Problem Set 6

Department of Economics at the University of Bern
International Monetary Economics

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Closing date:	May 23, 2017

1. Consider a simple monetary model of the nominal exchange rate. Let M_t denote nominal money supply, P_t, the price level, i_t the nominal net interest rates, Y_t real output, and ε_t the nominal exchange rate (in domestic currency units per foreign currency units). Small letters indicate the logs of the corresponding level-variables and foreign variables are distinguished by asteriks. Furthermore, use $\mathbf{i}_t \equiv log(1+i_t)$ and $e_t = log(\varepsilon_t)$. Finally, assume $\eta > 0$ and $\phi > 0$. The money demand equation, the purchasing power parity (PPP) and the uncovered interest rate parity (UIP) are given by

$$\frac{M_t}{P_t} = \frac{Y_t^{\phi}}{(1+i_t)^{\eta}} \tag{1}$$

$$P_t = \varepsilon_t P_t^{\star} \tag{2}$$

$$(1+i_t) = (1+i_t^{\star})\mathbb{E}_t\left(\frac{\varepsilon_{t+1}}{\varepsilon_t}\right) \tag{3}$$

(a) Suppose that the exchange rate in time $t+1$ is known for sure as of period t. Express the model in logs.

Solution:

$$m_t - p_t = -\eta\mathbf{i}_t + \phi y_t \tag{4}$$

$$p_t = e_t + p_t^{\star} \tag{5}$$

$$\mathbf{i}_t = \mathbf{i}_t^{\star} + e_{t+1} - e_t \tag{6}$$

(b) Now, suppose that agents have to make a forecast of the exchange rate in time $t+1$. The model in logs is then approximated by

$$m_t - p_t = -\eta \mathbf{i}_t + \phi y_t \tag{7}$$

$$p_t = e_t + p_t^\star \tag{8}$$

$$\mathbf{i}_t = \mathbf{i}_t^\star + \mathbb{E}_t e_{t+1} - e_t \tag{9}$$

Why is this model representation only an approximation of the original model? You may want to resort to the definition of concavity (a function $u(x)$ is strictly concave if $\mathbb{E}(u(x)) < u(\mathbb{E}(x))$ and use $v(\varepsilon_{t+1}) = log\left(\frac{\varepsilon_{t+1}}{\varepsilon_t}\right)$).

(c) Let m_t, y_t, and all foreign variables be exogenous variables. Find the solution for e_t. What is the (qualitative) response of e_t to an increase in the nominal money supply m_s?

(d) What is the (quantitative) response of e_t and $\mathbf{i}_t$ to an increase in the contemporaneous nominal money supply m_t (assuming $m_t \sim$ *i.i.d.*)?

Solution: *First, take the total differential with respect to e_t and m_t on the result of the previous exercise.*

$$de_t = \frac{1}{1+\eta} dm_t \tag{10}$$

The nominal exchange rate rises by $\frac{1}{1+\eta} dm_t$ in m_t. Second, take the total differential on the PPP equation with respect to p_t and e_t.

$$dp_t = de_t \tag{11}$$

$$dp_t = \frac{1}{1+\eta} dm_t \tag{12}$$

Third, take the total differential on the money demand equation *with respect to m_t, p_t and $\mathbf{i}_t$.*

$$dm_t - dp_t = -\eta d\mathbf{i}_t \tag{13}$$

$$dm_t - \frac{1}{1+\eta} dm_t = -\eta d\mathbf{i}_t \tag{14}$$

$$d\mathbf{i}_t = -\frac{1}{1+\eta} dm_t < 0 \tag{15}$$

The nominal interest rate falls by $\frac{1}{1+\eta} dm_t$ in m_t.

(e) What is the (qualitative) response of e_t to a decrease in the nominal interest rate? How do you explain that an interest rate shock induces $corr(e_t, \mathbf{i}_t) > 0$ while a nominal money supply shock implies $corr(e_t, \mathbf{i}_t) < 0$?

Solution: *The interest rate shock is conceptually different from the nominal money supply shock because it does not induce a change in the (exogenous) nominal money supply. Because nominal money supply is exogenous (and constant when we consider an interest rate shock), the money demand equation requires that $corr(p_t, \mathbf{i}_t) > 0$. In other words, a lower nominal interest rate translates into a lower price level. In turn, an decrease in p_t induces an decrease in e_t (an appreciation) via the PPP equation.*
In contrast, the nominal money supply shock affects the (endogenous) interest rate. In this sense, only a nominal money supply shock is in effect expansionary.

