Lecture Notes Monetary Exchange Rate Models

April 2017

1 Monetary Exchange Rate Models

These notes closely follow Obstfeld,Rogoff, pp. 530-557 and pp. 579-584. A detailed mathematical derivation and the interpretation will be done in class.

We consider a case, where money enters the utility function of the representative agent:

$$U_t = \sum_{s=t}^{\infty} \beta^{s-t} u\left(C_s, \frac{M_s}{P_s}\right) \tag{1}$$

M_t is the nominal money stock. We assume that $u_C > 0, u_M > 0$ and strict concavity. Purchasing power parity (PPP) holds: $P_t = \epsilon_t P^*$.

The budget constraint is given by:

$$B_{t+1} + \frac{M_t}{P_t} = (1+r)B_t + \frac{M_{t-1}}{P_t} + Y_t - C_t - T_t \tag{2}$$

Insert the budget constraint into the utility function:

$$U_t = \sum_{s=t}^{\infty} \beta^{s-t} u\left((1+r)B_s + \frac{M_{s-1}}{P_s} + Y_s - T_s - B_{s+1} - \frac{M_s}{P_s}, \frac{M_s}{P_s}\right) \tag{3}$$

The first-order conditions are:

$$u_C\left(C_t, \frac{M_t}{P_t}\right) = (1+r)\beta u_C\left(C_{t+1}, \frac{M_{t+1}}{P_{t+1}}\right) \tag{4}$$

$$\frac{1}{P_t}u_C\left(C_t, \frac{M_t}{P_t}\right) = \frac{1}{P_t}u_{M/P}\left(C_t, \frac{M_t}{P_t}\right) + \frac{1}{P_{t+1}}\beta u_C\left(C_{t+1}, \frac{M_{t+1}}{P_{t+1}}\right) \tag{5}$$

For example, the utility function could take the form:

$$u\left(C, M/P\right) = \frac{\left(C^{\gamma}(M/P)^{1-\gamma}\right)^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} \tag{6}$$

Or,

$$u\left(C, M/P\right) = \log(C) + \log\left(\frac{M}{P}\right) \tag{7}$$

The two first-order conditions can be combined to get:

$$\frac{u_{M/P}}{u_C} = 1 - \frac{P_t/P_{t+1}}{1+r} \tag{8}$$

The government budget constraint is:

$$G_t = T_t + \frac{M_t - M_{t-1}}{P_t} \tag{9}$$

Using equation (9) to substitute for T_t in equation (2), we get:

$$B_{t+1} = (1+r)B_t + Y_t - G_t - C_t \tag{10}$$

2 More advanced models

We study the Obstfeld-Rogoff Redux model (Obstfeld-Rogoff, chapter 10, Walsh (2010), chapter 9) and the small open New Keynesian model in Walsh (2010): Monetary Theory and Policy, ch. 9. The password for the password-protected lecture notes can be obtained from guido.baldi@vwi.unibe.ch.