Problem Set 2

Department of Economics at the University of Bern

International Monetary Economics

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1. The current account and government spending:[1]

(a) Set up a pure endowment model with two periods $t = \{1, 2\}$, (unproductive) government spending, and zero initial debt. Use a time separable log-utility function.

Solution: *The objective function*

$$U = u(c_1) + \beta u(c_2) \tag{1}$$
$$U = \log c_1 + \beta \log c_2 \tag{2}$$

The dynamic budget constraints (DBC) of the household, with $b_1 = b_1^{int} + b_1^G > 0$ *being an asset*

$$y_1 = c_1 + t_1 + b_1 \tag{3}$$
$$y_2 + (1+r)b_1 = c_2 + t_2 \tag{4}$$

The dynamic budget constraints (DBC) of the government, with $d_1^G > 0$ *being debt*

$$t_1 + d_1^G = g_1 \tag{5}$$
$$t_2 = g_2 + (1+r)d_1^G \tag{6}$$

The intertemporal budget constraint (IBC) of the household and the government

$$y_1 + \frac{y_2}{1+r} = c_1 + \frac{c_2}{1+r} + t_1 + \frac{t_2}{1+r} \tag{7}$$
$$g_1 + \frac{g_2}{1+r} = t_1 + \frac{t_2}{1+r} \tag{8}$$

Combine

$$y_1 + \frac{y_2}{1+r} = c_1 + \frac{c_2}{1+r} + g_1 + \frac{g_2}{1+r} \tag{9}$$

[1] cf. ime_slides_20170217, p. 15.

The market clearing condition (MCC) for (internal) debt[2]

$$d_1^G = b_1^G \tag{10}$$

(b) Derive the Euler equation

Solution:

$$u'(c_1) = \beta(1+r)u'(c_2) \tag{11}$$

$$u'(c_1) = \beta(1+r)u'((1+r)(y_1 - c_1 - g_1) + (y_2 - g_2)) \tag{12}$$

$$c_2 = \beta(1+r)c_1 \tag{13}$$

(c) Use the definition of the current account in period 1 to show that it is equal to the change in external assets (b_1^{int}). Under what conditions is it equal to the change in household assets b_1?

Solution: *By defintion*

$$CA_1 = tb_1 - rb_0 \tag{14}$$

$$CA_1 = y_1 - c_1 - g_1 - rb_0 \tag{15}$$

$$CA_1 = y_1 - c_1 - g_1 \tag{16}$$

Use the DBC of the government

$$CA_1 = y_1 - c_1 - t_1 - d_1^G \tag{17}$$

$$CA_1 + d_1^G = y_1 - c_1 - t_1 \tag{18}$$

Plug the above equation into the DBC of the household

$$CA_1 + d_1^G = b_1 \tag{19}$$

[2]Here we assume that the government cannot access the international asset market.

Use $b_1 = b_1^{int} + b_1^G$ with $b_1^G = d_1^G$.

$$CA_1 + d_1^G = b_1^{int} + b_1^G \tag{20}$$
$$CA_1 = b_1^{int} \tag{21}$$

Since $b_0^{int} = 0$, we have shown that the current account is equal to the change in external assets. The change in external assets is equal to the change in household assets only if $b_1^G = 0$, i.e. only if the (exogenous) variables for government spending and taxation happend to coincide with a balanced budget $t_i = g_i \ \forall i$.

(d) How does the current account in period 1 change if the government increases its spending in period 1?

Solution: *By defintion*

$$CA_1 = tb_1 - rb_0 \tag{22}$$
$$CA_1 = y_1 - c_1 - g_1 - rb_0 \tag{23}$$
$$CA_1 = y_1 - c_1 - g_1 \tag{24}$$
$$\frac{dCA_1}{dg_1} = -\frac{dc_1}{dg_1} - 1 \tag{25}$$

Approach 1: *Using the total differential on the (implicit) Euler equation with $c_2 = (1+r)(y_1 - c_1 - g_1) + (y_2 - g_2)$*

$$u''(c_1)dc_1 = -u''(c_2)\beta(1+r)^2 dc_1 - u''(c_2)\beta(1+r)^2 dg_1 \tag{26}$$
$$\frac{dc_1}{dg_1} = -\frac{u''(c_2)\beta(1+r)^2}{u''(c_1) + u''(c_2)\beta(1+r)^2} \tag{27}$$

