International Monetary Economics

Lecture Note 2: Models of the current account

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Overview

The determination of macroeconomic variables: Y, C, I, CA, B, ToT, ..

- Model 1
  1. Small open economy
  2. Endowments (no production)
  3. A single, perishable good
- Model 2
  1. Small open economy
  2. Production
  3. A single, non-perishable good (investment)
- Model 3
  1. Two (large) economies
  2. Endowments (no production)
  3. A single, perishable good
- Model 4
  1. Two (large) economies
  2. Production
  3. A single, non-perishable good (investment)

- Model 5
  1. Two (large) economies
  2. Endowments (no production)
  3. (Two) Country specific, perishable goods

Assumptions shared across models:

- Abstract from monetary (nominal) considerations
- No uncertainty
- Representative agent
- Full commitment to repay international loans
- No uncertainty
- No credit constraints
- Two periods

A useful starting point: A closed economy

Preferences

$$u(C_1) + \beta u(C_2)$$

The budget constraints

$$\begin{aligned} Y_1 &= C_1 + B_1 \\ Y_2 + (1+r)B_1 &= C_2 + B_2 \rightarrow \\ Y_1 + \frac{Y_2}{1+r} &= C_1 + \frac{C_2}{1+r} \end{aligned}$$

No credit constraint.
$B_2 = 0$
Optimal savings-consumption choice

$$u_{c_1} = \beta(1+r)u_{C_2}$$

The equilibrium

$$B_1 = 0$$
$$C_1 = Y_1$$
$$C_2 = Y_2$$

$r : B_1(r) = 0$

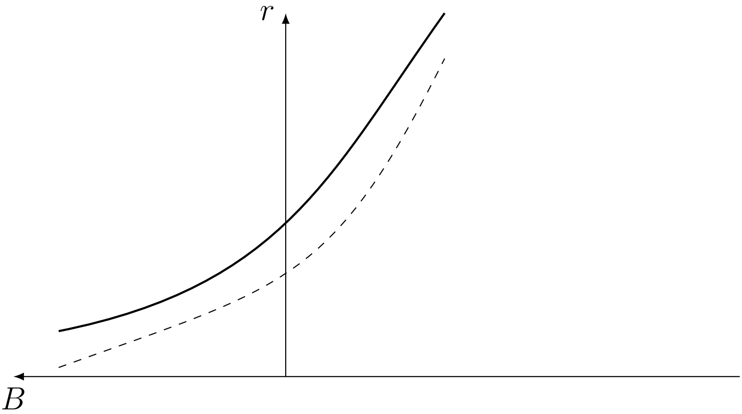
$$u_{c_1}(Y_1) = \beta(1+r)u_{c_2}(Y_2) \Rightarrow r = \frac{u_{c_1}(Y_1)}{\beta u_{c_2}(Y_2)} - 1$$

The General Equilibrium of the model determines the endogenous variables as a function of the exogenous variables:

$\{Y_1, Y_2\} \quad \Rightarrow \quad \{C_1(Y_1), C_2(Y_2), B_1 = 0, r(Y_1, Y_2)\}$

Comparative statics

- $\frac{dc_1}{dY_1} > 0 \quad dC_1 = dY_1 \quad dC_2 = dY_2$
- $\frac{dr}{dY_1} < 0$
- $\frac{dr}{dY_2} > 0$
- $\frac{dr}{d\beta} < 0$


r
B

The Small Open Economy

Small (and committed to repay) means inability to affect r.

$$Y_t + (1+r)B_{t-1} = C_t + B_t$$
$$CA_t = B_t - B_{t-1} = \underbrace{Y_t - C_t}_{TB} + r_{t-1}B_{t-1}$$

Specialize to two periods

$$B_0(1+r) + Y_1 = C_1 + B_1$$
$$(1+r)B_1 + Y_2 = C_2 + B_2 \rightarrowtail$$
$$B_0(1+r) + Y_1 + \frac{Y_2}{1+r} = C_1 + \frac{C_2}{1+r} + B_2$$

