Sovereign debt

Case 1: Full commitment to pay

Case 2: Limited commitment to pay

Case 1: State-Contingent Contracts

Case 2: Non-State-Contingent Contracts

Model: Single good, uncertainty, 2 dates

T=1: Trading Assets

T=2: Consumption

$Y_2 = \overline{Y} + \varepsilon$ with $\varepsilon \in \{\underline{\varepsilon} = \varepsilon_1 < \varepsilon_2 < \ldots < \varepsilon_{N-1} < \varepsilon_N = \overline{\varepsilon}\}$, and prob($\varepsilon_i$)=$\pi(\varepsilon_i)$ with $\sum_{i=1}^{N} \pi(\varepsilon_i) = 1$. The shock ε has a mean of zero, is observable and $\underline{\varepsilon}$ is such that $\overline{Y} + \underline{\varepsilon} > 0$.

Agents can contract with risk neutral competitive foreign insurers.

State contingent contract delivers $P(\varepsilon)$ at date 2 so that:
$C = \bar{Y} + \epsilon - P(\epsilon)$

- $P(\varepsilon) < 0$: insurers pay
- $P(\varepsilon) > 0$: insurers receive

Risk neutrality + perfect competition imply that profits are

$$\sum_{i=1}^{N} \pi(\varepsilon_i) P(\varepsilon_i) = 0$$

Payment is an issue for the country if $P(\varepsilon) > 0$. This raises the question of *Credibility*.

CASE 1: Full Commitment

- A simple example: $Y_2 = \{Y_{21}, Y_{22}\}$
- $Y_{21} = \bar{Y} + \epsilon$, $Y_{22} = \bar{Y} - \epsilon$, $Prob(\epsilon > 0) = 0.5$
- Schedule of payments,P_1, P_2. Zero profit condition and risk neutrality on the part of the insurers means that $P_1 + P_2 = 0 \Rightarrow P_1 = -P_2 = P$.
- $\max Eu(c) = 0.5u(\bar{Y} + \epsilon - P) + 0.5u(\bar{Y} - \epsilon + P)$
- Concavity of the utility function implies that $P = \epsilon$ so that $C_{12} = C_{22} = \bar{Y}$. Consumption is independent of the state of nature. Perfect consumption smoothing

The more general case with commitment

$$\max U = \sum_{i=1}^{N} \pi(\varepsilon_i) U(C_i)$$

$$\text{s.t. } C_i = \overline{Y} + \varepsilon_i - P(\varepsilon_i)$$

$$\sum_{i=1}^{N} \pi(\varepsilon_i) P(\varepsilon_i) = 0$$

$$\mathcal{L} = \sum_{i=1}^{N} \pi(\varepsilon_i) \left(U(\overline{Y} + \varepsilon_i - P(\varepsilon_i)) + \mu P(\varepsilon_i) \right)$$

FOC

$$\pi(\varepsilon_i) \left(-U'(C_i) + \mu \right) = 0 \iff U'(C_i) = \mu \forall i = 1, \dots, N$$

$$C_i = \overline{Y} \text{ and } P(\varepsilon_i) = \varepsilon_i \ \forall i = 1, \dots, N$$

There is full insurance.

CASE 2: Imperfect commitment to pay

- If the borrower lacks commitment to pay and if international insurers are competitive and the cost of not paying is **zero** then there will be no int'l asset trade.

$$P(\varepsilon_i) = 0 \ \forall i = 1, \ldots, N \qquad C_i = \overline{Y} + \varepsilon_i$$