Assume $\beta(1+r) = 1$

$$\frac{dc_1}{dg_1} = -\frac{1+r}{1+(1+r)} \tag{28}$$

$$\frac{dc_1}{dg_1} = -\frac{\frac{1}{\beta}}{1+\frac{1}{\beta}} \tag{29}$$

$$\frac{dc_1}{dg_1} = -\frac{1}{1+\beta} < 0 \tag{30}$$

Approach 2: *Using the (explicit) Euler equation in the intertemporal budget constraint, assuming log-utility*

$$c_1 = \frac{1}{1+\beta}\left(y_1 + \frac{y_2}{1+r} - \left(g_1 + \frac{g_2}{1+r}\right)\right) \tag{31}$$

$$\frac{dc_1}{dg_1} = -\frac{1}{1+\beta} < 0 \tag{32}$$

with $|\frac{dc_1}{dg_1}| < 1$. *The effect on the current account is*

$$\frac{dCA_1}{dg_1} = \frac{1}{1+\beta} - 1 \tag{33}$$

$$\frac{dCA_1}{dg_1} = -\frac{\beta}{1+\beta} < 0 \tag{34}$$

with $|\frac{dCA_1}{dg_1}| < 1$.

(e) Suppose the country has no access to the international (asset) market. How is the current account affected by an increase in g_1? How does the response of c_1 change compared to a small open economy model?

(f) Now assume that government spending is productive (i.e. it raises income). In particular, $y_i = A_i g_i^\alpha$ with $\alpha > 0$. What is the effect of a government spending shock on the current account?

(g) Someone tells you that his model (with productive goverment spending) features $\frac{\mathrm{d}CA_1}{\mathrm{d}g_1} > 0$. What would that imply?

Solution: *The current account reacts positively to a (productive) government spending shock if and only if* $\alpha \frac{y_1}{g_1} > 1$*, i.e. if and only if the marginal product of a (productive) government spending is greater than one.*

2. Consumption, savings, investment (Obstfeld, Rogoff, et al. (1996), chapter 1, exercise 3): Assume date 1 home output is a strictly concave function of the capital stock in place multiplied by a productivity parameter.[3] Investment is defined as the change in capital (no depreciation). Time starts at $t = 0$ and ends in $t = 1$.[4]

$$y_1 = a_1 k_1^{\alpha} \tag{35}$$

$$i_0 = k_1 - k_0 \tag{36}$$

(a) Claim: Investment is determined so that the marginal product of capital equals r. Why? Derive the optimality condition formally.

(b) Solve for k_1

Solution:

$$r = \alpha a_1 k_1^{\alpha-1} \tag{37}$$

$$k_1^{1-\alpha} = \frac{\alpha a_1}{r} \tag{38}$$

$$k_1 = \left(\frac{\alpha a_1}{r}\right)^{\frac{1}{1-\alpha}} \tag{39}$$

(c) Find the solution for i_0

[3] Date 0 output is exogenous because it depends on the initial capital stock $k_0 > 0$.
[4] In Obstfeld, Rogoff, et al. (1996), $t = \{1, 2\}$.

Solution:

$$i_0 = k_1 - k_0 \tag{40}$$

$$i_0 = \left(\frac{\alpha a_1}{r}\right)^{\frac{1}{1-\alpha}} - k_0 \tag{41}$$

If $\alpha \in (0,1)$, i.e. if the production function is strictly concave in capital, investment decreases in r.[5]

(d) Assume log-utility and $b_0 = 0$. Show that c_0 can be written as

$$c_0 = \frac{1}{1+\beta}\left(y_0 + \frac{y_1}{1+r} - \left(i_0 + \frac{i_1}{1+r}\right)\right) \tag{42}$$