$B_2 = 0$

$$u(C_1) + \beta u(C_2)$$

Optimal consumption satisfies the Euler equation

$$u_{c_1} = \beta(1+r)u_{c_2}$$

$c_i = c(B_0, Y_1, Y_2, r)$

$$\begin{aligned} CA_1 = B_1 - B_0 &= Y_1 - C_1 + rB_0 \\ CA_2 = B_2 - B_1 = -B_1 &= Y_2 - C_2 + rB_1 \\ CA_1 + CA_2 = B_1 - B_0 - B_1 &= -B_0 \end{aligned}$$

if $B_0 = 0 \quad \rightarrow \quad CA_1 + CA_2 = 0$
For stationarity in consumption: $\beta(1+r) = 1$
A key property of the model: $u_{c_1} = u_{c_2} \;\Rightarrow\; C_1 = C_2$
Perfect consumption smoothing over time

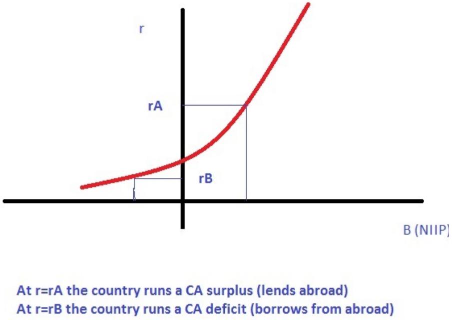


FIGURE : The Current Account

Question: Changes in the economic environment and the CA?

A temporary increase in current output $(Y_1 \uparrow, Y_2 \rightarrow)$

$$CA = B_1 - B_0 = Y_1 - C_1 + rB_0 \rightarrow \frac{dCA}{dY_1} = 1 - \frac{dC_1}{dY_1}$$

$$0 < \frac{dC_1}{dY_1} = \frac{(1+r)U_{c_2c_2}}{u_{c_1c_1} + (1+r)u_{c_2c_2}} < 1$$

The CA_1 improves

A temporary increase in future output $(Y_2 \uparrow, Y_1 \rightarrow)$
From the FOC $0 < \frac{dC_1}{dY_2} < 1$. From the definition of the CA,
$CA = Y_1 - C_1 + rB_0 \Rightarrow \frac{dCA_1}{dY_2} = -\frac{dC_1}{dY_2} < 0$
The CA_1 deteriorates

A permanent increase in output
$Y_1 \uparrow \quad Y_2 \uparrow$: $dY_1 = dY_2 = dY$
From the FOC: $u_{c_1c_1}dC_1 = u_{c_2c_2}dC_2 \Rightarrow dC_1 = dC_2 = dC$.
From the budget constraint: $dC = dY$. Hence
$\frac{dC_1}{dY_1} = 1 \qquad \frac{dCA}{dY} = 0$

Computation of the general equilibrium of the model in a specific case
Preferences

$$u(C_1) + \beta u(C_2) = log(C_1) + \beta log(C_2)$$

subject to

$$(1 + r) + Y_1 + \frac{Y_2}{1 + r} = C_1 + \frac{C_2}{1 + r}$$

FOC

$$C_2 = \beta(1 + r)C_1$$

Solutions
$C_1 = ((1 + r)Y_1 + Y_2)/(1 + \beta)(1 + r))$,
$C_2 = ((1 + r)Y_1 + Y_2)\beta/(1 + \beta)$,
$CA = B = (Y_1(1 + r)\beta - Y_2)/(1 + \beta)(1 + r))$

The big picture: The permanent income hypothesis tells us how changes in income (output) affect consumption-savings decisions and shape the response of the CA.

