Sovereign debt

Case 1: Full commitment to pay

Case 2: Limited commitment to pay

Case 1: State-Contingent Contracts

Case 2: Non-State-Contingent Contracts

Model: Single good, uncertainty, 2 dates

T=1: Trading Assets

T=2: Consumption

$Y_2 = \overline{Y} + \varepsilon$ with $\varepsilon \in \{\underline{\varepsilon} = \varepsilon_1 < \varepsilon_2 < \ldots < \varepsilon_{N-1} < \varepsilon_N = \overline{\varepsilon}\}$, and prob($\varepsilon_i$)=$\pi(\varepsilon_i)$ with $\sum_{i=1}^{N} \pi(\varepsilon_i) = 1$. The shock ε has a mean of zero, is observable and $\underline{\varepsilon}$ is such that $\overline{Y} + \underline{\varepsilon} > 0$.

Agents can contract with risk neutral competitive foreign insurers.

Contracts deliver $P(\varepsilon)$ on date 2 so that: $C = \bar{Y} + \epsilon - P(\epsilon)$

- $P(\varepsilon) < 0$: insurers pay
- $P(\varepsilon) > 0$: insurers receive

Risk neutrality + perfect competition imply that profits are

$$\sum_{i=1}^{N} \pi(\varepsilon_i) P(\varepsilon_i) = 0$$

Payment is an issue for the country if $P(\varepsilon) > 0$. This raises the question of *Credibility*.

CASE 1: Full Commitment

- A simple example: $Y_2 = \{Y_{21}, Y_{22}\}$
- $Y_{21} = \bar{Y} + \epsilon$, $Y_{22} = \bar{Y} - \epsilon$, $Prob(\epsilon > 0) = 0.5$
- Schedule of payments,P_1, P_2. Zero profit condition and risk neutrality on the part of the insurers means that $P_1 + P_2 = 0 \Rightarrow P_1 = -P_2 = P$.
- $\max Eu(c) = 0.5u(\bar{Y} + \epsilon - P) + 0.5u(\bar{Y} - \epsilon + P)$
- Concavity of the utility function implies that $P = \epsilon$ so that $C_{12} = C_{22} = \bar{Y}$. Consumption is independent of the state of nature. Perfect consumption smoothing

The more general case with commitment

$$\max U = \sum_{i=1}^{N} \pi(\varepsilon_i) U(C_i)$$
$$\text{s.t. } C_i = \overline{Y} + \varepsilon_i - P(\varepsilon_i)$$
$$\sum_{i=1}^{N} \pi(\varepsilon_i) P(\varepsilon_i) = 0$$

$$\mathcal{L} = \sum_{i=1}^{N} \pi(\varepsilon_i) \left(U(\overline{Y} + \varepsilon_i - P(\varepsilon_i)) + \mu P(\varepsilon_i) \right)$$

FOC

$$\pi(\varepsilon_i) \left(-U'(C_i) + \mu \right) = 0 \iff U'(C_i) = \mu \forall i = 1, \ldots, N$$

$$C_i = \overline{Y} \text{ and } P(\varepsilon_i) = \varepsilon_i \ \forall i = 1, \ldots, N$$

There is full insurance.

CASE 2: Imperfect commitment to pay

- If the borrower lacks commitment to pay and if international insurers are competitive and the cost of not paying is **zero** then there will be no int'l asset trade.

$$P(\varepsilon_i) = 0 \ \forall i = 1, \dots, N \qquad C_i = \overline{Y} + \varepsilon_i$$

- Zero consumption smoothing/insurance: $C_{2i} = Y_{2i}$
- Suboptimal due to the concavity of utility
- In order to support international international asset trade (debt) we need to introduce a cost of not paying the contracted amount (of default), L. Let it be a function of output: $L = \eta Y_2$ with $\eta \in (0, 1)$.
- Incentive compatibility constraint (pay only when the payment is less than the sanction):

$$P(\varepsilon_i) \leqslant \eta Y_2 = \eta(\overline{Y} + \varepsilon_i)$$

An example with two states

- Let η be sufficiently small as to make the commitment equilibrium with $P = \epsilon$ infeasible.
  $\epsilon > \eta(\bar{Y} + \epsilon)$.
- The maximum payment that the sovereign will make in the good state 1 for fear of sanctions is
  $P = \eta(\bar{Y} + \epsilon) < \epsilon$.
- Let $\epsilon - \eta(\bar{Y} + \epsilon) = m > 0$.
  $C_{21} = \bar{Y} + \epsilon - P = \bar{Y} + \epsilon - \eta(\bar{Y} + \epsilon) = \bar{Y} + m$ and
  $C_{22} = \bar{Y} - m$
- Lower welfare than in the case of commitment as some idiosyncratic risk remains

