Sovereign debt

Case 1: Full commitment to pay

Case 2: Limited commitment to pay

Case 1: State-Contingent Contracts

Case 2: Non-State-Contingent Contracts

Model: Single good, uncertainty, 2 dates

T=1: Trading Assets

T=2: Consumption

$Y_2 = \overline{Y} + \varepsilon$ with $\varepsilon \in \{\underline{\varepsilon} = \varepsilon_1 < \varepsilon_2 < \ldots < \varepsilon_{N-1} < \varepsilon_N = \overline{\varepsilon}\}$, and prob($\varepsilon_i$)=$\pi(\varepsilon_i)$ with $\sum_{i=1}^{N} \pi(\varepsilon_i) = 1$. The shock ε has a mean of zero, is observable and $\underline{\varepsilon}$ is such that $\overline{Y} + \underline{\varepsilon} > 0$.

Agents can contract with risk neutral competitive foreign insurers.

State contingent contract delivers $P(\varepsilon)$ at date 2 so that: $C = \bar{Y} + \epsilon - P(\epsilon)$

- $P(\varepsilon) < 0$: insurers pay
- $P(\varepsilon) > 0$: insurers receive

Risk neutrality + perfect competition imply that profits are

$$\sum_{i=1}^{N} \pi(\varepsilon_i) P(\varepsilon_i) = 0$$

Payment is an issue for the country if $P(\varepsilon) > 0$. This raises the question of *Credibility*.

CASE 1: Full Commitment

- A simple example: $Y_2 = \{Y_{21}, Y_{22}\}$
- $Y_{21} = \bar{Y} + \epsilon$, $Y_{22} = \bar{Y} - \epsilon$, $Prob(\epsilon > 0) = 0.5$
- Schedule of payments,P_1, P_2. Zero profit condition and risk neutrality on the part of the insurers means that $P_1 + P_2 = 0 \Rightarrow P_1 = -P_2 = P$.
- $\max Eu(c) = 0.5u(\bar{Y} + \epsilon - P) + 0.5u(\bar{Y} - \epsilon + P)$
- Concavity of the utility function implies that $P = \epsilon$ so that $C_{12} = C_{22} = \bar{Y}$. Consumption is independent of the state of nature. Perfect consumption smoothing

The more general case with commitment

$$\max U = \sum_{i=1}^{N} \pi(\varepsilon_i) U(C_i)$$
$$\text{s.t. } C_i = \overline{Y} + \varepsilon_i - P(\varepsilon_i)$$
$$\sum_{i=1}^{N} \pi(\varepsilon_i) P(\varepsilon_i) = 0$$

$$\mathcal{L} = \sum_{i=1}^{N} \pi(\varepsilon_i) \left(U(\overline{Y} + \varepsilon_i - P(\varepsilon_i)) + \mu P(\varepsilon_i) \right)$$

FOC

$$\pi(\varepsilon_i) \left(-U'(C_i) + \mu \right) = 0 \iff U'(C_i) = \mu \forall i = 1, \ldots, N$$

$$C_i = \overline{Y} \text{ and } P(\varepsilon_i) = \varepsilon_i \ \forall i = 1, \ldots, N$$

There is full insurance.

CASE 2: Imperfect commitment to pay

- If the borrower lacks commitment to pay and if international insurers are competitive and the cost of not paying is **zero** then there will be no int'l asset trade.

$$P(\varepsilon_i) = 0 \ \forall i = 1, \ldots, N \qquad C_i = \overline{Y} + \varepsilon_i$$

- Zero consumption smoothing/insurance: $C_{2i} = Y_{2i}$
- Suboptimal due to the concavity of utility
- In order to support international international asset trade (debt) we need to introduce a cost of not paying the contracted amount (of default), L. Let it be a function of output: $L = \eta Y_2$ with $\eta \in (0, 1)$.
- Incentive compatibility constraint (pay only when the payment is less than the sanction):

$$P(\varepsilon_i) \leqslant \eta Y_2 = \eta(\overline{Y} + \varepsilon_i)$$

An example with two states

- Let η be sufficiently small as to make the commitment equilibrium with $P = \epsilon$ infeasible.
  $\epsilon > \eta(\bar{Y} + \epsilon)$.
- The maximum payment that the sovereign will make in the good state 1 for fear of sanctions is
  $P = \eta(\bar{Y} + \epsilon) < \epsilon$.
- Let $\epsilon - \eta(\bar{Y} + \epsilon) = m > 0$.
  $C_{21} = \bar{Y} + \epsilon - P = \bar{Y} + \epsilon - \eta(\bar{Y} + \epsilon) = \bar{Y} + m$ and
  $C_{22} = \bar{Y} - m$
- Lower welfare than in the case of commitment as some idiosyncratic risk remains

