Problem Set 3

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International Monetary Economics

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1. A two period model with full and limited commitment: The social planner maximizes

$$\max_{C_1,C_2} \big(u(C_1) + \beta u(C_2)\big) \tag{1}$$

subject to the dynamic budget constraints (DBC)

$$Y_1 + d_1 = C_1 \tag{2}$$
$$Y_2 = C_2 + (1+r)d_1 \tag{3}$$

with $d_1 > 0$ ($<$) representing liabilities (assets).

(a) Why must we not include the natural debt limit (NDL) as a constraint in the optimization problem?

(b) Suppose that defaulting comes at a cost of kY_2. Write the optimization problem as a Lagrangian. Comment on the Lagrange multiplier of the new constraint.

Solution: *We (additionally) impose the incentive compatibility constraint (ICC)* $kY_2 \geq (1+r)d_1$ *to capture the fact that the lender knows that the borrower will default on all debt exceeding* kY_2 *(because it is cheaper for the borrower to pay the cost of defaulting than to actually serve the debt). From the borrower's perspective, the ICC is a borrowing constraint.*

$$\mathcal{L} = u(Y_1 + d_1) + \beta u(Y_2 - (1+r)d_1) + \mu(kY_2 - (1+r)d_1) \tag{4}$$

The constraint is less tight if kY_2 *rises. Consequently, we can interpret* μ *as the marginal utility of a marginally less tight constraint. Because a looser constraint cannot make you worse off, it must be that* $\mu \geq 0$.

(c) Derive the Euler equation and the complementary slackness condition.

Solution:

$$u'(C_1) = \beta(1+r)u'(C_2) + \mu(1+r) \tag{5}$$

$$\mu(kY_2 - (1+r)d_1) = 0 \tag{6}$$

with $\mu > 0$ *if* $kY_2 = (1+r)d_1$ *and vice versa.*

(d) What are the implications of $\mu > 0$ for C_1 (compared to a situation of full commitment)?

Solution: *The marginal utility of* C_1 *increases (compared to the full commitment case) if* $\mu > 0$. *Because the felicity function is (by assumption) strictly concave in* C, *a higher marginal utility of consumption is associated to a lower consumption level. Consequently,* C_1 *is lower under limited commitment than under full commitment if the borrowing constraint binds.*

(e) Can the natural debt limit (NDL) ever bind if $\mu > 0$? Why (not)?