The more general case

$$\max U = \sum_{i=1}^{N} \pi(\varepsilon_i) U(C_i)$$

$$\text{s.t. } C_i = \overline{Y} + \varepsilon_i - P(\varepsilon_i)$$

$$\sum_{i=1}^{N} \pi(\varepsilon_i) P_i = 0$$

$$P(\varepsilon_i) \leqslant \eta(\overline{Y} + \varepsilon_i)$$

$$\sum_{i=1}^{N} \pi(\varepsilon_i) \left(U(\overline{Y} + \varepsilon_i - P(\varepsilon_i)) + \mu P(\varepsilon_i) \right) + \lambda(\varepsilon_i) \left(\eta(\overline{Y} + \varepsilon_i) - P(\varepsilon_i) \right)$$

FOC

$$-\pi(\varepsilon_i) U'(C_i) + \mu \pi(\varepsilon_i) - \lambda(\varepsilon_i) = 0$$

Slackness condition

$$\lambda(\varepsilon_i) \left(\eta(\overline{Y} + \varepsilon_i) - P(\varepsilon_i) \right) = 0$$

Two possibilities.
The incentive compatibility constraint (ICC) is binding (satisfied with equality), $\lambda(\varepsilon_i) > 0$
ICC is not binding, $\lambda(\varepsilon_i) = 0$.

1. If $\lambda(\varepsilon_i) = 0$, then $P(\varepsilon_i) < \eta(\overline{Y} + \varepsilon_i)$ and

$$u'(C_i) = \mu \ \forall i = 1, \ldots, N$$

2. If $P(\varepsilon_i) = \eta(\overline{Y} + \varepsilon_i) \Longrightarrow \lambda(\varepsilon_i) > 0$

$$U'(C_i) = \mu - \frac{\lambda(\varepsilon_i)}{\pi(\varepsilon_i)} \neq \mu$$

   Imperfect consumption insurance. Consumption is not constant across states of nature. It depends on ε_i

How much consumption smoothing can a sovereign achieve? Guess: The ICC will not bind for low values of ε (because the country receives rather than pays) then

$$\lambda(\varepsilon_i) = 0 \Longrightarrow U'(C_2(\varepsilon_i)) = \mu$$

For low values of ε, period 2 consumption, C_2, is constant. Hence

$$C_2 = \overline{Y} + \varepsilon_i - P(\varepsilon_i) = \text{constant} \Longleftrightarrow P(\varepsilon_i) = \underbrace{\overline{Y} - \text{constant}}_{P_0} + \varepsilon_i$$

Hence

$$P(\varepsilon_i) = P_0 + \varepsilon_i$$

Let $\widetilde{\varepsilon}$ be such that the country is indifferent between paying or not paying (and suffering the sanction)

Default	Pay
$(1-\eta)(\overline{Y}+\widetilde{\varepsilon})$	$\overline{Y}+\varepsilon_i - P(\varepsilon_i) = \overline{Y}+\widetilde{\varepsilon} - P_0 - \widetilde{\varepsilon} = \overline{Y} - P_0$