- $Y_1 \uparrow, Y_2 \rightarrow \Rightarrow B \uparrow CA \uparrow$
- $Y_2 \uparrow, Y_1 \Rightarrow \rightarrow B \downarrow CA \downarrow$
- $Y_1 \uparrow, Y_2 \uparrow dY_1 = dY_2 \Rightarrow B \rightarrow CA \rightarrow$

Exercise: The effects on the CA of output growth:
$Y_1 \uparrow, Y_2 \uparrow dY_2 > dY_1$

Exercise: Redo the analysis in a model which also contains a labor-leisure choice and production (utility is u(c, n) and production is y=f(n) where n is labor).

Government Spending and the Current Account

The households' budget constraints

$$Y_1 = C_1 + T_1 + B$$
$$Y_2 + (1+r)B = C_2 + T_2 \mapsto$$
$$C_1 + \frac{C_2}{1+r} + T_1 + \frac{T_2}{1+r} = Y_1 + \frac{Y_2}{1+r}$$

The government's budget constraint

$$T_1 + B_1^G = G_1$$
$$T_2 = (1+r)B_1^G + G_2 \mapsto$$
$$T_1 + \frac{T_2}{1+r} = G_1 + \frac{G_2}{1+r}$$

Private plus public budget constraints

$$Y_1 + \frac{Y_2}{1+r} = C_1 + \frac{C_2}{1+r} + G_1 + \frac{G_2}{1+r}$$

Taxes do not appear in the constraints faced by the households

Ricardian equivalence: Timing of taxes does not matter.

Assumptions: Perfect credit markets, bequest motives,..

Government spending and the determination of the CA

$U(C_1)+\beta U(C_2) = U(C_1)+\beta U((1+r)Y_1+Y_2-(1+r)C_1-(1+r)G_1-G_2)$

$$U_{C_1} = \beta(1+r)U_{C_2}$$

Assume again $\beta(1+r)$. The effect of a change in G_1 on the CA_1 is
$\frac{\mathrm{d}(CA_1)}{\mathrm{d}G_1} = -\frac{dC_1}{dG_1} - 1.$

$$\frac{\mathrm{d}C_1}{\mathrm{d}G_1} = -\frac{(1+r)U_{C_2C_2}}{U_{C_1C_1} + (1+r)U_{C_2C_2}} < 0 \text{ and } \left|\frac{\mathrm{d}C_1}{\mathrm{d}G_1}\right| < 1$$

$\frac{dCA_1}{dG_1} < 0$. $CA_1 \downarrow (-), CA_2 \uparrow (+)$, $CA_1 + CA_2 = 0$

Government spending and the determination of the CA (cont'ed)

The effect of an expected increase in G_2 on the CA.
$\frac{\mathrm{d}C_1}{\mathrm{d}G_2} < 0$

$$0 = \frac{\mathrm{d}C_1}{\mathrm{d}G_2} + 0 + \frac{\mathrm{d}B}{\mathrm{d}G_2} \Longrightarrow \frac{\mathrm{d}CA_1}{\mathrm{d}G_2} = \frac{\mathrm{d}B}{\mathrm{d}G_2} = -\frac{\mathrm{d}C_1}{\mathrm{d}G_2} > 0$$

- The permanent income hypothesis can be used to understand the effects of changes in the level of government spending on savings and the current account.
- Because government spending is completely useless (from the point of view of the households, an increase in government spending is equivalent to a reduction in their output (income). Hence, temporary increases in G make savings and the CA change (exercise 1).
- The basic picture does not change when government spending is useful (for private consumption and production), as long as an extra unit of government spending does not generate more than one unit of consumption-production gains.

Exercise 1: (a)Determine the effects on the CA of a change in current G when government spending is useful as a consumption substitute, $u(C, G)$. Also speculate on the same relationship when government spending improves the production capacity of the economy, $Y = F(G)$.

Exercise 2: Derive the effects on the CA of a permanent increase in G.