- Zero consumption smoothing/insurance: $C_{2i} = Y_{2i}$
- Suboptimal due to the concavity of utility
- In order to support international international asset trade (debt) we need to introduce a cost of not paying the contracted amount (of default), L. Let it be a function of output: $L = \eta Y_2$ with $\eta \in (0, 1)$.
- Incentive compatibility constraint (pay only when the payment is less than the sanction):

$$P(\varepsilon_i) \leqslant \eta Y_2 = \eta(\overline{Y} + \varepsilon_i)$$

An example with two states

- Let η be sufficiently small as to make the commitment equilibrium with $P = \epsilon$ infeasible.
  $\epsilon > \eta(\bar{Y} + \epsilon)$.
- The maximum payment that the sovereign will make in the good state 1 for fear of sanctions is
  $P = \eta(\bar{Y} + \epsilon) < \epsilon$.
- Let $\epsilon - \eta(\bar{Y} + \epsilon) = m > 0$.
  $C_{21} = \bar{Y} + \epsilon - P = \bar{Y} + \epsilon - \eta(\bar{Y} + \epsilon) = \bar{Y} + m$ and
  $C_{22} = \bar{Y} - m$
- Lower welfare than in the case of commitment as some idiosyncratic risk remains

The more general case

$$\max U = \sum_{i=1}^{N} \pi(\varepsilon_i) U(C_i)$$

$$\text{s.t. } C_i = \overline{Y} + \varepsilon_i - P(\varepsilon_i)$$

$$\sum_{i=1}^{N} \pi(\varepsilon_i) P_i = 0$$

$$P(\varepsilon_i) \leqslant \eta(\overline{Y} + \varepsilon_i)$$

$$\sum_{i=1}^{N} \pi(\varepsilon_i) \left(U(\overline{Y} + \varepsilon_i - P(\varepsilon_i)) + \mu P(\varepsilon_i) \right) + \lambda(\varepsilon_i) \left(\eta(\overline{Y} + \varepsilon_i) - P(\varepsilon_i) \right)$$

FOC

$$-\pi(\varepsilon_i) U'(C_i) + \mu \pi(\varepsilon_i) - \lambda(\varepsilon_i) = 0$$

Slackness condition

$$\lambda(\varepsilon_i) \left(\eta(\overline{Y} + \varepsilon_i) - P(\varepsilon_i) \right) = 0$$

Two possibilities.
The incentive compatibility constraint (ICC) is binding (satisfied with equality), $\lambda(\varepsilon_i) > 0$
ICC is not binding, $\lambda(\varepsilon_i) = 0$.

1. If $\lambda(\varepsilon_i) = 0$, then $P(\varepsilon_i) < \eta(\overline{Y} + \varepsilon_i)$ and

$$u'(C_i) = \mu \ \forall i = 1, \dots, N$$

2. If $P(\varepsilon_i) = \eta(\overline{Y} + \varepsilon_i) \implies \lambda(\varepsilon_i) > 0$

$$U'(C_i) = \mu - \frac{\lambda(\varepsilon_i)}{\pi(\varepsilon_i)} \neq \mu$$

   Imperfect consumption insurance. Consumption is not constant across states of nature. It depends on ε_i

How much consumption smoothing can a sovereign achieve? Guess: The ICC will not bind for low values of ε (because the country receives rather than pays) then

$$\lambda(\varepsilon_i) = 0 \Longrightarrow U'(C_2(\varepsilon_i)) = \mu$$

For low values of ε, period 2 consumption, C_2, is constant. Hence

$$C_2 = \overline{Y} + \varepsilon_i - P(\varepsilon_i) = \text{constant} \Longleftrightarrow P(\varepsilon_i) = \underbrace{\overline{Y} - \text{constant}}_{P_0} + \varepsilon_i$$

Hence

$$P(\varepsilon_i) = P_0 + \varepsilon_i$$

Let $\widetilde{\varepsilon}$ be such that the country is indifferent between paying or not paying (and suffering the sanction)

Default	Pay
$(1-\eta)(\overline{Y}+\widetilde{\varepsilon})$	$\overline{Y}+\varepsilon_i - P(\varepsilon_i) = \overline{Y}+\widetilde{\varepsilon} - P_0 - \widetilde{\varepsilon} = \overline{Y} - P_0$