Indifference implies that

$$(1-\eta)(\overline{Y}+\widetilde{\varepsilon}) = \overline{Y} - P_0 \tag{1}$$

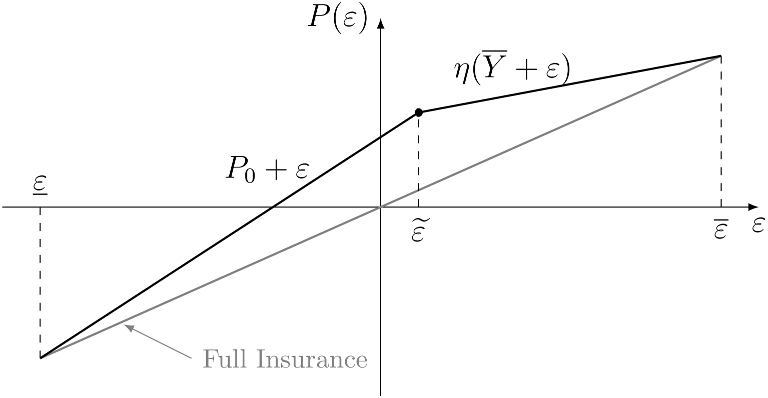
For $\varepsilon > \widetilde{\varepsilon}$ the country will never pay more than the sanction, $\eta(\overline{Y}+\varepsilon_i)$. Hence

$$P(\varepsilon) = \begin{cases} P_0 + \varepsilon & \text{if } \varepsilon \leqslant \widetilde{\varepsilon} \\ \eta(\overline{Y}+\varepsilon) & \text{if } \varepsilon > \widetilde{\varepsilon} \end{cases}$$

$$\int_{\underline{\varepsilon}}^{\widetilde{\varepsilon}} (P_0 + \varepsilon_i)\mathrm{d}f(\varepsilon_i) + \int_{\widetilde{\varepsilon}}^{\overline{\varepsilon}} \eta(\overline{Y}+\varepsilon_i)\mathrm{d}f(\varepsilon_i) = 0 \tag{2}$$

Equations (1)-(2) are two equations in the two unknown, P_0 and $\widetilde{\varepsilon}$.

Figure : Debt Contract



Properties of equilibrium under state contingent contracts: Default incentives stronger during good times

Introduce non-contingent contracts

Sanction: Exclusion from credit markets in case of default in addition to the standard cost of default (output loss)

Example: 2 periods with outstanding debt in the first period

$C_1 = Y_1 - \aleph b_1 - (1 - \aleph) k Y_1 + \aleph q b_2, \qquad C_2 = Y_2 - \aleph b_2$

$\aleph$ is indicator of repayment. Y_2 is known in advance. Let $k_2 = 1 \Rightarrow b_2 \leq Y_2$ so the sovereign always repays in period 2 debt up to Y_2 and $q = \beta$ (risk free loan).

In period 1 if $\aleph < 0$ then $\aleph = 0$

Utility of default and no-default

$D : u(Y_1 - kY_1) + \delta u(Y_2)$

$ND : u(Y_1 - b_1 + q b_2) + \delta u(Y_2 - b_2)$

Assume the borrower is risk neutral.
$D : Y_1 - kY_1 + \delta Y_2$
$ND : Y_1 - b_1 + qb_2 + \delta(Y_2 - b_2) = Y_1 - b_1 + qb_2$
$b_2 = Y_2$ due to the linearity of utility and the fact that $\beta > \delta$.
Default if $b_1 > kY_1 + (\beta - \delta)Y_2$.

- Low current level of income
- Low income growth prospects
- Large outstanding level of debt

For more general treatment see: Eaton and Gersovitz, 1981, Arellano, 2008, Uribe, 2013 ch 8.

A model with investment

$Y_1 = Y_1$, $Y_2 = F(K_2)$, $K_1 = 0$, $F' > 0$,$F" < 0$.

$Y_1 + D - C_1 - K_2 = 0$,

$F(K_2) + K_2 - C_2 - \aleph(1 + r)D - (1 - \aleph)k(F(K_2) + K_2) = 0$

CASE 1: After borrowing the country enjoys discretion over the level of investment.

Given debt, D, and an investment decision, K_2, the debtor defaults if $(1 + r)D < k(F(K_2) + K_2)$.

Given D, optimal investment decision K_2 maximizes

$$u(Y_1+D-K_2)+\delta u(F(K_2)+K_2-min\{(1+r)D, k(F(K_2)+K_2)\})$$

Solve under default and no default, K_2^d and K_2^{nd}. Default if $U(D, K_2^d(D)) > U(D, K_2^{nd}(D))$.