Small open economy with investment

An additional means of saving, physical capital

- Production technology: $Y = F(K)$
- Capital Accumulation: $K_{t+1} = I_t + K_t$, $\delta = 0$
- Budget Constraint:

$$F(K_t) + K_t + (1+r)B_t + (1+r)B_t^G = \\ C_t + T_t + B_{t+1} + B_{t+1}^G + K_{t+1} \Rightarrow \\ Y_t + (1+r)B_t = C_t + T_t + I_t + B_{t+1}$$

The government budget constraint is

$$(1+r)B_t^G + G_t = T_t + B_{t+1}^G$$

Combining the private and government budget constraint gives the current account as:

$$CA_t \equiv B_{t+1} - B_t = Y_t + rB_t - \underbrace{(C_t + I_t + G_t)}_{\text{absorption}}$$

Private savings is $S_t = Y_t + rB_t - C_t - T_t$.
Substituting in the expression for the CA gives the twin deficit expression
$CA_t = S_t - I_t + T_t - G_t$.

Social planning problem

$$\begin{aligned} F(K_1) + K_1 &= C_1 + K_2 + G_1 + B \\ F(K_2) + K_2 + (1+r)B &= C_2 + K_3 + G_2 \mapsto \\ C_1 + I_1 + G_1 + \frac{C_2 + I_2 + G_2}{1+r} &= F(K_1) + \frac{F(I_1 + K_1)}{1+r} \\ &\equiv Y_1 + \frac{Y_2}{1+r} \end{aligned}$$

$I_1 = K_2 - K_1,\ I_2 = K_3 - K_2 = -K_2$

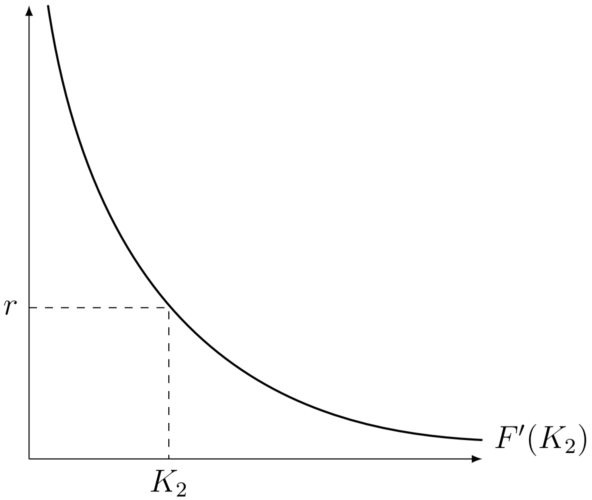
$$U(C_1) + \beta U(C_2) = U(C_1) + \beta U\left((1+r)(F(K_1) - C_1 - G_1 - I_1) + F(I_1 + K_1) - G_2 + \underbrace{I_1 + K_1}_{K_2}\right)$$

$$\begin{aligned} /C_1 &: \ U_{C_1} = \beta(1+r)U_{C_2} \\ /I_1 &: \ \beta U_{C_2}\left(-(1+r) + F_{K_2} + 1\right) = 0 \Longrightarrow F_{K_2} = r \end{aligned}$$

where $F_{K_2} = F'(K_2)$. $F'' dK_2 = dr$ so $dK_2/dr = 1/F'' < 0$.

The role of uncertainty

Figure : Capital choice



$$\left.\begin{array}{l} I_1 \\ K_2 \end{array}\right\} \Longleftarrow r$$

Main points:
Because r is exogenous the desired investment and capital stock are independent of demand conditions (preferences). There is complete separation of savings from investment decisions. This requires the following

1. Small open economy
2. Perfect capital mobility

The Feldstein–Horioka puzzle: A strong positive correlation between national savings and investment rates. If capital is very mobile across countries, then the correlation between savings and investment should be close to zero, as the preceding analysis shows.