Indifference implies that

$$(1-\eta)(\overline{Y}+\widetilde{\varepsilon}) = \overline{Y} - P_0 \tag{1}$$

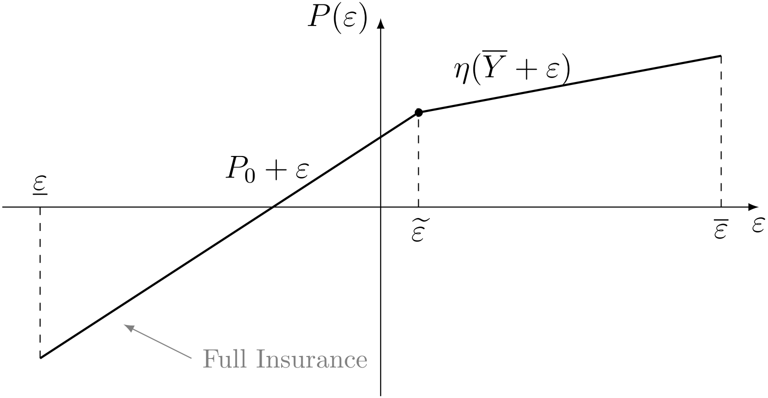
For $\varepsilon > \widetilde{\varepsilon}$ the country will never pay more than the sanction, $\eta(\overline{Y}+\varepsilon_i)$. Hence

$$P(\varepsilon) = \begin{cases} P_0 + \varepsilon & \text{if } \varepsilon \leqslant \widetilde{\varepsilon} \\ \\ \eta(\overline{Y}+\varepsilon) & \text{if } \varepsilon > \widetilde{\varepsilon} \end{cases}$$

$$\int_{\underline{\varepsilon}}^{\widetilde{\varepsilon}} (P_0 + \varepsilon_i)\mathrm{d}f(\varepsilon_i) + \int_{\widetilde{\varepsilon}}^{\overline{\varepsilon}} \eta(\overline{Y}+\varepsilon_i)\mathrm{d}f(\varepsilon_i) = 0 \tag{2}$$

Equations (1)-(2) are two equations in the two unknown, P_0 and $\widetilde{\varepsilon}$.

FIGURE : Debt Contract



Non-contingent contracts

- Properties of equilibrium under state contingent contracts: Default incentives stronger during good times.
- It seems counterfactual (but according to Tomz and Wright's (2007) many defaults occur during boom periods)
- Can the model produce countercyclical default if debt contract are non-contingent?

Example: 2 periods with outstanding debt in the first period; concave utility
Sanction: Exclusion from credit markets in case of default in addition to the standard output cost of default (k*Y)
$C_1 = Y_1 - \aleph b_1 - (1 - \aleph)kY_1 + \aleph q b_2, \qquad C_2 = Y_2 - \aleph b_2$
$\aleph$ is indicator of repayment ($= 1$ full, $= 0$ zero repayment).
Y_2 is known in advance. Let $k_2 = 1 \Rightarrow$ can borrow up to $b_2 \leq Y_2$. The sovereign always repays in period 2 and $q = \beta$ (risk free loan).
In period 1 if $\aleph < 1$ then $\aleph = 0$ (due to fixed sanction)

Utility of default and no-default
$D : u(Y_1 - kY_1) + \delta u(Y_2)$
$ND : u(Y_1 - b_1 + qb_2) + \delta u(Y_2 - b_2)$
Assume the borrower is risk neutral.
$D : Y_1 - kY_1 + \delta Y_2$
$ND : Y_1 - b_1 + qb_2 + \delta(Y_2 - b_2) = Y_1 - b_1 + qb_2$
$b_2 = Y_2$ due to the linearity of utility and the fact that $\beta > \delta$.
Default if $b_1 > kY_1 + (\beta - \delta)Y_2$.

- Low current level of income
- Low income growth prospects
- Large outstanding level of debt

For more general treatment see: Eaton and Gersovitz, 1981, Arellano, 2008, Uribe, 2013 ch 8.

