Problem Set 6

Department of Economics at the University of Bern

International Monetary Economics

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1. Consider a simple monetary model of the nominal exchange rate. Let M_t denote nominal money supply, P_t, the price level, i_t the nominal net interest rates, Y_t real output, and ε_t the nominal exchange rate (in domestic currency units per foreign currency units). Small letters indicate the logs of the corresponding level-variables and foreign variables are distinguished by asteriks. Furthermore, use $\mathbf{i}_t \equiv log(1 + i_t)$ and $e_t = log(\varepsilon_t)$. Finally, assume $\eta > 0$ and $\phi > 0$. The money demand equation, the purchasing power parity (PPP) and the uncovered interest rate parity (UIP) are given by

$$\frac{M_t}{P_t} = \frac{Y_t^{\phi}}{(1+i_t)^{\eta}} \tag{1}$$

$$P_t = \varepsilon_t P_t^{\star} \tag{2}$$

$$(1+i_t) = (1+i_t^{\star})\mathbb{E}_t\left(\frac{\varepsilon_{t+1}}{\varepsilon_t}\right) \tag{3}$$

(a) Suppose that the exchange rate in time $t+1$ is known for sure as of period t. Express the model in logs.

Solution:

$$m_t - p_t = -\eta \mathbf{i}_t + \phi y_t \tag{4}$$

$$p_t = e_t + p_t^{\star} \tag{5}$$

$$\mathbf{i}_t = \mathbf{i}_t^{\star} + e_{t+1} - e_t \tag{6}$$

(b) Now, suppose that agents have to make a forecast of the exchange rate in time $t+1$. The model in logs is then approximated by

$$m_t - p_t = -\eta \mathbf{i}_t + \phi y_t \tag{7}$$
$$p_t = e_t + p_t^\star \tag{8}$$
$$\mathbf{i}_t = \mathbf{i}_t^\star + \mathbb{E}_t e_{t+1} - e_t \tag{9}$$

Why is this model representation only an approximation of the original model? You may want to resort to the definition of concavity (a function $u(x)$ is strictly concave if $\mathbb{E}(u(x)) < u(\mathbb{E}(x))$ and use $v(\varepsilon_{t+1}) = log\left(\frac{\varepsilon_{t+1}}{\varepsilon_t}\right)$).

Solution: *The model is only an approximation of the original model because*

$$\mathbb{E}_t\left(log\left(\frac{\varepsilon_{t+1}}{\varepsilon_t}\right)\right) < log\left(\mathbb{E}_t\left(\frac{\varepsilon_{t+1}}{\varepsilon_t}\right)\right) \tag{10}$$

The model in equation 9 takes the expected values of the logs (LHS of equation 10) instead of the log of the expected value (RHS of equation 10). The two approaches differ because $v(\varepsilon_{t+1})$ is strictly concave. The approximation is better, the lower the variance of the forecast error.

(c) Let m_t, y_t, and all foreign variables be exogenous variables. Find the solution for e_t. What is the (qualitative) response of e_t to an increase in the nominal money supply m_s?

Solution: *First, combine the model equations to eliminate the endogenous variables.*

$$m_t - (e_t + p_t^\star) = -\eta\left(\mathbf{i}_t^\star + \mathbb{E}_t e_{t+1} - e_t\right) + \phi y_t \tag{11}$$
$$(1+\eta)e_t = m_t - p_t^\star + \eta\left(\mathbf{i}_t^\star + \mathbb{E}_t e_{t+1}\right) - \phi y_t \tag{12}$$
$$e_t = \frac{1}{1+\eta}\left(m_t - p_t^\star + \eta \mathbf{i}_t^\star - \phi y_t\right) + \frac{\eta}{1+\eta}\mathbb{E}_t e_{t+1} \tag{13}$$

Second, iterate forward

$$e_t = \frac{1}{1+\eta} \sum_{s=t}^{\infty} \left(\frac{\eta}{1+\eta} \right)^{s-t} \mathbb{E}_t \left(m_s - p_s^\star + \eta \mathbf{i}_s^\star - \phi y_s \right) \tag{14}$$

e_t increases in m_s, i.e. the nominal exchange rate depreciates in the (current and expected) nominal money supply. A contemporaneous nominal money supply shock has a bigger effect on the nominal exchange rate than a nominal money supply shock which is expected to materialize in the future. The monetary authority can influence the nominal exchange rate by credibly announcing future actions without changing anything today.

(d) What is the (quantitative) response of e_t and $\mathbf{i}_t$ to an increase in the contemporaneous nominal money supply m_t (assuming $m_t \sim i.i.d.$)?

