International Monetary Economics

Lecture Note 3: International financial markets

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A key reason for int'l asset trade: Consumption smoothing

- over time (borrowing-lending)
- across states of nature (risk sharing).

IMPLICATIONS OF GLOBAL MARKETS

- a) The law of one price (people in different countries face the same asset prices)
- b) Consumption smoothing (people in different countries can pool national consumption risks)
- c) The efficient international allocation of investment (new savings, regardless of where it originates, is allocated to the country with the most productive investment opportunities)

2 periods, period 2 uncertain
Two possible outcomes (states of world) $s = 1, 2$

$$u = u(c_1) + \beta \{\pi(1)u(c_{21}) + \pi(2)u(c_{22})\}$$

$\pi(1)$ = probability of state $s = 1$ occurring
c_{21} = consumption in 2nd period if state $s = 1$ occurs
c_{22} = consumption in 2nd period if state $s = 2$ occurs

- An Arrow-Debreu security is an asset that delivers 1 unit of output in period 2 in state s and zero otherwise (the payoff is state contingent).
- A bond is a non-contingent asset. It delivers $1 + r \; \forall s$. It is equivalent to $1 + r$ units of A-D securities for $s = 1$ and $1 + r$ units of A-D for $s = 2$

Complete asset markets: There exist A-D securities $\forall s$.

Let $Q(s)$ be the period 1 price of an A-D security that delivers 1 unit of the good in the next period if state s materializes and zero otherwise
The individual's budget constraint in period 1 is

$$Y_1 = c_1 + B(1)Q(1) + B(2)Q(2)$$

$B(1)$ = number of securities that pay 1 unit in $s = 1$
$B(2)$ = number of securities that pay 1 unit in $s = 2$
Note that for the individual, B(s) can be either positive (when he gets paid in state s) or negative (when he pays out in state s)
The budget constraint in period 2 is state by state:

$$c_2(1) = Y_2(1) + B(1)$$
$$c_2(2) = Y_2(2) + B(2)$$

We have full insurance if $c_2(1) = c_2(2) = c_2$

Pricing of A-D securities

$$\max_{B(1),B(2)} u[Y_1 - Q(1)B(1) - Q(2)B(2)] + \beta\left\{\pi(1)[Y_2(1) + B1)] + \pi(2)[Y_2(2) + B(2)]\right\}$$

$$Q(s)u'(c_1) = \beta\pi(s)u'(c_2(s)) \qquad (1)$$

A –real– bond costs now 1 unit and pays out $(1+r)$ units in the next period, independent of the state.
In order to replicate this payout, one can buy 2 A-D securities, one that pays in state 1 and the other that pays in state 2.
One needs to buy (1+r) units of each type of these securities in order to get 1+r (recall that 1 unit of the asset pays 1 unit of the good).
How much do they cost? They cost $Q(1)(1+r) + Q(2)(1+r)$.
Arbitrage requires that the price of the real bond and of this portfolio of A-D securities should be the same
$Q(1)(1+r) + Q(2)(1+r) = 1 \Rightarrow Q(1) + Q(2) = 1/(1+r)$.
The interest rate on the real bond can be computed from equation 1. It is
$\frac{1}{(1+r)} = Q(1) + Q(2) = \beta\pi(1)\frac{\beta u_{c_2}(1)}{u_{c_1}} + \beta\pi(2)\frac{u_{c_2}(2)}{u_{c_1}} = \frac{\beta}{u_{c_1}}Eu_{c_2}$

Implications of complete asset markets for international risk sharing, the correlation of cross country consumption movements, the volatility of national consumption relative to the volatility of domestic output etc.

$$U_{c_1} Q(s) = \pi(s) \beta U_{c_2}(s)$$

$$\begin{aligned} 1 &: U_{c_1} Q(1) = \pi(1) \beta U_{c_2}(1) \\ 2 &: U_{c_1} Q(2) = \pi(2) \beta U_{c_2}(2) \end{aligned}$$

$$\frac{Q(1)}{Q(2)} = \frac{\pi(1)}{\pi(2)} \frac{U_{c_2}(1)}{U_{c_2}(2)}$$

If $\frac{Q(1)}{Q(2)} = \frac{\pi(1)}{\pi(2)}$, then $C_2(1) = C_2(2)$ and Arrow–Debreu prices are actuarially fair.

If Arrow–Debreu prices are actuarially fair then countries will insure themselves against all future consumption fluctuations.

Example: $U(C) = \frac{C^{1-\rho}}{1-\rho}$ and $U(C^\star) = \frac{C^{\star\,1-\rho}}{1-\rho}$, then

$$C_2(s) = \left(\frac{\beta\pi(s)}{Q(s)}\right)^{\frac{1}{\rho}} C_1$$

$$C_2^\star(s) = \left(\frac{\beta\pi(s)}{Q(s)}\right)^{\frac{1}{\rho}} C_1^\star$$

The resource constraints in each period and in each state of nature imply

$$C_2(s)+C_2^\star(s) = Y_2^W(s) = \left(\frac{\beta\pi(s)}{Q(s)}\right)^{\frac{1}{\rho}} (C_1+C_1^\star) = \left(\frac{\beta\pi(s)}{Q(s)}\right)^{\frac{1}{\rho}} Y_1^W$$

Hence

$$Q(s) = \pi(s)\beta \left(\frac{Y_2^W(s)}{Y_1^W}\right)^{-\rho} \quad \forall s$$

Observation 1: Under complete asset markets, the distribution of income (wealth) across countries does not matter for asset prices—*i.e.* world income, Y^W, matters, not $(Y_2, Y_2^\star)$.

$$\frac{Q(s_i)}{Q(s_j)} = \left(\frac{Y_2^W(s_i)}{Y_2^W(s_j)}\right)^{-\rho} \frac{\pi(s_i)}{\pi(s_j)}$$

Observation 2: Prices will be actuarially fair if $Y_2^W(s_i) = Y_2^W(s_j)\ \forall i, j$, *i.e.* world output is constant, independent of $s \implies$ NO AGGREGATE WORLD OUTPUT UNCERTAINTY.

Implications of the model under complete asset markets:

$$\frac{C_2(s)}{C_1} = \frac{C_2^\star(s)}{C_1^\star}$$

That is, consumption growth is perfectly correlated across countries

$$\text{corr}(C_{gr}, C_{gr}^\star) > \text{corr}(Y_{gr}, Y_{gr}^\star)$$

The correlation of consumption growth across countries greater than the correlation of outputs
Caveats: The existence of non-traded goods.

A simple numerical example

Case A: No aggregate uncertainty

	Y_H	Y_F	Y_{H+F}	C_H	C_F
R	200	100	300	150	150
NR	100	200	300	150	150

R corresponds to rain, NR to no rain.

Perfect consumption smoothing across states

$(C_H(i) = C_H(j), i, j = RvsNR)$

Perfect international risk sharing

$(C_H(i) = C_F(i), i = R, NR)$

Case B: Aggregate uncertainty

	Y_H	Y_F	Y_{H+F}	C_H	C_F
R	300	100	400	200	200
NR	100	200	300	150	150

Perfect international risk sharing
$(C_H(i) = C_F(i), i = R, NR)$
Imperfect consumption smoothing across states
$(C_H(i) \neq C_H(j), i, j = RvsNR)$
Unlike idiosyncratic risk which can be fully diversified (insured against) aggregate risk cannot be eliminated.