Two -large- country model, endowments

$$Y_1 = C_1 + B \tag{1}$$
$$Y_2 + (1+r)B = C_2 \tag{2}$$
$$Y_1^\star = C_1^\star + B^\star \tag{3}$$
$$Y_2^\star + (1+r)B^\star = C_2^\star \tag{4}$$
$$B + B^\star = 0 \tag{5}$$

Combining (1)–(5), we obtain the market clearing conditions for the two outputs

$$Y_1 + Y_1^\star = C_1 + C_1^\star \tag{6}$$
$$Y_2 + Y_2^\star = C_2 + C_2^\star \tag{7}$$

Combining (1)–(2) and (3)–(4)gives the intertemporal budget constraint for each country

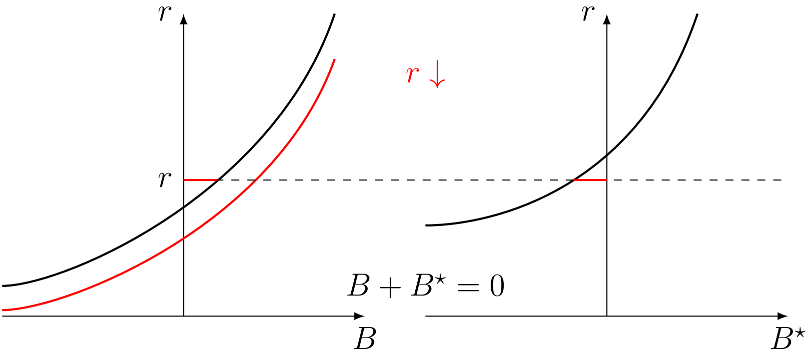
$$Y_1 + Y_2/(1+r) = C_1 + C_2/(1+r) \tag{8}$$
$$Y_1^\star + Y_2^\star/(1+r) = C_1^\star + C_2^\star/(1+r) \tag{9}$$

The FOCs

$$U_{C_1} = \beta(1+r)U_{C_2} \Longrightarrow C_2 = \mathcal{C}_2(C_1, r) \tag{10}$$
$$U^{\star}_{C_1} = \beta(1+r)U^{\star}_{C_2} \Longrightarrow C^{\star}_2 = \mathcal{C}^{\star}_2(C^{\star}_1, r) \tag{11}$$

Equations (6)-(11) are 6 equations in 5 unknowns, $\{C_1, C_2, C^{\star}_1, C^{\star}_2, r\}$.
To solve for the equilibrium of the model we use Walras law: If there are N markets we only need consider the equilibrium ($D = S$) in the N-1 markets. The N-th market will clear automatically when the N-1 markets clear.

Figure : Variation in Y_1, endowment world economy



- What is the main difference between a small and a large economy? In the former case r is exogenous. In the large economy: Smaller variations in the current account as a result of economic disturbances than in the small open economy due to the "dampening effect" of the induced change in r. *Small open economies exhibit greater macroeconomic volatility*.
- Growth and welfare. Is a country hurt by an increase in trading partners' growth rates? In Fig. 3 an increase in Y_2/Y_1 shifts the Home saving curve upwards, increasing world interest rates and making the foreign country (borrower) worse off.

Empirical project: Examine the empirical relationship between country size and macroeconomic volatility.

2 country model with investment

- Domestic technology: $Y = AF(K)$
- Foreign technology: $Y^\star = A^\star F(K^\star)$
- $S = B + I$
- Period $t = 1$: $K_1 + A_1 F(K_1) = C_1 + K_2 + B$ and $K_2 = I_1 + K_1$ imply: $Y_1 = C_1 + I_1 + B$
- Period $t = 2$: $K_2 + A_2 F(K_2) + (1 + r)B = C_2$

$$K_1 + A_1 F(K_1) - C_1 - (I_1 + K_1) = \frac{C_2 - K_1 - I_1 - A_2 F(K_1 + I_1)}{1 + r}$$