Lenders choose $\bar{D}$, $\bar{D} : U(\bar{D}, K_2^d(\bar{D})) = U(\bar{D}, K_2^{nd}(\bar{D}))$ (No default).

Kinky properties of the solution

Determination of optimal choice of K_2

$\Lambda = u(Y_1 + D - K_2) + \delta u(F(K_2) + K_2 - (1+r)D) - \lambda(D - \bar{D})$

The FOCs are

$$\begin{aligned} u'(C_1) &= (1+r)\delta u'(C_2) + \lambda \\ u'(C_1) &= (1 + F'(K_2))\delta u'(C_2) \\ 0 &= \lambda(\bar{D} - D) \end{aligned}$$

When the borrowing constraint binds ($D = \bar{D}, \lambda > 0$) consumption is tilted towards the future (C_1 is too low). But at the same time, C_2 is also below its level in the absence of default risk.

CASE 2. The country commits to a particular level of investment.
Loan such that: $(1+r)D = k(F(K_2)+K_2)$

$$u(Y_1+D-K_2)+\delta u(F(K_2)+K_2-(1+r)D)-\lambda((1+r)D-k(F(K_2)+K_2))$$

$$\begin{aligned} u'(C_1) &= (1+r)(\delta u'(C_2)+\lambda) \\ u'(C_1) &= (1+F'(K_2))(\delta u'(C_2)+k\lambda) \\ 0 &= \lambda((1+r)D-k(F(K_2)+K_2)) \end{aligned}$$

When the borrowing constraint binds ($\lambda > 0$) consumption is tilted towards the future (C_1 is too low).
The country invests less if there is default risk ($F' > r$) but more relative to the case of no investment commitment. Thus it can receive more funds relative to that case. The ability to tie one's hands helps.

- **Dellas-Niepelt**: A model with official and private creditors
- Probability of sovereign default depends on both the level and the composition of debt
- Higher exposure to official lenders improves incentives to repay but also carries extra costs such as reduced ex post flexibility.

The model accounts for sovereign debt crises:

- official lending to sovereigns takes place only in times of debt distress
- carries a favorable rate
- tends to *displace* private funding
- in the presence of debt overhang the availability of official funding increases the probability of default on outstanding debt

Justification for the key assumption. Club members

The model

$$G_1 = \max_{\aleph_1 \in [0,1],\ (b_2, b_2^e)} u(Y_1 - \aleph_1 b_1 - (1-\aleph_1) L_1 + d_1) + \delta E_1 [G_2)]$$

$$G_2 = \max_{\aleph_2 \in [0,1]} u(Y_2 - \aleph_2 \tilde{b}_2 - (1-\aleph_2)(L_2 + \mathcal{L}(b_2^e))).$$

$$d_1 \equiv b_2 q_1 - b_2^e \Delta_1, \qquad \Delta_1 \equiv q_1 - p_1, \qquad \tilde{b}_2 = b_{02} + b_2 + b_2^e$$

The Choice of Repayment in the First Period
The Choice of Repayment in the Second Period

$$\begin{aligned} r_2 &= 1 \quad if \qquad L_2 \geq \tilde{b}_2 - \mathcal{L}(b_2^e) \\ r_2 &= 0 \quad if \qquad L_2 < \tilde{b}_2 - \mathcal{L}(b_2^e) \end{aligned}$$

The Choice of Debt Issued to Private Lenders: Elasticity of debt offer curve
The Choice of Debt Issued to Official Lenders
Description of equilibria
Corner solutions

Examples:
$u'(c) = 1$, $\mathcal{L}'(b_2^e) = \mathcal{L}'$ with $0 \leq \mathcal{L}' < 1$, and $F_2'(L_2) = f_2$
Exogenous Price Discount, No Long-Term Debt Overhang
$(b_{02}\xi_1 = 0)$
Private creditors

$$b_2^{\mathrm{PR}} = \frac{1}{f_2}\frac{\beta - \delta}{2\beta - \delta}, \quad b_2^{e\,\mathrm{PR}} = 0, \quad G_1^{\mathrm{PR}} = \frac{1}{2f_2}\frac{(\beta - \delta)^2}{2\beta - \delta}.$$