Solution: *First, take the total differential with respect to e_t and m_t on the result of the previous exercise.*

$$de_t = \frac{1}{1+\eta} dm_t \tag{15}$$

The nominal exchange rate rises by $\frac{1}{1+\eta} dm_t$ in m_t. Second, take the total differential on the PPP equation with respect to p_t and e_t.

$$dp_t = de_t \tag{16}$$

$$dp_t = \frac{1}{1+\eta} dm_t \tag{17}$$

Third, take the total differential on the money demand equation *with respect to m_t, p_t and $\mathbf{i}_t$.*

$$dm_t - dp_t = -\eta d\mathbf{i}_t \tag{18}$$

$$dm_t - \frac{1}{1+\eta} dm_t = -\eta d\mathbf{i}_t \tag{19}$$

$$d\mathbf{i}_t = -\frac{1}{1+\eta} dm_t < 0 \tag{20}$$

The nominal interest rate falls by $\frac{1}{1+\eta} dm_t$ in m_t.

(e) What is the (qualitative) response of e_t to a decrease in the nominal interest rate? How do you explain that an interest rate shock induces $corr(e_t, \mathbf{i}_t) > 0$ while a nominal money supply shock implies $corr(e_t, \mathbf{i}_t) < 0$?

Solution: *The interest rate shock is conceptually different from the nominal money supply shock because it does not induce a change in the (exogenous) nominal money supply. Because nominal money supply is exogenous (and constant when we consider an interest rate shock), the money demand equation requires that $corr(p_t, \mathbf{i}_t) > 0$. In other words, a lower nominal interest rate translates into a lower price level. In turn, an decrease in p_t induces an decrease in e_t (an appreciation) via the PPP equation.*
In contrast, the nominal money supply shock affects the (endogenous) interest rate. In this sense, only a nominal money supply shock is in effect expansionary.

(f) Find a way to show that the nominal exchange rate depreciates more when the *change* in the nominal money supply exhibits positive persistence. Assume that $\eta \mathbf{i}_s^\star - p_s^\star - \phi y_s = 0$.[1]

Solution: *We can model positive persistence in the growth of nominal money supply as an AR(1) process with zero mean, $\rho \in$*

[1]cf. Obstfeld, Rogoff, et al. (1996, p. 526-530)

$(0, 1)$, *and constant innovation variance. Formally,*

$$m_t - m_{t-1} = \rho(m_{t-1} - m_{t-2}) + \zeta_t^m \tag{21}$$

with $\mathbb{E}_t \zeta_{t+i}^m = 0 \ \forall i$ *and* $V(\zeta_{t+i}^m) = \sigma_m^2 \ \forall i$. *The expected change in the nominal money supply in period* $t + S$ *is then given by*

$$\mathbb{E}_t \left(m_{t+S} - m_{t+S-1} \right) = p^S \left(m_t - m_{t-1} \right) \tag{22}$$

Lead the solution for e_t *(equation 14) by one period, take expectations as of period* t *and substract the solution for* e_t *to get*

$$\mathbb{E}_t e_{t+1} - e_t = \frac{1}{1+\eta} \sum_{s=t}^{\infty} \left(\frac{\eta}{1+\eta} \right)^{s-t} \mathbb{E}_t \left(m_{s+1} - m_s \right) \tag{23}$$

Combine the last two expressions

$$\mathbb{E}_t e_{t+1} - e_t = \frac{1}{1+\eta} \sum_{s=t}^{\infty} \left(\frac{\eta\rho}{1+\eta} \right)^{s-t} \rho \left(m_t - m_{t-1} \right) \tag{24}$$

$$\mathbb{E}_t e_{t+1} - e_t = \frac{\rho}{1+\eta-\eta\rho} \left(m_t - m_{t-1} \right) \tag{25}$$

$$\mathbb{E}_t e_{t+1} - e_t = \frac{\rho}{1+\eta(1-\rho)} \left(m_t - m_{t-1} \right) \tag{26}$$

Substitute this expression into the original model equation (equation 11) an solve for e_t

$$m_t - e_t = -\frac{\eta\rho}{1+\eta(1-\rho)} \left(m_t - m_{t-1} \right) \tag{27}$$

$$e_t = m_t + \frac{\eta\rho}{1+\eta(1-\rho)} \left(m_t - m_{t-1} \right) \tag{28}$$

It follows that

$$de_t = dm_t + \frac{\eta\rho}{1+\eta(1-\rho)} dm_t \tag{29}$$

$$de_t = \frac{1+\eta}{1+\eta(1-\rho)} dm_t \tag{30}$$

A change in the current nominal money supply has a greater effect on the current nominal exchange rate if the change in the nominal money supply exhibits positive persistence (compared to exercise 1.d. in which we assumed that $m_t \sim i.i.d.$). This is so because a shock to the current nominal money supply does not only increase current nominal money supply but also the expected future nominal money supply.