The maximization problem: Select C_1, I_1

$$U(C_1)+\beta U\left((1+r)(A_1F(K_1)-C_1-I_1)+A_2F(K_1+I_1)+K_1+I_1\right)$$

FOCs

$$/I_1 \;:\; \beta U'(C_2)\left(-(1+r)+A_2F'(K_2)+1\right)=0 \tag{12}$$

$$/C_1 \;:\; U'(C_1)=\beta(1+r)U'(C_2) \tag{13}$$

$$r = A_2F'(K_2)$$

Arbitrage condition between the two possible investments (I_1, B):

$$r = A_2F'(K_1+I_1) \iff I_1 = \mathcal{I}(\underset{-}{r}, \underset{+}{A_2}, \underset{-}{K_1})$$

Comparative exercises

An increase in current domestic productivity
$A_1 \uparrow$, $A_2 \rightarrow$, and $\{A_1^\star, A_2^\star\} \rightarrow$:

- I_1 curve does not shift
- $Y_1 - C_1$: $A_1 \uparrow \Longrightarrow Y_1 = A_1 F(K_1) \uparrow$. By the permanent income hypothesis, we have $\Delta C_1 < \Delta Y_1$, hence $Y_1 - C_1$ shifts to the right.

The world interest rate decreases. The foreign CA worsens further.

An increase in expected future domestic productivity can be analyzed similarly. Starting from a zero initial CA, the effect on the CA is negative.

FIGURE : An increase in A_1

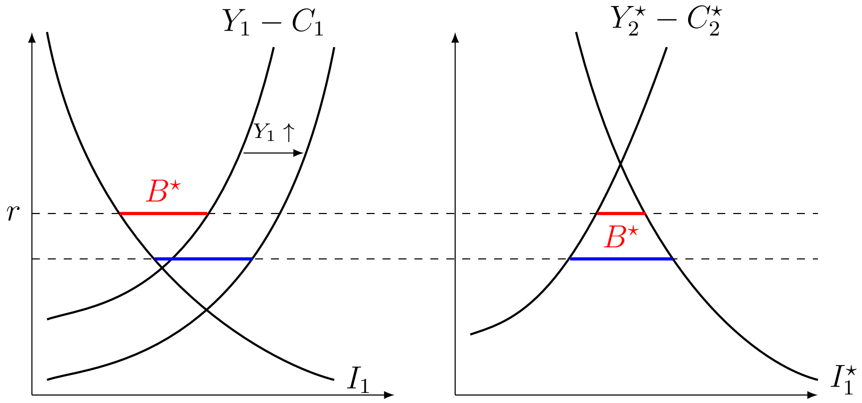
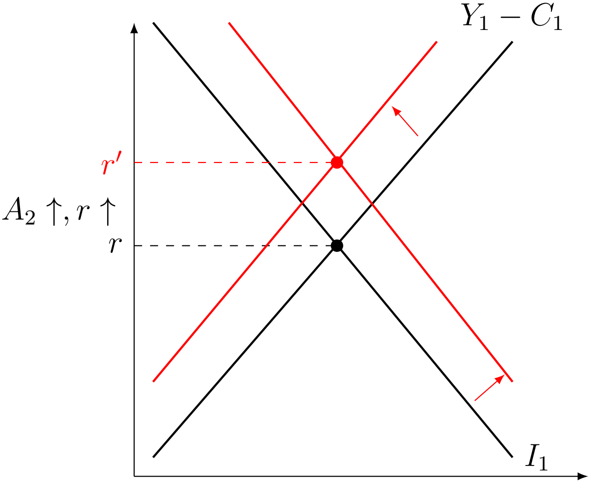


Figure : An increase in A_2



The non-separation of investment from savings
Can the model account for the Felstein-Horioka finding? How?
(Hint: Consider a combination of current and future productivity shocks)
Question: Can the model account for movements in real world interest rates? For instance, why rates were high in 80s? How?
An increase in the expected profitability of investment. It increases r and may increase or decrease I. Under log utility, world I goes down

A multiperiod version

$$\sum_{t=0}^{T} \beta^t U(C_t)$$

subject to

$$(1+r)B_{t-1} + Y_t = C_t + B_t$$

The intertemporal budget constraint

$$(1+r)B_{-1} + \sum_{t=0}^{T} \left(\frac{1}{1+r}\right)^t Y_t = \sum_{t=0}^{T} \left(\frac{1}{1+r}\right)^t C_t + \frac{B_T}{(1+r)^T}$$

Its infinite horizon version

$$(1+r)B_{-1} + \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t Y_t = \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t C_t + \lim_{T\to\infty} \frac{B_T}{(1+r)^T}$$

$$(1+r)B_{-1} + \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t Y_t = \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t C_t$$

because $\lim_{T\to\infty} \frac{B_T}{(1+r)^T} = 0$.