The more general case

$$\max U = \sum_{i=1}^{N} \pi(\varepsilon_i) U(C_i)$$

$$\text{s.t. } C_i = \overline{Y} + \varepsilon_i - P(\varepsilon_i)$$

$$\sum_{i=1}^{N} \pi(\varepsilon_i) P_i = 0$$

$$P(\varepsilon_i) \leqslant \eta(\overline{Y} + \varepsilon_i)$$

$$\sum_{i=1}^{N} \pi(\varepsilon_i) \left(U(\overline{Y} + \varepsilon_i - P(\varepsilon_i)) + \mu P(\varepsilon_i) \right) + \lambda(\varepsilon_i) \left(\eta(\overline{Y} + \varepsilon_i) - P(\varepsilon_i) \right)$$

FOC

$$-\pi(\varepsilon_i) U'(C_i) + \mu \pi(\varepsilon_i) - \lambda(\varepsilon_i) = 0$$

Slackness condition

$$\lambda(\varepsilon_i) \left(\eta(\overline{Y} + \varepsilon_i) - P(\varepsilon_i) \right) = 0$$

Two possibilities.
The incentive compatibility constraint (ICC) is binding (satisfied with equality), $\lambda(\varepsilon_i) > 0$
ICC is not binding, $\lambda(\varepsilon_i) = 0$.

1. If $\lambda(\varepsilon_i) = 0$, then $P(\varepsilon_i) < \eta(\overline{Y} + \varepsilon_i)$ and

$$u'(C_i) = \mu \ \forall i = 1, \dots, N$$

2. If $P(\varepsilon_i) = \eta(\overline{Y} + \varepsilon_i) \implies \lambda(\varepsilon_i) > 0$

$$U'(C_i) = \mu - \frac{\lambda(\varepsilon_i)}{\pi(\varepsilon_i)} \neq \mu$$

Imperfect consumption insurance. Consumption is not constant across states of nature. It depends on ε_i

How much consumption smoothing can a sovereign achieve? Guess: The ICC will not bind for low values of ε (because the country receives rather than pays) then

$$\lambda(\varepsilon_i) = 0 \Longrightarrow U'(C_2(\varepsilon_i)) = \mu$$

For low values of ε, period 2 consumption, C_2, is constant. Hence

$$C_2 = \overline{Y} + \varepsilon_i - P(\varepsilon_i) = \text{constant} \Longleftrightarrow P(\varepsilon_i) = \underbrace{\overline{Y} - \text{constant}}_{P_0} + \varepsilon_i$$

Hence

$$P(\varepsilon_i) = P_0 + \varepsilon_i$$

Let $\widetilde{\varepsilon}$ be such that the country is indifferent between paying or not paying (and suffering the sanction)

Default	Pay
$(1-\eta)(\overline{Y}+\widetilde{\varepsilon})$	$\overline{Y}+\varepsilon_i - P(\varepsilon_i) = \overline{Y}+\widetilde{\varepsilon} - P_0 - \widetilde{\varepsilon} = \overline{Y} - P_0$

Indifference implies that

$$(1-\eta)(\overline{Y}+\widetilde{\varepsilon}) = \overline{Y} - P_0 \tag{1}$$

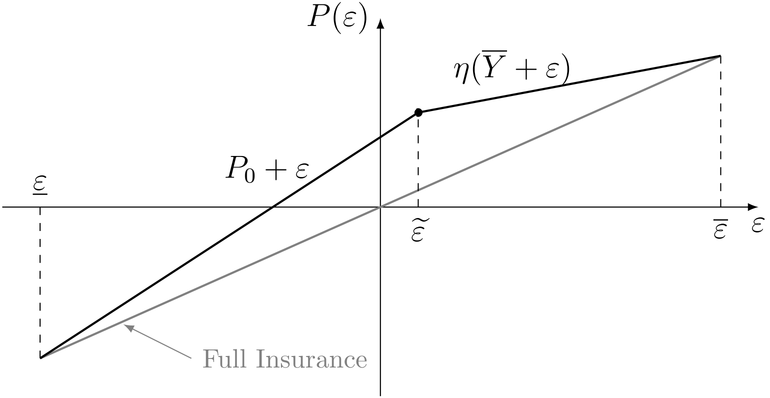
For $\varepsilon > \widetilde{\varepsilon}$ the country will never pay more than the sanction, $\eta(\overline{Y}+\varepsilon_i)$. Hence

$$P(\varepsilon) = \begin{cases} P_0 + \varepsilon & \text{if } \varepsilon \leqslant \widetilde{\varepsilon} \\ \\ \eta(\overline{Y}+\varepsilon) & \text{if } \varepsilon > \widetilde{\varepsilon} \end{cases}$$

$$\int_{\underline{\varepsilon}}^{\widetilde{\varepsilon}} (P_0 + \varepsilon_i)\mathrm{d}f(\varepsilon_i) + \int_{\widetilde{\varepsilon}}^{\overline{\varepsilon}} \eta(\overline{Y}+\varepsilon_i)\mathrm{d}f(\varepsilon_i) = 0 \tag{2}$$

Equations (1)-(2) are two equations in the two unknown, P_0 and $\widetilde{\varepsilon}$.