A –two period– model with investment
$Y_1 = Y_1$, $Y_2 = F(K_2)$, $K_1 = 0$, $F' > 0$,$F" < 0$.
$Y_1 + D - C_1 - K_2 = 0$,
$F(K_2) + K_2 - C_2 - \aleph(1 + r)D - (1 - \aleph)k(F(K_2) + K_2) = 0$

CASE 1: After borrowing the country enjoys discretion over the level of investment.
Given debt, D, and an investment decision, K_2, the debtor defaults if $(1 + r)D > k(F(K_2) + K_2)$.
Given D, optimal investment decision K_2 maximizes

$$u(Y_1+D-K_2)+\delta u(F(K_2)+K_2-min\{(1+r)D, k(F(K_2)+K_2)\})$$

Solve under default and no default, K_2^d and K_2^{nd}. Default if $U(D, K_2^d(D)) > U(D, K_2^{nd}(D))$.

Lenders choose $\bar{D}$, $\bar{D} : U(\bar{D}, K_2^d(\bar{D})) = U(\bar{D}, K_2^{nd}(\bar{D}))$ (No default).

Kinky properties of the solution

Determination of optimal choice of K_2

$\Lambda = u(Y_1 + D - K_2) + \delta u(F(K_2) + K_2 - (1+r)D) - \lambda(D - \bar{D})$

The FOCs are

$$\begin{aligned} u'(C_1) &= (1+r)\delta u'(C_2) + \lambda \\ u'(C_1) &= (1 + F'(K_2))\delta u'(C_2) \\ 0 &= \lambda(\bar{D} - D) \end{aligned}$$

When the borrowing constraint binds ($D = \bar{D}, \lambda > 0$) consumption is tilted towards the future (C_1 is too low). But at the same time, C_2 is also below its level in the absence of default risk.

CASE 2. The country commits to a particular level of investment.
Loan such that: $(1+r)D = k(F(K_2) + K_2)$ k = default cost

$$u(Y_1+D-K_2)+\delta u(F(K_2)+K_2-(1+r)D)-\lambda((1+r)D-k(F(K_2)+K_2))$$

$$\begin{aligned}
u'(C_1) &= (1+r)(\delta u'(C_2)+\lambda) \\
u'(C_1) &= (1+F'(K_2))(\delta u'(C_2)+k\lambda) \\
0 &= \lambda((1+r)D - k(F(K_2)+K_2))
\end{aligned}$$

When the borrowing constraint binds ($\lambda > 0$) consumption is tilted towards the future (C_1 is too low).
The country invests less if there is default risk ($F' > r$) but more relative to the case of no investment commitment. Why? (the role of $k\lambda$ Thus it can receive more funds relative to that case. The ability to tie one's hands helps.

Dellas-Niepelt: a model with official and private creditors and pari passu (no discrimination among creditors)
Key assumption: differential enforcement power across creditors.
Possible justification: club membership

- Probability of sovereign default depends on both the level and the composition of debt
- Higher exposure to official lenders improves incentives to repay but also carries extra costs such as reduced ex post flexibility (repay more often in the future; and suffer a bigger cost when not repaying).

The model accounts for several empirical features :

- official lending to sovereigns takes place in periods of large borrowing needs
- it carries a favorable rate
- in the presence of large debt overhang the availability of official funding increases the probability of default on outstanding debt

The model

$$G_1(b, b^e) = u(y_1 + qb) + \delta E_1 G_2(b, b^e)$$
$$G_2(b, b^e) = \max(u(y_2 - min\{b, L + \phi(b^e)\})$$

where $L + \phi(b^e)$ is the cost of default, L is a random variable whose value is unknown in period 1 and is drawn from the distribution $F(L)$, $b =$ total and $b^e =$ official debt, ϕ sanction associated with default on official debt. Note that privately held debt is $b - b^e$. The debtor defaults when $L + \phi(b^e) < b$ that is for realizations of $L : L < b - \phi(b^e)$. Hence, the probability of default is $F(L) = F(b - \phi(b^e))$ and the probability of repayment $1 - F()$.