CA deficit sustainability: **Can a country run perpetual current account deficits when $B_{-1} < 0$?**

$$C_t + B_t = (1+r)B_{t-1} + Y_t \Rightarrow B_t = (1+r)B_{t-1} + TB_t$$

Let $TB = -\alpha r B_{t-1}$ with $\alpha < 1$.

$$B_t = (1 + r - \alpha r)B_{t-1}$$

If $B_{-1} < 0$ then $B_t < 0 \; \forall t$

$$B_t = (1 + r(1-\alpha))^t B_{-1}$$

then

$$CA_t = B_t - B_{t-1} = r(1-\alpha)B_{t-1} < 0$$

Is this feasible? Under what conditions?

$$\lim_{t\to\infty} \frac{B_t}{(1+r)^t} = \frac{(1+r-\alpha r)^t}{(1+r)^t} B_{-1} = 0 \iff \left(\frac{1 + r(1-\alpha)}{1+r} \right) < 1$$

TB grows[2] at rate $r(1-\alpha)$, Y must grow at least at $r(1-\alpha)$ to prevent $C < 0$.

[2] $TB_t = -\alpha r B_{t-1}$, $TB_{t-1} = -\alpha r B_{t-2} \Rightarrow \frac{TB_t}{TB_{t-1}} - 1 = r(1-\alpha)$

Figure : Foreign accounts

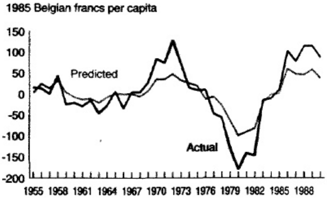


Figure 2.4
Belgium: Actual and predicted current accounts

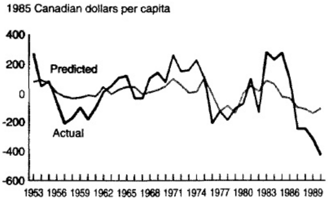


Figure 2.5
Canada: Actual and predicted current accounts

Figure : Foreign accounts

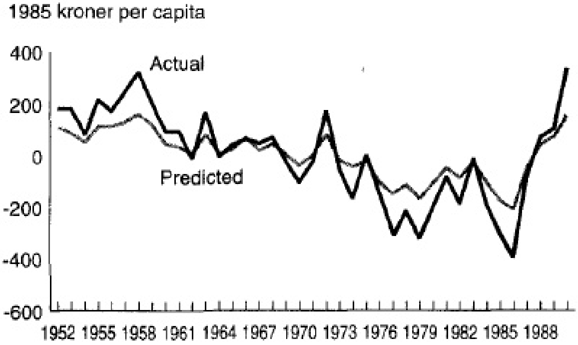


Figure 2.6

Figure : Foreign accounts

95 2.3 A Stochastic Current Account Model

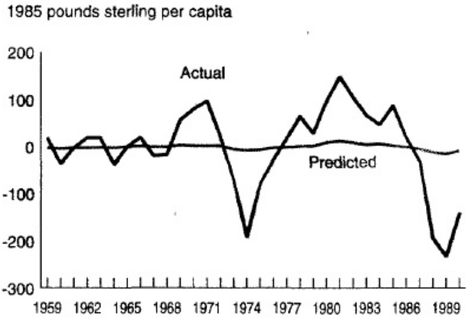


Figure 2.8
United Kingdom: Actual and predicted current accounts