

Problem Set 2

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International Monetary Economics

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1. The current account and government spending:¹

- (a) Set up a pure endowment model with two periods $t = \{1, 2\}$, (unproductive) government spending, and zero initial debt. Use a time separable log-utility function.

Solution: *The objective function*

$$U = u(c_1) + \beta u(c_2) \quad (1)$$

$$U = \log c_1 + \beta \log c_2 \quad (2)$$

The dynamic budget constraints (DBC) of the household, with $b_1 = b_1^{int} + b_1^G$

$$y_1 = c_1 + t_1 + b_1 \quad (3)$$

$$y_2 + (1 + r)b_1 = c_2 + t_2 \quad (4)$$

where $b_1 > 0$ is an asset and $b_1 < 0$ is debt. The dynamic budget constraints (DBC) of the government

$$t_1 + d_1^G = g_1 \quad (5)$$

$$t_2 = g_2 + (1 + r)d_1^G \quad (6)$$

where $d_1^G > 0$ is debt and $d_1^G < 0$ is an asset. The intertemporal budget constraint (IBC) of the household and the government

$$y_1 + \frac{y_2}{1 + r} = c_1 + \frac{c_2}{1 + r} + t_1 + \frac{t_2}{1 + r} \quad (7)$$

$$g_1 + \frac{g_2}{1 + r} = t_1 + \frac{t_2}{1 + r} \quad (8)$$

Combine

$$y_1 + \frac{y_2}{1 + r} = c_1 + \frac{c_2}{1 + r} + g_1 + \frac{g_2}{1 + r} \quad (9)$$

¹cf. ime_slides_20170217, p. 15.

The market clearing condition (MCC) for (internal) debt²

$$d_1^G = b_1^G \quad (10)$$

(b) Derive the Euler equation

Solution:

$$u'(c_1) = \beta(1+r)u'(c_2) \quad (11)$$

$$u'(c_1) = \beta(1+r)u'((1+r)(y_1 - c_1 - g_1) + (y_2 - g_2)) \quad (12)$$

$$c_2 = \beta(1+r)c_1 \quad (13)$$

(c) Use the definition of the current account in period 1 to show that it is equal to the change in external assets (b_1^{int}). Under what conditions is it equal to the change in household assets b_1 ?

Solution: *By definition*

$$CA_1 = tb_1 + rb_0 \quad (14)$$

$$CA_1 = y_1 - c_1 - g_1 + rb_0 \quad (15)$$

$$CA_1 = y_1 - c_1 - g_1 \quad (16)$$

Use the DBC of the government

$$CA_1 = y_1 - c_1 - t_1 - d_1^G \quad (17)$$

$$CA_1 + d_1^G = y_1 - c_1 - t_1 \quad (18)$$

Plug the above equation into the DBC of the household

$$CA_1 + d_1^G = b_1 \quad (19)$$

²Here we assume that the government cannot access the international asset market.

Use $b_1 = b_1^{int} + b_1^G$ with $b_1^G = d_1^G$.

$$CA_1 + d_1^G = b_1^{int} + b_1^G \quad (20)$$

$$CA_1 = b_1^{int} \quad (21)$$

Since $b_0^{int} = 0$, we have shown that the current account is equal to the change in external assets. The change in external assets is equal to the change in household assets only if $b_1^G = 0$, i.e. only if the (exogenous) variables for government spending and taxation happen to coincide with a balanced budget $t_i = g_i \forall i$.

- (d) How does the current account in period 1 change if the government increases its spending in period 1?

Solution: By definition

$$CA_1 = tb_1 + rb_0 \quad (22)$$

$$CA_1 = y_1 - c_1 - g_1 + rb_0 \quad (23)$$

$$CA_1 = y_1 - c_1 - g_1 \quad (24)$$

$$\frac{dCA_1}{dg_1} = -\frac{dc_1}{dg_1} - 1 \quad (25)$$

Approach 1: Using the total differential on the (implicit) Euler equation with $c_2 = (1+r)(y_1 - c_1 - g_1) + (y_2 - g_2)$

$$u''(c_1)dc_1 = -u''(c_2)\beta(1+r)^2dc_1 - u''(c_2)\beta(1+r)^2dg_1 \quad (26)$$

$$\frac{dc_1}{dg_1} = -\frac{u''(c_2)\beta(1+r)^2}{u''(c_1) + u''(c_2)\beta(1+r)^2} \quad (27)$$