Solution: *Combining the (two) dynamic budget constraints (DBC), the initial conditions ($k_0 > 0$ and known, $b_0 = 0$), the transversality conditions ($b_2 \leq 0$, $k_2 \leq 0$), the No Ponzi condition ($b_2 \geq 0$) and the constraint on capital ($k_i \geq 0\ \forall i$).*

$$y_0 + k_0 = c_0 + k_1 + b_1 \tag{43}$$

$$y_1 + k_1 + (1+r)b_1 = c_1 \tag{44}$$

Combine

$$y_1 + k_1 + (1+r)(y_0 + k_0 - c_0 - k_1) = c_1 \tag{45}$$

Use $i_0 = k_1 - k_0$ and $i_1 = -k_1$

$$y_1 - i_1 + (1+r)(y_0 - c_0 - i_0) = c_1 \tag{46}$$

$$c_0 + \frac{c_1}{1+r} = y_0 + \frac{y_1}{1+r} - \left(i_0 + \frac{i_1}{1+r}\right) \tag{47}$$

[5] cf. ime_slides_20170217, p. 33.

Use the Euler equation

$$c_0(1+\beta) = y_0 + \frac{y_1}{1+r} - \left(i_0 + \frac{i_1}{1+r}\right) \tag{48}$$

$$c_0 = \frac{1}{1+\beta}\left(y_0 + \frac{y_1}{1+r} - \left(i_0 + \frac{i_1}{1+r}\right)\right) \tag{49}$$

(e) Find the solution for c_0

Solution: *First, re-express the no-arbitrage condtion as*

$$k_1 = \left(\frac{\alpha a_1}{r}\right)^{\frac{1}{1-\alpha}} \tag{50}$$

$$a_1 = \frac{r}{\alpha} k_1^{1-\alpha} \tag{51}$$

$$a_1 k_1^{\alpha} = \frac{r}{\alpha} k_1 \tag{52}$$

Second, re-express the result from d)

$$c_0 = \frac{1}{1+\beta}\left(y_0 + \frac{y_1}{1+r} - (k_1 - k_0) + \frac{k_1}{1+r}\right) \tag{53}$$

$$c_0 = \frac{1}{1+\beta}\left(y_0 + k_0 + \frac{y_1}{1+r} - \frac{r}{1+r}k_1\right) \tag{54}$$

$$c_0 = \frac{1}{1+\beta}\left(y_0 + k_0 + \frac{1}{1+r}\left(y_1 - rk_1\right)\right) \tag{55}$$

$$c_0 = \frac{1}{1+\beta}\left(y_0 + k_0 + \frac{1}{1+r}\left(a_1 k_1^{\alpha} - rk_1\right)\right) \tag{56}$$

$$c_0 = \frac{1}{1+\beta}\left(y_0 + k_0 + \frac{1}{1+r}\left(\frac{r}{\alpha}k_1 - rk_1\right)\right) \tag{57}$$

$$c_0 = \frac{1}{1+\beta}\left(y_0 + k_0 + k_1\frac{r(1-\alpha)}{\alpha(1+r)}\right) \tag{58}$$

Third, use the no-arbitrage condtion

$$c_0 = \frac{1}{1+\beta}\left(y_0 + k_0 + \left(\frac{\alpha a_1}{r}\right)^{\frac{1}{1-\alpha}} \frac{r(1-\alpha)}{\alpha(1+r)}\right) \tag{59}$$

$$c_0 = \frac{1}{1+\beta}\left(y_0 + k_0 + \left(\frac{\alpha}{r}\right)^{\frac{\alpha}{1-\alpha}} \frac{1-\alpha}{1+r} a_1^{\frac{1}{1-\alpha}}\right) \tag{60}$$

(f) Find the solution for s_0

Solution: *By definition*

$$s_0 = y_0 - c_0 \tag{61}$$

$$s_0 = y_0 - \frac{1}{1+\beta}\left(y_0 + k_0 + \left(\frac{\alpha}{r}\right)^{\frac{\alpha}{1-\alpha}} \frac{1-\alpha}{1+r} a_1^{\frac{1}{1-\alpha}}\right) \tag{62}$$

If $\alpha \in (0,1)$, i.e. if the production function is strictly concave in capital, savings increase in r.[6]

[6] cf. ime_slides_20170217, p. 33.

References

Obstfeld, M., K. S. Rogoff, et al. (1996): *Foundations of international macroeconomics*, vol. 30. MIT press Cambridge, MA.