Problem Set 1

Department of Economics at the University of Bern

International Monetary Economics

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1. The relationship between the current account and the financial account: In the absence of debt forgiveness and migrants' transfers, the current account and the financial account sum up to zero.[1] We will proof this formally.

   (a) Set up the household problem with infinitely many periods.[2] Write the dynamic budget constraint (DBC) with d_t being (external) debt (instead of assets).

   (b) Derive the Euler equation.

   (c) Iterate on the DBC and discuss the No ponzi condition in the infinte period model.

   (d) Assume $\beta(1+r) = 1$. What are the implications for the consumption path?

   (e) Re-express the iterated DBC with constant consumption. Define $y_t^P = \frac{r}{1+r}\sum_{j=0}^{\infty}\frac{y_{t+j}}{(1+r)^j}$. What is the interpretation of y_t^P?

   Solution:

$$(1+r)d_{t-1} = \sum_{j=0}^{\infty}\frac{y_{t+j}}{(1+r)^j} - \sum_{j=0}^{\infty}\frac{c_t}{(1+r)^j} \tag{1}$$

$$\frac{1+r}{r}c_t = \sum_{j=0}^{\infty}\frac{y_{t+j}}{(1+r)^j} - (1+r)d_{t-1} \tag{2}$$

$$c_t = \frac{r}{1+r}\sum_{j=0}^{\infty}\frac{y_{t+j}}{(1+r)^j} - rd_{t-1} \tag{3}$$

   y_t^P *is a weighted average of the (expected) endowments over time (the weights given to each period's endowment* $(\frac{r}{1+r}\frac{1}{(1+r)^j})$ *sum up*

[1] cf. ime_slides_1_20170217, p.2.
[2] cf. Math Review, exercise 1.

to unity). If you do not see this, take equation 1 with $s = 1$. Also, assume (without loss of generality) $d_{t-1} = 0$.

$$c_t + \frac{c_{t+1}}{1+r} = y_t + \frac{y_{t+1}}{1+r} \tag{4}$$

$$c_t \frac{2+r}{1+r} = y_t + \frac{y_{t+1}}{1+r} \tag{5}$$

$$c_t = \frac{1+r}{2+r}\left(y_t + \frac{y_{t+1}}{1+r}\right) \tag{6}$$

The weights sum up to unity.

$$\frac{1+r}{2+r} + \frac{1+r}{(2+r)(1+r)} \stackrel{?}{=} 1 \tag{7}$$

$$\frac{1+r}{2+r} + \frac{1}{2+r} \stackrel{?}{=} 1 \tag{8}$$

$$\frac{2+r}{2+r} \stackrel{?}{=} 1 \tag{9}$$

$$1 \stackrel{!}{=} 1 \tag{10}$$

(f) Combine

$$y_t^P = c_t + rd_{t-1} \tag{11}$$

$$y_t + d_t = c_t + (1+r)d_{t-1} \tag{12}$$

to get an expression that relates the change in debt to $y_t^P - y_t$. What is the interpretation of this result?

Solution:

$$y_t^P - y_t = d_t - d_{t-1} \tag{13}$$

Increase borrowing if your current income is below the weighted average of (expected) endowments over time.

(g) Use the definition of the current account $ca_t \equiv tb_t - rd_{t-1}$ and

$tb_t = y_t - c_t$ to show that the current account and the financial account $(d_t - d_{t-1})$ sum up to zero.

Solution:

$$ca_t \equiv tb_t - rd_{t-1} \tag{14}$$
$$ca_t = y_t - (c_t + rd_{t-1}) \tag{15}$$
$$ca_t = y_t - y_t^P \tag{16}$$
$$ca_t = -(d_t - d_{t-1}) \tag{17}$$
$$ca_t + (d_t - d_{t-1}) = 0 \tag{18}$$

2. Two country model with logarithmic utility (Obstfeld, Rogoff, et al. (1996), chapter 1, exercise 2): Consider a pure endowment model with two periods $t = \{1, 2\}$ and two (large open) economies (home and foreign, foreign distinguished by asteriks) with analogous utility functions but different endowments and different time preferences. In equilibrium, the international asset market clears, i.e.

$$S_t + S_t^\star = 0 \ \forall t \tag{19}$$
$$TB_t + TB_t^\star = 0 \ \forall t \tag{20}$$
$$(Y_t - C_t) + (Y_t^\star - C_t^\star) = 0 \ \forall t \tag{21}$$

The utility functions are

$$U_1 = \log C_1 + \beta \log C_2 \tag{22}$$
$$U_1^\star = \log C_1^\star + \beta^\star \log C_2^\star \tag{23}$$

(a) Home receives perishable endowments Y_1 and Y_2. Its initial asset holding is zero $(B_0 = 0)$. Show that the solution for home's period 1 consumption can be written as

$$C_1(Y_1, Y_2; \beta, r) = \frac{1}{1+\beta}\left(Y_1 + \frac{Y_2}{1+r}\right) \tag{24}$$

(b) Find the solution for $S_1(Y_1, Y_2; \beta, r)$

(c) Compute the equilibrium world interest rate (r). Then assume $\beta = \beta^\star$ and comment on what determines the value of $\beta(1+r)$.