Use $\beta(1+r)^2 = -\frac{u''(c_1)}{u''(c_2)}$

$$\frac{dc_1}{dg_1} = -\frac{1+r}{1+(1+r)} \quad (28)$$

$$\frac{dc_1}{dg_1} = -\frac{\frac{1}{\beta}}{1+\frac{1}{\beta}} \quad (29)$$

$$\frac{dc_1}{dg_1} = -\frac{1}{1+\beta} < 0 \quad (30)$$

Approach 2: Using the (explicit) Euler equation in the intertemporal budget constraint, assuming log-utility

$$c_1 = \frac{1}{1+\beta} \left(y_1 + \frac{y_2}{1+r} - \left(g_1 + \frac{g_2}{1+r} \right) \right) \quad (31)$$

$$\frac{dc_1}{dg_1} = -\frac{1}{1+\beta} < 0 \quad (32)$$

with $|\frac{dc_1}{dg_1}| < 1$. The effect on the current account is

$$\frac{dCA_1}{dg_1} = \frac{1}{1+\beta} - 1 \quad (33)$$

$$\frac{dCA_1}{dg_1} = -\frac{\beta}{1+\beta} < 0 \quad (34)$$

with $|\frac{dCA_1}{dg_1}| < 1$.

- (e) Suppose the country has no access to the international (asset) market. How is the current account affected by an increase in g_1 ? How does the response of c_1 change compared to a small open economy model?
- (f) Now assume that government spending is productive (i.e. it raises income). In particular, $y_i = A_i g_i^\alpha$ with $\alpha > 0$. What is the effect of a government spending shock on the current account?
- (g) Someone tells you that his model (with productive government spending) features $\frac{dCA_1}{dg_1} > 0$. What would that imply?

Solution: The current account reacts positively to a (productive) government spending shock if and only if $\alpha \frac{y_1}{g_1} > 1$, i.e. if and only if the marginal product of a (productive) government spending is greater than one.

2. Consumption, savings, investment (Obstfeld, Rogoff, et al. (1996), chapter 1, exercise 3): Assume date 1 home output is a strictly concave function of the capital stock in place multiplied by a productivity parameter.³ Investment is defined as the change in capital (no depreciation). Time starts at $t = 0$ and ends in $t = 1$.⁴

$$y_1 = a_1 k_1^\alpha \quad (35)$$

$$i_0 = k_1 - k_0 \quad (36)$$

- (a) Claim: Investment is determined so that the marginal product of capital equals r . Why? Derive the optimality condition formally.
- (b) Solve for k_1

Solution:

$$r = \alpha a_1 k_1^{\alpha-1} \quad (37)$$

$$k_1^{1-\alpha} = \frac{\alpha a_1}{r} \quad (38)$$

$$k_1 = \left(\frac{\alpha a_1}{r} \right)^{\frac{1}{1-\alpha}} \quad (39)$$

- (c) Find the solution for i_0

³Date 0 output is exogenous because it depends on the initial capital stock $k_0 > 0$.

⁴In Obstfeld, Rogoff, et al. (1996), $t = \{1, 2\}$.

Solution:

$$i_0 = k_1 - k_0 \quad (40)$$

$$i_0 = \left(\frac{\alpha a_1}{r} \right)^{\frac{1}{1-\alpha}} - k_0 \quad (41)$$

If $\alpha \in (0, 1)$, i.e. if the production function is strictly concave in capital, investment decreases in r .⁵

(d) Assume log-utility and $b_0 = 0$. Show that c_0 can be written as

$$c_0 = \frac{1}{1+\beta} \left(y_0 + \frac{y_1}{1+r} - \left(i_0 + \frac{i_1}{1+r} \right) \right) \quad (42)$$

Solution: Combining the (two) dynamic budget constraints (DBC), the initial conditions ($k_0 > 0$ and known, $b_0 = 0$), the transversality conditions ($b_2 \leq 0$, $k_2 \leq 0$), the No Ponzi condition ($b_2 \geq 0$) and the constraint on capital ($k_i \geq 0 \forall i$).

