3. The Behaviour of households with markets for Commodities and Credit

Problem 3.1. (Barro 3.10: Wealth Effects)

Consider the households' s budget constraint in real terms over an infinite horizon,
$y_1 + y_2/(1+R) + ... + b_0 \cdot (1+R)/P = c_1 + c_2/(1+R) + ...$
Using this condition, evaluate the wealth effect of the following:

a) An increase in the Price level P, for a household, that has a positive value of initial bonds, b_0. (The result has implications for the effects of unexpected price changes on the wealth of nominal creditors and nominal debitors.

b) An increase in the interest rate, R; for a household that has $b_0 = 0$ and $c_t = y_t$ in each period.

c) An increase in the interest rate R, for a household that has $b_0 = 0$, $c_t > y_t$ for t>T, and $c_t < y_t$ for t<T, where T is some date in the future.

Problem 3.2 (Barro 3.11: Short Term and long term interest rates)

Assume that \$1 worth of one period bonds issued at the end of period 0 pays out \$ $(1+R_1)$ during period 1, that is, the principal of \$1 plus interest payment of R1. Assume that \$1 worth of one-period bonds issued at the end of period 1 will pay you $(1+R_2)$ during period 2. Suppose that people also market a two period bond an the end of period 0. One dollars's worth of this asset pays out \$ (1+2R) during period 2. Lenders from date 0 to date 2 have the option of holding a two-period bond or a succession of one-period bonds. Borrowers have a similar choice between negotiating a two- period loan or two successive one period loans.

a) What must be the relations of R to R_1 and R_2? Explain the answer from the standpoint of borrowers and lenders.

b) If $R_1 > R_2$, what is the relation between R (The current long term interest rate) and R_1 (The current short term interest rate)? The answer is an important result about the term structure of interest rates.

Problem 3.3.

Consider a two-period model with

$U(c_1, c_2) = u(c_1) + \beta u(c_2)$ where period utility is $u(c_t) = (c_t)^{1/2}$

a) Determine the Euler Equation.
b) Determine the household's optimal choice of c_1^*, c_2^*, b_1^*, where b is bond bought at period t.
c) Determine the Equilibrium interest rate R^*.

d) Determine the effect on the equilibrium interest rate R^* of a permanent negative shock to the income of the representative household. How does this relate to the case in which $u(c_t) = \ln c_t$

Problem 3.4.

In Problem 3.3. we have determined the equilibrium interest rate R for the two-period model.

a) Suppose the representative household becomes more impatient. Determine the direction of the change in the equilibrium interest rate.

b) Suppose the representative household gets a temporary negative shock to its income in period 1, y_1. Determine the change in the equilibrium interest rate.

Problem 3.5

Consider a person who consumes and works in two periods and has the utility function

$$U(c_1, c_2, n_1, n_2) = \left(\ln c_1 - \frac{1}{1-\varphi} n_1^{1+\varphi} \right) + \beta \left(\ln c_2 - \frac{1}{1-\varphi} n_2^{1+\varphi} \right)$$

where c_1 is consumption in period 1, n_1 is labour in period 1 and $\varphi > 0$ is a parameter.

a) Let λ be the Lagrange Multiplier associated with the budget constraint

$$c_1 + \frac{1}{1+r} c_2 = w_1 n_1 + \frac{1}{1+r} w_2 n_2$$

where w is the real wage. Derive the F.O.C.s.

b) We may think of the resulting optimality conditions as giving the demand for goods and the supply of labor given the marginal utility of wealth λ. Suppose for the time being that $\beta = (1+r)^{-1}$.

Derive an expression for the relative labor supply, n_1 / n_2. Comment.

c) Now, suppose that β may differ from $(1+r)^{-1}$. Derive an expression for the relative labor supply n_1 / n_2 .Consider an increase in (1 + r).

d) *Optional:* Show that $1/\vartheta$ is the elasticity of substitution of labor across periods.