Figure: Debt Contract



Non-contingent contracts

- Properties of equilibrium under state contingent contracts: Default incentives stronger during good times.
- It seems counterfactual (but according to Tomz and Wright's (2007) many defaults occur during boom periods)
- Can the model produce countercyclical default if debt contract are non-contingent?

Example: 2 periods with outstanding debt in the first period; concave utility

Sanction: Exclusion from credit markets in case of default in addition to the standard output cost of default (k*Y)

$C_1 = Y_1 - \aleph b_1 - (1-\aleph)kY_1 + \aleph q b_2, \qquad C_2 = Y_2 - \aleph b_2$

$\aleph$ is indicator of repayment (= 1 full, = 0 zero repayment).

Y_2 is known in advance. Let $k_2 = 1 \Rightarrow$ can borrow up to $b_2 \leq Y_2$. The sovereign always repays in period 2 and $q = \beta$ (risk free loan).

In period 1 if $\aleph < 1$ then $\aleph = 0$ (due to fixed sanction)

Utility of default and no-default

$D : u(Y_1 - kY_1) + \delta u(Y_2)$

$ND : u(Y_1 - b_1 + qb_2) + \delta u(Y_2 - b_2)$

Assume the borrower is risk neutral.

$D : Y_1 - kY_1 + \delta Y_2$

$ND : Y_1 - b_1 + qb_2 + \delta(Y_2 - b_2) = Y_1 - b_1 + qb_2$

$b_2 = Y_2$ due to the linearity of utility and the fact that $\beta > \delta$.

Default if $b_1 > kY_1 + (\beta - \delta)Y_2$.

- Low current level of income
- Low income growth prospects
- Large outstanding level of debt

For more general treatment see: Eaton and Gersovitz, 1981, Arellano, 2008, Uribe, 2013 ch 8.

A –two period– model with investment
$Y_1 = Y_1$, $Y_2 = F(K_2)$, $K_1 = 0$, $F' > 0$,$F" < 0$.
$Y_1 + D - C_1 - K_2 = 0$,
$F(K_2) + K_2 - C_2 - \aleph(1 + r)D - (1 - \aleph)k(F(K_2) + K_2) = 0$

CASE 1: After borrowing the country enjoys discretion over the level of investment.
Given debt, D, and an investment decision, K_2, the debtor defaults if $(1 + r)D < k(F(K_2) + K_2)$.
Given D, optimal investment decision K_2 maximizes

$$u(Y_1+D-K_2)+\delta u(F(K_2)+K_2-min\{(1+r)D, k(F(K_2)+K_2)\})$$

Solve under default and no default, K_2^d and K_2^{nd}. Default if $U(D, K_2^d(D)) > U(D, K_2^{nd}(D))$.

Lenders choose $\bar{D}$, $\bar{D} : U(\bar{D}, K_2^d(\bar{D})) = U(\bar{D}, K_2^{nd}(\bar{D}))$ (No default).

Kinky properties of the solution

Determination of optimal choice of K_2

$\Lambda = u(Y_1 + D - K_2) + \delta u(F(K_2) + K_2 - (1+r)D) - \lambda(D - \bar{D})$

The FOCs are

$$\begin{aligned} u'(C_1) &= (1+r)\delta u'(C_2) + \lambda \\ u'(C_1) &= (1+F'(K_2))\delta u'(C_2) \\ 0 &= \lambda(\bar{D} - D) \end{aligned}$$

When the borrowing constraint binds ($D = \bar{D}, \lambda > 0$) consumption is tilted towards the future (C_1 is too low). But at the same time, C_2 is also below its level in the absence of default risk.

CASE 2. The country commits to a particular level of investment.
Loan such that: $(1+r)D = k(F(K_2) + K_2)$ k = default cost

$$u(Y_1+D-K_2)+\delta u(F(K_2)+K_2-(1+r)D)-\lambda((1+r)D-k(F(K_2)+K_2))$$

$$\begin{aligned}
u'(C_1) &= (1+r)(\delta u'(C_2)+\lambda) \\
u'(C_1) &= (1+F'(K_2))(\delta u'(C_2)+k\lambda) \\
0 &= \lambda((1+r)D-k(F(K_2)+K_2))
\end{aligned}$$

When the borrowing constraint binds ($\lambda > 0$) consumption is tilted towards the future (C_1 is too low).
The country invests less if there is default risk ($F' > r$) but more relative to the case of no investment commitment. Thus it can receive more funds relative to that case. The ability to tie one's hands helps.

