

# Problem Set 3

**Department of Economics at the University of Bern**  
International Monetary Economics

Please bring errors to the author's attention. Contact:  
[fabio.canetg@vwi.unibe.ch](mailto:fabio.canetg@vwi.unibe.ch)

---

Author: Fabio Canetg  
Address: Schanzeneckstrasse 1  
3001 Bern  
Switzerland  
E-Mail: [fabio.canetg@vwi.unibe.ch](mailto:fabio.canetg@vwi.unibe.ch)  
Closing date: May 23, 2017

---

1. A two period model with full and limited commitment: The social planner maximizes

$$\max_{C_1, C_2} (u(C_1) + \beta u(C_2)) \quad (1)$$

subject to the dynamic budget constraints (DBC)

$$Y_1 + d_1 = C_1 \quad (2)$$

$$Y_2 = C_2 + (1 + r)d_1 \quad (3)$$

with  $d_1 > 0$  ( $<$ ) representing liabilities (assets).

- (a) Why must we not include the natural debt limit (NDL) as a constraint in the optimization problem?
- (b) Suppose that defaulting comes at a cost of  $kY_2$ . Write the optimization problem as a Lagrangian. Comment on the Lagrange multiplier of the new constraint.

**Solution:** We (additionally) impose the incentive compatibility constraint (ICC)  $kY_2 \geq (1+r)d_1$  to capture the fact that the lender knows that the borrower will default on all debt exceeding  $kY_2$  (because it is cheaper for the borrower to pay the cost of defaulting than to actually serve the debt). From the borrower's perspective, the ICC is a borrowing constraint.

$$\mathcal{L} = u(Y_1 + d_1) + \beta u(Y_2 - (1 + r)d_1) + \mu(kY_2 - (1 + r)d_1) \quad (4)$$

The constraint is less tight if  $kY_2$  rises. Consequently, we can interpret  $\mu$  as the marginal utility of a marginally less tight constraint. Because a looser constraint cannot make you worse off, it must be that  $\mu \geq 0$ .

- (c) Derive the Euler equation and the complementary slackness condition.

**Solution:**

$$u'(C_1) = \beta(1+r)u'(C_2) + \mu(1+r) \quad (5)$$

$$\mu(kY_2 - (1+r)d_1) = 0 \quad (6)$$

with  $\mu > 0$  if  $kY_2 = (1+r)d_1$  and  $\mu = 0$  if  $kY_2 > (1+r)d_1$ .

- (d) What are the implications of  $\mu > 0$  for  $C_1$  (compared to a situation of full commitment)?

**Solution:** *The marginal utility of  $C_1$  increases (compared to the full commitment case) if  $\mu > 0$ . Because the felicity function is (by assumption) strictly concave in  $C$ , a higher marginal utility of consumption is associated to a lower consumption level. Consequently,  $C_1$  is lower under limited commitment than under full commitment if the borrowing constraint binds.*

- (e) Can the natural debt limit (NDL) ever bind if  $\mu > 0$ ? Why (not)?