$$y_0 + k_0 = c_0 + k_1 + b_1 \quad (43)$$

$$y_1 + k_1 + (1+r)b_1 = c_1 \quad (44)$$

Combine

$$y_1 + k_1 + (1+r)(y_0 + k_0 - c_0 - k_1) = c_1 \quad (45)$$

Use $i_0 = k_1 - k_0$ and $i_1 = -k_1$

$$y_1 - i_1 + (1+r)(y_0 - c_0 - i_0) = c_1 \quad (46)$$

$$c_0 + \frac{c_1}{1+r} = y_0 + \frac{y_1}{1+r} - \left(i_0 + \frac{i_1}{1+r} \right) \quad (47)$$

⁵cf. ime_slides_20170217, p. 33.

Use the Euler equation

$$c_0(1 + \beta) = y_0 + \frac{y_1}{1 + r} - \left(i_0 + \frac{i_1}{1 + r} \right) \quad (48)$$

$$c_0 = \frac{1}{1 + \beta} \left(y_0 + \frac{y_1}{1 + r} - \left(i_0 + \frac{i_1}{1 + r} \right) \right) \quad (49)$$

(e) Find the solution for c_0

Solution: First, re-express the no-arbitrage condition as

$$k_1 = \left(\frac{\alpha a_1}{r} \right)^{\frac{1}{1-\alpha}} \quad (50)$$

$$a_1 = \frac{r}{\alpha} k_1^{1-\alpha} \quad (51)$$

$$a_1 k_1^\alpha = \frac{r}{\alpha} k_1 \quad (52)$$

Second, re-express the result from d)

$$c_0 = \frac{1}{1 + \beta} \left(y_0 + \frac{y_1}{1 + r} - (k_1 - k_0) + \frac{k_1}{1 + r} \right) \quad (53)$$

$$c_0 = \frac{1}{1 + \beta} \left(y_0 + k_0 + \frac{y_1}{1 + r} - \frac{r}{1 + r} k_1 \right) \quad (54)$$

$$c_0 = \frac{1}{1 + \beta} \left(y_0 + k_0 + \frac{1}{1 + r} (y_1 - r k_1) \right) \quad (55)$$

$$c_0 = \frac{1}{1 + \beta} \left(y_0 + k_0 + \frac{1}{1 + r} (a_1 k_1^\alpha - r k_1) \right) \quad (56)$$

$$c_0 = \frac{1}{1 + \beta} \left(y_0 + k_0 + \frac{1}{1 + r} \left(\frac{r}{\alpha} k_1 - r k_1 \right) \right) \quad (57)$$

$$c_0 = \frac{1}{1 + \beta} \left(y_0 + k_0 + k_1 \frac{r(1 - \alpha)}{\alpha(1 + r)} \right) \quad (58)$$

Third, use the no-arbitrage condition

$$c_0 = \frac{1}{1+\beta} \left(y_0 + k_0 + \left(\frac{\alpha a_1}{r} \right)^{\frac{1}{1-\alpha}} \frac{r(1-\alpha)}{\alpha(1+r)} \right) \quad (59)$$

$$c_0 = \frac{1}{1+\beta} \left(y_0 + k_0 + \left(\frac{\alpha}{r} \right)^{\frac{\alpha}{1-\alpha}} \frac{1-\alpha}{1+r} a_1^{\frac{1}{1-\alpha}} \right) \quad (60)$$

(f) Find the solution for s_0

Solution: By definition

$$s_0 = y_0 - c_0 \quad (61)$$

$$s_0 = y_0 - \frac{1}{1+\beta} \left(y_0 + k_0 + \left(\frac{\alpha}{r} \right)^{\frac{\alpha}{1-\alpha}} \frac{1-\alpha}{1+r} a_1^{\frac{1}{1-\alpha}} \right) \quad (62)$$

If $\alpha \in (0, 1)$, i.e. if the production function is strictly concave in capital, savings increase in r .⁶

⁶cf. ime_slides.20170217, p. 33.

References

OBSTFELD, M., K. S. ROGOFF, ET AL. (1996): *Foundations of international macroeconomics*, vol. 30. MIT press Cambridge, MA.