > 1$ and if $Y_1 = Y_2 \quad \Rightarrow \quad q > 1$. Country 2 is richer.

$$c_{21} > c_{11} \quad \text{and } c_{22} > c_{12}$$

Exercise: Let $u = c_1$, $u^\star = c_2$ with

$$c_1 = \left[\gamma^{\frac{1}{\theta}} c_{11}^{\frac{\theta-1}{\theta}} + (1-\gamma)^{\frac{1}{\theta}} c_{12}^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}} \qquad \theta > 0$$

$$c_2 = \left[\gamma^{\frac{1}{\theta}} c_{21}^{\frac{\theta-1}{\theta}} + (1-\gamma)^{\frac{1}{\theta}} c_{22}^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}} \qquad \theta > 0$$

Note that there is no home bias in consumption.
Determine the terms of trade $q = p_2/p_1$.

Dynamic extension

- 2 period
- storable goods Y_{1t}, Y_{2t}

Question: How does the relative price in any period respond to changes in current or future economic conditions?
For instance, describe the following effects
dq_t/dY_{t+1}, $dq_t/d\theta_{t+1}$ (for the log) and $dq_t/d\gamma_{t+1}$ (for the CES)
Project: There is a popular claim that countries with a strong CA position also have a strong real exchange rate (terms of trade). Use the 2 country, 2 good, 2 period model with various shocks to derive the correlation between the CA and q. Under what conditions do you get this positive correlation? Do these correlations imply anything for international trade competitiveness?

B. Non-traded goods and the real exchange rate

$$\text{Real exchange rate} \begin{cases} \frac{p_N}{p_T} \\ \\ \frac{P}{P^\star} \end{cases}$$

The Balassa-Samuelson effect: $\rightarrow$ Productivity growth and real exchange rate appreciation

- Fast growing countries experience a real exchange rate appreciation
- But there are no implications for international competitiveness
- (Does the experience of a country like Greece fit to the B-S framework?)
- $\rightarrow$ Rich countries have higher price levels

Demonstrate the B-S result using the Ricardian model
Assumptions:

- 2 sectors
- 1 factor of production (L)
- perfect L mobility across sectors
- linear production technology ($AP = MP$ =constant)

$$L = L_T + L_N$$
$$Y_T = \alpha_T L_T$$
$$Y_N = \alpha_N L_N$$

Perfect competition: $P = MC \Rightarrow P_T = \alpha_T w$ and $P_N = \alpha_N w$

Observation: $\frac{P_N}{P_T} = \frac{\alpha_N}{\alpha_T}$ That is, the real exchange rate is independent of demand (due to the linearity of production). The price index

$$P = P_T^{(1-\theta)} P_N^{\theta}$$

$$\left.\begin{array}{l} P = P_T \left(\frac{P_N}{P_T}\right)^{\theta} \\ P^{\star} = P_T^{\star} \left(\frac{P_N^{\star}}{P_T^{\star}}\right)^{\theta} \end{array}\right\} \frac{P}{P^{\star}} = \frac{P_T}{P_T^{\star}} \left(\frac{\alpha_N}{\alpha_N^{\star}}\right)^{\theta} \left(\frac{\alpha_T^{\star}}{\alpha_T}\right)^{\theta}$$

$$\left.\begin{array}{l} P_T = P_T^{\star} \\ \alpha_N = \alpha_N^{\star} \end{array}\right\} \begin{array}{l} \alpha_T^{\star} > \alpha_T \\ \frac{1}{\alpha_T} > \frac{1}{\alpha_T^{\star}} \end{array}$$

Then $P > P^{\star}$

- Rich countries are more productive (have lower α_T) $\Rightarrow$ will have higher price levels
- Source of price differences: difference in prices of NON traded goods
  $P_T = \alpha_T w \qquad\qquad P_N = \alpha_N w$
- Similarly, if a country experiences fast productivity growth (typically in manufacturing goods which tend to be traded) then it will experience an appreciation in its real exchange rate (defined either as $\frac{p_N}{p_T} or \frac{P}{P^\star}$)
  $\frac{P_N}{P_T} \uparrow$ and $\frac{P}{P^\star} \uparrow$ as a country grows fast
- Note: If the production function is concave (say, C-D) demand conditions, for instance, the level of government spending matter for the real exchange rate

Empirical project: To what extent the changes in the real exchange rates in the various countries of the EU following monetary integration (EMU) reflect B-S factors? To what extent real appreciations (Greece, Spain,...) represent a worsening in international competitiveness?