Solution: *An equilibrium is defined as a set of prices and quantities which 1) satisfy the agents' optimality condition and 2) clear (all) markets. The market clearing condition (MCC) on the international asset market requires*

$$S_1 + S_1^\star = 0 \tag{25}$$

$$\frac{\beta Y_1}{1+\beta} - \frac{Y_2}{(1+\beta)(1+r)} + \frac{\beta^\star Y_1^\star}{1+\beta^\star} - \frac{Y_2^\star}{(1+\beta^\star)(1+r)} = 0 \tag{26}$$

$$\frac{\beta Y_1}{1+\beta} + \frac{\beta^\star Y_1^\star}{1+\beta^\star} = \frac{Y_2}{(1+\beta)(1+r)} + \frac{Y_2^\star}{(1+\beta^\star)(1+r)} \tag{27}$$

$$1 + r = \frac{\frac{Y_2}{1+\beta} + \frac{Y_2^\star}{1+\beta^\star}}{\frac{\beta Y_1}{1+\beta} + \frac{\beta^\star Y_1^\star}{1+\beta^\star}} \tag{28}$$

For $\beta = \beta^\star$

$$\beta(1+r) = \frac{Y_2 + Y_2^\star}{Y_1 + Y_1^\star} \tag{29}$$

$\beta(1+r)$ *is pinned down by aggregated income over time. Put differently, assuming* $\beta(1+r) = 1$ *corresponds to assuming that aggregated income (across the two countries) is constant over time. If future aggregated income is higher than current aggregated income, both countries want to borrow. Since the MCC on the international asset market has to hold in equilibrium, the interest rate must adjusts accordingly (i.e. must be higher, the greater the difference in* $(Y_2 + Y_2^\star) - (Y_1 + Y_1^\star)$*) to incentivize savings.*

(d) Check that the world interest rate lies between the autarky rates r^A and $r^{A\star}$.

Solution: *Because households are all alike within a country, they can only transfer wealth across periods if they have a foreign counter-party. In other words, they cannot transfer wealth across periods in autarky. Hence*

$$S_1(r^A) = 0 \tag{30}$$

$$\frac{\beta Y_1}{1+\beta} - \frac{Y_2}{(1+\beta)(1+r^A)} = 0 \tag{31}$$

$$(1+r^A) = \frac{Y_2}{\beta Y_1} \tag{32}$$

$$Y_2 = (1+r^A)\beta Y_1 \tag{33}$$

and equivalently for the foreign country.[3] *Replace Y_2 and $Y_2^\star$ in equation 28 with the above expression*

$$1+r = \frac{\frac{(1+r^A)\beta Y_1}{1+\beta} + \frac{(1+r^{A\star})\beta^\star Y_1^\star}{1+\beta^\star}}{\frac{\beta Y_1}{1+\beta} + \frac{\beta^\star Y_1^\star}{1+\beta^\star}} \tag{34}$$

$$1+r = (1+r^A)\frac{\frac{\beta Y_1}{1+\beta}}{\frac{\beta Y_1}{1+\beta} + \frac{\beta^\star Y_1^\star}{1+\beta^\star}} + (1+r^{A\star})\frac{\frac{\beta^\star Y_1^\star}{1+\beta^\star}}{\frac{\beta Y_1}{1+\beta} + \frac{\beta^\star Y_1^\star}{1+\beta^\star}} \tag{35}$$

$$r = r^A\frac{\frac{\beta Y_1}{1+\beta}}{\frac{\beta Y_1}{1+\beta} + \frac{\beta^\star Y_1^\star}{1+\beta^\star}} + r^{A\star}\frac{\frac{\beta^\star Y_1^\star}{1+\beta^\star}}{\frac{\beta Y_1}{1+\beta} + \frac{\beta^\star Y_1^\star}{1+\beta^\star}} \tag{36}$$

Because the world interest rate is a weighted average of the two autarky interest rates, it must lie between them.

(e) Confirm that the country with an autarky interest rate below r will run a current account surplus on date 1 while the one with an autarky rate above r will run a deficit.

Solution: *Since $B_0 = 0$, the current account in period 1 is equal*

[3] Another way to obtain the above result would have been to realize that $S_1(r^A) = 0 \leftrightarrow C_1 = Y_1$, which we could have used in the Euler equation with r^A.

to the trade balance.

$$CA_1 = Y_1 - C_1 \tag{37}$$
$$CA_1 = \frac{\beta Y_1}{1+\beta} - \frac{Y_2}{(1+\beta)(1+r)} \tag{38}$$

Replace Y_1 with $\frac{Y_2}{(1+r^A)\beta}$ (see equation 33).

$$CA_1 = \frac{Y_2}{(1+\beta)(1+r^A)} - \frac{Y_2}{(1+\beta)(1+r)} \tag{39}$$
$$CA_1 = \frac{Y_2}{1+\beta}\left(\frac{1}{1+r^A} - \frac{1}{1+r}\right) \tag{40}$$
$$CA_1 = \frac{Y_2}{1+\beta}\left(\frac{r-r^A}{(1+r^A)(1+r)}\right) \tag{41}$$

