

Exercises Chapter 2: A simple Robinson Crusoe Economy

Problem 2.1

Here we consider a specific functional form for a production function, the Cobb-Douglas function, and check whether it satisfies the properties typical of a production function. The Cobb-Douglas production function is:

$$Y = Z \cdot K^\alpha \cdot L^{1-\alpha} \text{ where } 0 < \alpha < 1.$$

- Define Z , K , and L .
- What does it mean that Y is proportional to Z ? Show, for the Cobb-Douglas case, that Y is proportional to Z .
- What does it mean that the marginal product of capital (or labor), MPK (or MPL), is positive? Show that the marginal products are positive in the Cobb-Douglas case.
- What does it mean that the marginal product of capital (or labor), MPK (or MPL), is decreasing? Show that the marginal products are diminishing in the Cobb-Douglas case.
- Explain the concept of constant returns to scale, CRS ? Does the Cobb-Douglas production function satisfy this property? Show it.

Problem 2.2.

Assume a simple Robinson Crusoe Economy with no investment. Suppose the preferences are given by $u(c, l) = c^\gamma (1-l)^{1-\gamma}$ where γ is between zero and one. Technology is

$$y = f(l) = Zl^\alpha.$$

Solve for optimal choices of consumption c and labour l .

Problem 2.3.

Walrasian (or General) equilibrium in a one-consumer, one-producer economy:

One way to solve for a general equilibrium when outcomes are (Pareto)optimal is to directly maximize utility subject to production constraints. Consider the problem of maximizing

$$u(c, l) = \log(c) + \log(l)$$

subject to $c = zn^\alpha$ and $n + l = L$

where n is the quantity of labor services and z, L, α are parameters.

- Form the Lagrangian suitable to this problem. Attach w to the constraint $n + l = L$ and p to the constraint $c = zn^\alpha$. Give an economic interpretation of the two multipliers. Given this interpretation, why does it make sense that the implicit real wage in consumption units is w/p ? Write down and interpret the FOC with respect to n, l , and c . Show the outcomes graphically in a diagram that contains an indifference curve and the production possibility frontier $c = zn^\alpha$.
- Does the optimal choice of leisure depend on the level of productivity Z ?

Problem 2.4. (Barro 2.8 Effects of Shifts in the Production Function on the Choice of work effort)

Assume that the Production Function is $y = Z \cdot l^{0.5} + B$. What are the effects on a household's work effort, l , output y , and consumption from

- a) An increase in the coefficient Z ?
- b) An increase in the coefficient B ?

Problem 2.5. (Barro 2.11 Productivity)

A popular measure of productivity is the ratio of output (say, real GDP) to employment (say, worker-hours). In the Graph of the production function that follows, this concept of productivity at the employment level l' is given by the ratio y' / l' . Productivity at this point equals the slope of the dashed line that is drawn from the origin to intersect the production function at the employment level l' .

- a) For the Production function shown and for the employment level l , show graphically that productivity y/l , always exceeds the marginal product of labor, MPL.
- b) Consider a technological change that shifts the production function upward proportionally at all levels of l (Figure 2.10). What happens to the choices of work effort, l , and output, y ? What happens to productivity y/l ?
- c) Assume now that the form of the production function does not change. But suppose that people shift their tastes and become more willing to work. What happens here to the choices of work effort, l , and output, y ? What happens to productivity y/l ?