(g) Suppose that $\rho = 0$. What kind of process does the nominal money supply follow? How does that affect the impact of a nominal money supply shock on the exchange rate? Compare your result to your finding in exercise 1.d.

Solution: *If the change in the nominal money supply exhibits zero persistence, the nominal money supply follows a random walk process in levels. Formally,*

$$m_t = m_{t-1} + \zeta_t^m \tag{31}$$

Using equation 30 with $\rho = 0$, we find that the effect of a nominal money supply shock on the exchange rate is

$$de_t = dm_t \tag{32}$$

i.e. the exchange rate responds one-for-one to the nominal money supply shock. The reason is that an innovation to a random walk process is constant.[2] *The exchange rate is more responsive to a nominal money supply shock, the higher the persistence in the nominal money supply.*

2. According to rational expectations models of the nominal exchange rate, such as the Monetary Model, a increase in the domestic [nominal] money supply is expected to cause an appreciation in the exchange rate, but the exchange rate depreciates [on impact]. Explain why the

[2] The result could have been derived directly from equation 14 with $\mathbb{E}_t m_s = dm_t \ \forall s$

Monetary Model is nonetheless correct (Wickens (2012), chapter 12, exercise 1). Assume, for simplicity, that the domestic nominal money supply follows an *i.i.d.* process.

Solution: *In exercise 1.c., we have shown that the solution for the exchange rate is*

$$e_t = \frac{1}{1+\eta} \sum_{s=t}^{\infty} \left(\frac{\eta}{1+\eta} \right)^{s-t} \mathbb{E}_t \left(m_s - p_s^\star + \eta \mathbf{i}_s^\star - \phi y_s \right) \tag{33}$$

An increase in the domestic nominal money supply induces a depreciation of the exchange rate.[3] *Formally,*

$$de_t = \frac{1}{1+\eta} dm_t \tag{34}$$

Furthermore, an increase in the domestic nominal money supply comes with a fall in the domestic interest rate.[4] *Formally,*

$$d\mathbf{i}_t = -\frac{1}{1+\eta} dm_t \tag{35}$$

Let us now use the uncovered interest rate parity (UIP) to proof that an increase in the domestic nominal money supply causes an expected appreciation of the exchange rate. We can re-express the UIP as

$$\mathbf{i}_t = \mathbf{i}_t^\star + \mathbb{E}_t e_{t+1} - e_t \tag{36}$$

$$e_t = \mathbb{E}_t e_{t+1} + \mathbf{i}_t^\star - \mathbf{i}_t \tag{37}$$

$$e_t = \mathbb{E}_t \sum_{j=0}^{\infty} \left(\mathbf{i}_{t+j}^\star - \mathbf{i}_{t+j} \right) \tag{38}$$

Assume that the domestic interest rate equals the foreign interest rate in steady state, i.e. $\mathbf{i}^\star = \mathbf{i}$. *Furthermore, remember that the nominal money supply shock follows an i.i.d. process. Together with the lack*

[3] cf. exercise 1.d.

[4] cf. exercise 1.d.

of endogenous persistence, this implies that the expected exchange rate equals zero in period $t+1$. Formally,

$$e_t = \frac{1}{1+\eta} dm_t \tag{39}$$

$$\mathbb{E}_t e_{t+1} = 0 \tag{40}$$

Consequently, we have that

$$\mathbb{E}_t e_{t+1} - e_t < 0 \tag{41}$$

i.e. an expected appreciation. In conclusion, we have shown that the monetary model of the nominal exchange rate implies 1) that the exchange rate depreciates in response to a domestic nominal money supply shock ("impact effect") and 2) that the exchange rate is expected to appreciate after a nominal money supply shock ("dynamics back to steady state").

3. Consider the following model of the nominal exchange rate. $\bar{y}$ denotes the output level consistent with stable prices, z_t is a zero mean money supply shock, bars indicate constant variables, and foreign variables are assumed to be zero at all times. All remaining notation and interpretation is as in the previous exercises.

$$\bar{m} + z_t - p_t = \phi y_t^d - \eta \mathbf{i}_t \tag{42}$$

$$\mathbf{i}_t = \mathbf{i}_t^\star + \mathbb{E}_t e_{t+1} - e_t \tag{43}$$

$$p_t - p_{t-1} = \pi(y_t^d - \bar{y}) \tag{44}$$

$$y_t^d = \delta(e_t + p_t^\star - p_t) \tag{45}$$

The aim of the exercise is to re-express the model such that it is compatible with standard procedures for solving dynamic stochastic general equilibrium (DSGE) models on a computer.

(a) Provide an economic interpretation of the last two model equations.

__Solution:__ The third model equation suggests that prices are adjusted upwards (downwards) if the current output y_t^d is above (below) the steady state output $\bar{y}$. It may be interpreted as a Phillips curve relationship. The fourth model equation determines the current output y_t^d as a (positive) function of the real exchange rate. In other words, domestic output is higher, the lower the real value of the domestic currency is, compared to the value suggested by the purchasing power parity.