- **Dellas-Niepelt**: A model with official and private creditors
- Probability of sovereign default depends on both the level and the composition of debt
- Higher exposure to official lenders improves incentives to repay but also carries extra costs such as reduced ex post flexibility (repay more often in the future; and suffer a bigger cost when not repaying).

The model accounts for several features of sovereign debt crises:

- official lending to sovereigns takes place in periods of large borrowing needs
- it carries a favorable rate
- in the presence of large debt overhang the availability of official funding increases the probability of default on outstanding debt

Justification for the key assumption (differential enforcement power). Club membership

The model

$$
\begin{aligned}
G_1(b, b^e) &= u(y_1 + qb) + \delta E_1 G_2(b, b^e) \\
G_2(b, b^e) &= \max_{r_2} u(y_2 - br_2 - \Xi r_2 < 1(L_2 + \phi(b^e))
\end{aligned}
$$

Ξx is the indicator function that takes the value of one when choice x has been made and zero otherwise

$$G_1 = u(Y_1 + \beta qb) + \delta \int^{b-\phi(b^e)} u(Y_2 - L - \phi(b^e)) f(L) dL + \delta u(Y_2 - b)(1 - F(b - \phi(b^e$$

Debt price $q = \beta E_1 r_2 = \beta(1 - F(b - \phi(b^e)))$ (creditors are risk neutral and competitive), $F(L) =$ probability of default, $b =$ total and $b^e =$ official debt, ϕ sanction associated with default on official debt.

- The Choice of Repayment in the Second Period

$$
\begin{aligned}
r_2 &= 1 \quad if \qquad L_2 \geq \tilde{b}_2 - \phi(b_2^e) \\
r_2 &= 0 \quad if \qquad L_2 < \tilde{b}_2 - \phi(b_2^e)
\end{aligned}
$$

- The Choice of Debt Issued to Private Lenders: Elasticity of debt offer curve
- The Choice of Debt Issued to Official Lenders

FOCs

- Private, b
- $dG/db = u_1'\beta(1 - F - fb) - \delta u_{2N}'(1 - F)$
-
- Official, b^e
- $u_1'\beta fb\phi' - \delta\phi' \int^{b-\phi} u_{2D}' f dL$

$f = F'$ and Y_2 certain

A simple example with an interior solution

- Two realizations of L , 0 (with 1-π)) and m (with π)
- Cost of default $= L + \phi b^e$
- $u(c) = ln(c)$
- $log(y_1 + \beta\pi b) + \delta\pi log(y_2 - b) + \delta(1-\pi)log(y_2 - \phi b^e)$

$\pi =$ prob of default, $\phi =$ constant

$$\frac{\beta\pi}{y_1 + \beta\pi b} - \frac{\delta\pi}{y_2 - b} - \frac{\delta(1-\pi)}{y_2 + \bar{L}_2 - b} = 0 \tag{3}$$

$$b^e - \frac{b - \bar{L}_2}{\phi} = 0. \tag{4}$$

Properties of equilibrium

- With private only, $max(b) = m = 0.4$
- With official only, $max(b) = m/(1 - f) = 0.57$
- An interior solution with $b, b^e > 0$ and $b > m$
- A numerical example $\beta = 0.9, \delta = 0.5, \pi = 0.6, y_1 = 1, y_2 = 1.5, \bar{L}_2 = 0.4, \phi = 0.3 \rightarrow b = 0.47$ and $b^e = 0.23$.

Intuition: Official gives the debtor to overcome the strict borrowing constraint, m. But because of its higher cost in the case of default, the debtor makes limited use of it.

Long-term debt overhang, b_{02}

- Outstanding in first period, maturing in second
- Let $\tilde{b}_2 \equiv b_2 + b_{02} r_1$

Marginal effect of b_2^e, given b_2

Interaction between debt overhang, refinancing and default choice

- Overhang changes price elasticity of private and official debt, increasing probability of default
- Higher probability of default increases the future cost of *official* funds
- Overhang reduces relative attractiveness of official funds
- When official refinancing is available and credibility very valuable, overhang may increase incentive to default

"Dynamic" default decision in first period (benefits of default accrue in both periods)

- Default wipes out b_1 and b_{02}
- The latter implies direct increases in q_1 and G_2

With larger debt overhang, private debt more likely under no default, even with large borrowing needs

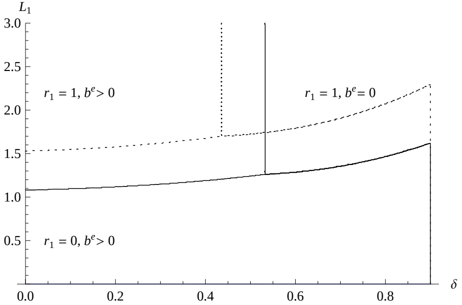


FIGURE: Default and official lending regions with debt overhang