Clearly, the current account of the home country is positive if the world interest rate is above the autarky interest rate of the home country (i.e. if $r > r^A$).
Suppose, for the sake of the argument, that $\beta = \beta^\star$ and $\beta(1+r) = 1$ (that is, suppose that aggregated income across countries is constant over time). If $r > r^A$, it must be that $\beta(1+r^A) < 1$, i.e. the current endowment in the home country is higher than the future endowment in the home country. As a consequence of strict concavity of the utility function (i.e. the desire to smooth consumption), agents in the home country want to transfer some resources to the future period. They lend on the international asset market.
Since we know that the world interest rate is between the two autarky interest rates, the situation in the foreign country is exactly vice versa. That is, agents in the foreign country want to bring resources into the current period. They do so by borrowing on the international asset market.

(f) How does an increase in foreign's rate of output growth affect home's utility? Observe that a rise in the ratio $Y_2^\star/Y_1^\star$ raises the equilibrium world interest rate. Then show that the derivative of U_1 with respect to r is

$$\frac{\mathrm{d}U_1}{\mathrm{d}r} = \frac{\beta}{1+r}\left[\frac{r - r^A}{(1+r) + \beta(1+r^A)}\right] \tag{42}$$

What is your conclusion?

Solution: *First, re-express the IBC*

$$C_1 + \frac{C_2}{1+r} = Y_1 + \frac{Y_2}{1+r} \tag{43}$$
$$C_2 = (1+r)(Y_1 - C_1) + Y_2 \tag{44}$$

then plug it into the (implicit) utility function

$$U_1 = U(C_1, C_2) \tag{45}$$
$$U_1 = U(C_1, (1+r)(Y_1 - C_1) + Y_2) \tag{46}$$

Take the total derivative

$$dU_1 = \left(\frac{\partial U_1}{\partial C_1} - \frac{\partial U_1}{\partial C_2}(1+r)\right) dC_1 + \frac{\partial U_1}{\partial C_2}(Y_1 - C_1)\, dr \tag{47}$$
$$\frac{dU_1}{dr} = \left(\frac{\partial U_1}{\partial C_1} - \frac{\partial U_1}{\partial C_2}(1+r)\right)\frac{dC_1}{dr} + \frac{\partial U_1}{\partial C_2}(Y_1 - C_1) \tag{48}$$

Realize that the term in brackets is the Euler equation, i.e. the term in brackets is zero

$$\frac{dU_1}{dr} = \frac{\partial U_1}{\partial C_2}(Y_1 - C_1) \tag{49}$$
$$\frac{dU_1}{dr} = \beta\frac{1}{C_2}(Y_1 - C_1) \tag{50}$$
$$\tag{51}$$

Use the Euler equation

$$\frac{dU_1}{dr} = \frac{1}{1+r}\frac{Y_1 - C_1}{C_1} \tag{52}$$

Use results from a) and b)

$$\frac{dU_1}{dr} = \frac{1}{1+r}\left[\frac{\frac{\beta Y_1}{1+\beta} - \frac{Y_2}{(1+\beta)(1+r)}}{\frac{1}{1+\beta}\left(Y_1 + \frac{Y_2}{1+r}\right)}\right] \tag{53}$$

$$\frac{dU_1}{dr} = \frac{1}{1+r}\left[\frac{\beta Y_1 - \frac{Y_2}{(1+r)}}{Y_1 + \frac{Y_2}{1+r}}\right] \tag{54}$$

$$\frac{dU_1}{dr} = \frac{\beta}{1+r}\left[\frac{Y_1 - \frac{Y_2}{\beta(1+r)}}{Y_1 + \frac{Y_2}{1+r}}\right] \tag{55}$$

$$\frac{dU_1}{dr} = \frac{\beta}{1+r}\left[\frac{(1+r)Y_1 - \frac{Y_2}{\beta}}{(1+r)Y_1 + Y_2}\right] \tag{56}$$

$$\frac{dU_1}{dr} = \frac{\beta}{1+r}\left[\frac{(1+r) - \frac{Y_2}{\beta Y_1}}{(1+r) + \frac{Y_2}{Y_1}}\right] \tag{57}$$

Use results from d)

$$\frac{dU_1}{dr} = \frac{\beta}{1+r}\left[\frac{r - r^A}{(1+r) + \beta(1+r^A)}\right] \tag{58}$$

First, note that the the world interest rate increases because of the higher aggregated income in period 2 (cf. exercise c). Furthermore, as seen in exercise e, home runs a current account surplus in period 1 if $r > r^A$. That is, home saves on the international asset market if $r > r^A$. Home benefits (in terms of utility) from a (even) higher r because it is a saver; had it been a borrower (i.e. $r < r^A$), it would have suffered from a (marginal) increase in the world interest rate. We say that an increase in r enhances home's terms of trade if $r > r^A$ and reduces it if $r < r^A$.

References

Obstfeld, M., K. S. Rogoff, et al. (1996): *Foundations of international macroeconomics*, vol. 30. MIT press Cambridge, MA.