The current price of debt that matures tomorrow and pays 1 unit is q. If the creditors are risk neutral and the price of a risk free discount bond is β then the price of risky debt is $q = \beta$*(Prob of repayment) $=\beta * (1 - F())$.
Expected utility is then given by

$$G_1 = u(Y_1 + \beta q b) + \delta \int^{b-\phi(b^e)} u(Y_2 - L - \phi(b^e)) f(L) dL + \delta u(Y_2 - b)(1 - F(b - \phi(b^e$$

- The Choice of Debt Issued to Private Lenders: Elasticity of debt offer curve
- The Choice of Debt Issued to Official Lenders

FOCs

- Private, b
- $dG/db = u'_1\beta(1 - F - fb) - \delta u'_{2N}(1 - F)$
-
- Official, b^e
- $u'_1\beta fb\phi' - \delta\phi' \int^{b-\phi} u'_{2D} f dL$

$f = F'$ and Y_2 certain

A simple example with an interior solution

- Two realizations of L: 0 with probability 1-π and $\bar{L}_2$ with probability π. The only interesting case is when there is default if $L = 0$ and no default otherwise. Consequently, $\pi =$ is also the prob of default.
- Cost of default $= L + \phi b^e$
- $u(c) = ln(c)$
- $log(y_1 + \beta\pi b) + \delta\pi log(y_2 - b) + \delta(1 - \pi)log(y_2 - \phi b^e)$

$\phi =$ constant

$$\frac{\beta\pi}{y_1 + \beta\pi b} - \frac{\delta\pi}{y_2 - b} - \frac{\delta(1-\pi)}{y_2 + \bar{L}_2 - b} = 0 \quad (3)$$

$$b^e - \frac{b - \bar{L}_2}{\phi} = 0. \quad (4)$$

where we have used the fact that a binding borrowing constraint means that $b = \bar{L}_2 + \phi b^e \rightarrow \phi b^e = b - \bar{L}_2$.

Properties of equilibrium

- With private only, $max(b) = \bar{L}_2 = 0.4$
- With official only, $max(b) = \bar{L}_2/(1-f) = 0.57$
- An interior solution with $b, b^e > 0$ and $b > \bar{L}_2$
- A numerical example $\beta = 0.9, \delta = 0.5, \pi = 0.6, y_1 = 1, y_2 = 1.5, \bar{L}_2 = 0.4, \phi = 0.3 \rightarrow b = 0.47$ and $b^e = 0.23$.

Intuition: Official gives the debtor to overcome the strict borrowing constraint, m. But because of its higher cost in the case of default, the debtor makes limited use of it.

Long-term debt overhang, b_{02}

- Outstanding in first period, maturing in second
- Let $\tilde{b}_2 \equiv b_2 + b_{02} r_1$

Marginal effect of b_2^e, given b_2

Interaction between debt overhang, refinancing and default choice

- Overhang changes price elasticity of private and official debt, increasing probability of default
- Higher probability of default increases the future cost of *official* funds
- Overhang reduces relative attractiveness of official funds
- When official refinancing is available and credibility very valuable, overhang may increase incentive to default

"Dynamic" default decision in first period (benefits of default accrue in both periods)

- Default wipes out b_1 and b_{02}
- The latter implies direct increases in q_1 and G_2

With larger debt overhang, private debt more likely under no default, even with large borrowing needs

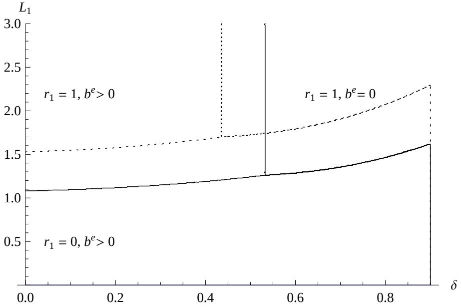


FIGURE : Default and official lending regions with debt overhang