(b) Reduce the model to a two-equation model in the endogenous variables e_t and p_t.

__Solution:__ Combine the money demand equation (MD), the uncovered interest rate parity (UIP) and the Phillips curve (PC) to obtain

$$\bar{m} + z_t - p_t = \phi y_t^d - \eta(\mathbb{E}_t e_{t+1} - e_t) \tag{46}$$

$$\mathbb{E}_t e_{t+1} - e_t = \frac{1}{\eta}\left(\phi y_t^d + p_t - \bar{m}\right) - \frac{z_t}{\eta} \tag{47}$$

$$\mathbb{E}_t e_{t+1} - e_t = \frac{1}{\eta}\left(\phi\delta(e_t - p_t) + p_t - \bar{m}\right) - \frac{z_t}{\eta} \tag{48}$$

Combine the PC and the y_t^d-equation to obtain

$$p_t - p_{t-1} = \pi(\delta(e_t - p_t) - \bar{y}) \tag{49}$$

(c) Solve for the steady state values e_{ss} and p_{ss}.

__Solution:__ In steady state, all variables are constant. More specif-

ically, $\mathbb{E}_t e_{t+1} = e_t$*,* $p_t = p_{t-1}$*, and* $z_t = 0$*. Hence*

$$0 = \frac{1}{\eta}\left(\phi\delta(e_{ss} - p_{ss}) + p_{ss} - \bar{m}\right) \tag{50}$$

$$0 = \pi(\delta(e_{ss} - p_{ss}) - \bar{y}) \tag{51}$$

This is a problem with two equations and two unknowns. The steady state values are given by

$$e_{ss} = \bar{m} + \frac{1 - \phi\delta}{\delta}\bar{y} \tag{52}$$

$$p_{ss} = \bar{m} - \phi\bar{y} \tag{53}$$

(d) Solve for $\bar{m}$ and $\bar{y}$ in terms of the two steady state values.

Solution: *Combine the solutions for the two steady state values from exercise c. to obtain*

$$e_{ss} - \frac{1 - \phi\delta}{\delta}\bar{y} = p_{ss} + \phi\bar{y} \tag{54}$$

$$\bar{y} = -\delta(p_{ss} - e_{ss}) \tag{55}$$

$$\bar{y} = \delta(e_{ss} - p_{ss}) \tag{56}$$

Plug this result into equation 52

$$e_{ss} = \bar{m} + (1 - \phi\delta)(e_{ss} - p_{ss}) \tag{57}$$

$$\bar{m} = (1 - \phi\delta)p_{ss} + \phi\delta e_{ss} \tag{58}$$

(e) Re-express the model in log-deviations from steady state.

Solution: *First, replace* $\bar{m}$ *and* $\bar{y}$ *in the model equations of exercise b. with the expressions derived in exercise d. Then add and subtract steady state values. Let hats denote log-deviations from* *steady state. For the exchange rate equation:*

$$\mathbb{E}_t e_{t+1} - e_t = \frac{1}{\eta}\left(\phi\delta(e_t - p_t) + p_t - (1-\phi\delta)p_{ss} - \phi\delta e_{ss}\right) - \frac{z_t}{\eta} \tag{59}$$

$$\mathbb{E}_t e_{t+1} - e_t = \frac{1}{\eta}\left(\phi\delta e_t + (1-\phi\delta)p_t - (1-\phi\delta)p_{ss} - \phi\delta e_{ss}\right) - \frac{z_t}{\eta} \tag{60}$$

$$\mathbb{E}_t \hat{e}_{t+1} - \hat{e}_t = \frac{1}{\eta}\left(\phi\delta\hat{e}_t + (1-\phi\delta)\hat{p}_t\right) - \frac{z_t}{\eta} \tag{61}$$

For the price level equation:

$$p_t - p_{t-1} = \pi(\delta(e_t - p_t) - \delta(e_{ss} - p_{ss}) \tag{62}$$

$$\hat{p}_t - \hat{p}_{t-1} = \pi\delta(\hat{e}_t - \hat{p}_t) \tag{63}$$

Collect terms on both model equations

$$\left(1 + \frac{\phi\delta}{\eta}\right)\hat{e}_t = \mathbb{E}_t\hat{e}_{t+1} - \frac{1-\phi\delta}{\eta}\hat{p}_t + \frac{1}{\eta}z_t \tag{64}$$

$$(1+\pi\delta)\hat{p}_t = \hat{p}_{t-1} + \pi\delta\hat{e}_t \tag{65}$$

We can solve this model using Sims' method, developed in Sims (2002). We will discuss the model dynamics and the computer code in class.

References

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