(f) Find a way to show that the nominal exchange rate depreciates more when the *change* in the nominal money supply exhibits positive persistence. Assume that $\eta \mathbf{i}_s^\star - p_s^\star - \phi y_s = 0$.[1]

Solution: *We can model positive persistence in the growth of nominal money supply as an AR(1) process with zero mean, $\rho \in$*

[1] cf. Obstfeld, Rogoff, et al. (1996, p. 526-530)

$(0, 1)$, *and constant innovation variance. Formally,*

$$m_t - m_{t-1} = \rho(m_{t-1} - m_{t-2}) + \zeta_t^m \tag{16}$$

with $\mathbb{E}_t\zeta_{t+i}^m = 0\ \forall i$ *and* $V(\zeta_{t+i}^m) = \sigma_m^2\ \forall i$. *The expected change in the nominal money supply in period* $t + S$ *is then given by*

$$\mathbb{E}_t\left(m_{t+S} - m_{t+S-1}\right) = p^S\left(m_t - m_{t-1}\right) \tag{17}$$

Lead the solution for e_t *(equation* **??***) by one period, take expectations as of period* t *and substract the solution for* e_t *to get*

$$\mathbb{E}_t e_{t+1} - e_t = \frac{1}{1+\eta}\sum_{s=t}^{\infty}\left(\frac{\eta}{1+\eta}\right)^{s-t}\mathbb{E}_t\left(m_{s+1} - m_s\right) \tag{18}$$

Combine the last two expressions

$$\mathbb{E}_t e_{t+1} - e_t = \frac{1}{1+\eta}\sum_{s=t}^{\infty}\left(\frac{\eta\rho}{1+\eta}\right)^{s-t}\rho\left(m_t - m_{t-1}\right) \tag{19}$$

$$\mathbb{E}_t e_{t+1} - e_t = \frac{\rho}{1+\eta-\eta\rho}\left(m_t - m_{t-1}\right) \tag{20}$$

$$\mathbb{E}_t e_{t+1} - e_t = \frac{\rho}{1+\eta(1-\rho)}\left(m_t - m_{t-1}\right) \tag{21}$$

Substitute this expression into the original model equation (equation **??***) an solve for* e_t

$$m_t - e_t = -\frac{\eta\rho}{1+\eta(1-\rho)}\left(m_t - m_{t-1}\right) \tag{22}$$

$$e_t = m_t + \frac{\eta\rho}{1+\eta(1-\rho)}\left(m_t - m_{t-1}\right) \tag{23}$$

It follows that

$$de_t = dm_t + \frac{\eta\rho}{1+\eta(1-\rho)}dm_t \tag{24}$$

$$de_t = \frac{1+\eta}{1+\eta(1-\rho)}dm_t \tag{25}$$

A change in the current nominal money supply has a greater effect on the current nominal exchange rate if the change in the nominal money supply exhibits positive persistence (compared to exercise 1.d. in which we assumed that $m_t \sim i.i.d.$). This is so because a shock to the current nominal money supply does not only increase current nominal money supply but also the expected future nominal money supply.

(g) Suppose that $\rho = 0$. What kind of process does the nominal money supply follow? How does that affect the impact of a nominal money supply shock on the exchange rate? Compare your result to your finding in exercise 1.d.

2. According to rational expectations models of the nominal exchange rate, such as the Monetary Model, a increase in the domestic [nominal] money supply is expected to cause an appreciation in the exchange rate, but the exchange rate depreciates [on impact]. Explain why the Monetary Model is nonetheless correct (Wickens (2012), chapter 12, exercise 1). Assume, for simplicity, that the domestic nominal money supply follows an *i.i.d.* process.

3. Consider the following model of the nominal exchange rate. $\bar{y}$ denotes the output level consistent with stable prices, z_t is a zero mean money supply shock, bars indicate constant variables, and foreign variables are assumed to be zero at all times. All remaining notation and interpretation is as in the previous exercises.

$$\bar{m} + z_t - p_t = \phi y_t^d - \eta \mathbf{i}_t \tag{26}$$

$$\mathbf{i}_t = \mathbf{i}_t^\star + \mathbb{E}_t e_{t+1} - e_t \tag{27}$$

$$p_t - p_{t-1} = \pi(y_t^d - \bar{y}) \tag{28}$$

$$y_t^d = \delta(e_t + p_t^\star - p_t) \tag{29}$$

The aim of the exercise is to re-express the model such that it is compatible with standard procedures for solving dynamic stochastic general equilibrium (DSGE) models on a computer.

(a) Provide an economic interpretation of the last two model equations.

Solution: *The third model equation suggests that prices are adjusted upwards (downwards) if the current output* y_t^d *is above (below) the steady state output* $\bar{y}$*. It may be interpreted as a Phillips curve relationship. The fourth model equation determines the current output* y_t^d *as a (positive) function of the real exchange rate. In other words, domestic output is higher, the lower the real value of the domestic currency is, compared to the value suggested by the purchasing power parity.*

(b) Reduce the model to a two-equation model in the endogenous variables e_t and p_t.

(c) Solve for the steady state values e_{ss} and p_{ss}.

(d) Solve for $\bar{m}$ and $\bar{y}$ in terms of the two steady state values.

(e) Re-express the model in log-deviations from steady state.

References

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