Official creditors only

$$b_2^{\mathrm{OF}} = \frac{1}{f_2}\frac{\beta\kappa - \delta}{2\beta\kappa - \delta(1 - \mathcal{L}')}\frac{1}{1 - \mathcal{L}'}, \quad b_2^{e\,\mathrm{OF}} = b_2^{\mathrm{OF}}$$

$$G_1^{\mathrm{OF}} = \frac{1}{2f_2}\frac{(\beta\kappa - \delta)^2}{2\beta\kappa - \delta(1 - \mathcal{L}')}\frac{1}{1 - \mathcal{L}'}$$

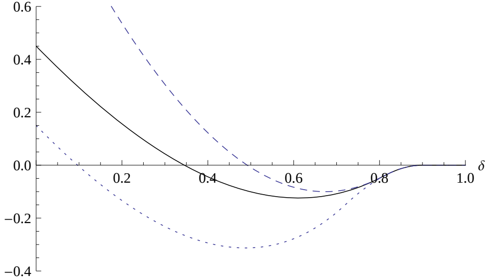


FIGURE : $G_1^{\text{OF}} - G_1^{\text{PR}}$ as function of δ. Higher $\mathcal{L}'$ shifts the curve up (dashed line), lower κ shifts the curve down (dotted line)

Official is favored when financing needs are high (low δ), enforcement is strong (high $\mathcal{L}'$) and official charges are low (high κ).

Exogenous Price Discount, Long-Term Debt Overhang

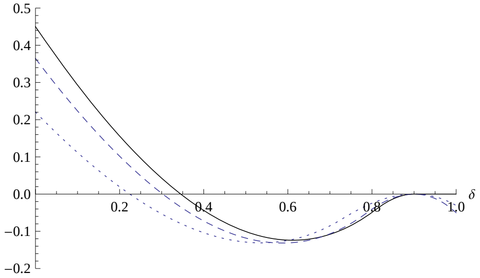


FIGURE : $G_1^{\mathrm{OF}} - G_1^{\mathrm{PR}}$ as function of δ. Higher debt overhang $b_{02}\xi_1$ reduces $\delta^{\star}$.

The sovereign's incentive to choose private debt for refinancing increases with the stock of outstanding long-term debt (Figure 3)

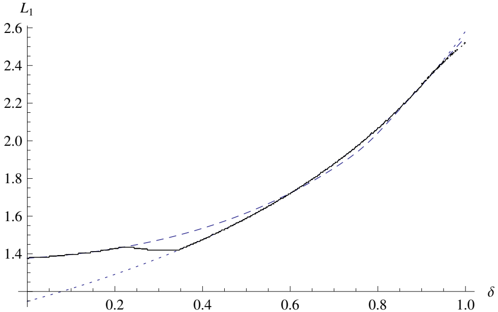


FIGURE : $\hat{L}_1^{\mathrm{PR}}$ (dotted), $\hat{L}_1^{\mathrm{OF}}$ (dashed), $\hat{L}_1$ (solid) as functions of δ.

Default is more likely when official debt is available for fresh funds.

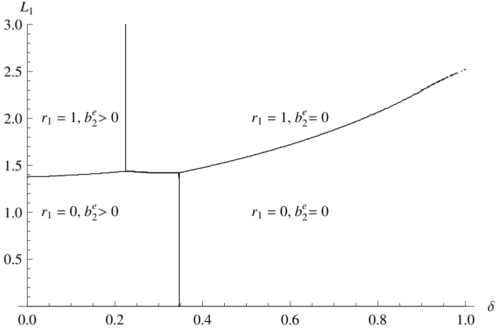


FIGURE : Default and official lending regions.

Interaction of the default decision and debt ownership structure. Variation with δ and L_1 (Figure 5).

- When the country can choose whether to default or not and which source of funds to use (when both private and official sources are available) then default occurs more often (with lower realizations of L_1) relative to the case where only private funds are available.
- The availability of official funding may increase default risk on outstanding debt when refinancing needs are high.
- Interestingly, it may not only be the borrowing country that favors default in these circumstances, but also the official creditors. For they may profit from the debt they buy as long as $\kappa < 1$ and, as a consequence, from a default because it increases the demand for official funds.