Chapter 3 – Additional Exercise

Robinson Crusoe lives three periods. He consumes coconuts and owns a coconut tree. In addition, there is a bank from which he can borrow more coconuts or to which he can lend coconuts.
The bank offers three different kinds of contracts. In period 1, RC can borrow or lend coconuts with fixed interest rate, r_1. The settlement is during the period 2. In addition, RC can borrow or lend coconuts with fixed interest rate, r_{12}. The settlement is during the period 3. Finally, in period 2, RC can borrow or lend coconuts with fixed interest rate, r_2 and the settlement is during the period 3.

RC's objective function is as follows:

$$u(c_1) + \beta u(c_2) + \beta^2 u(c_3)$$

Budget constraints are:

$$c_1 + s_1 + s_{12} \leq e_1$$
$$c_2 + s_2 \leq e_2 + (1 + r_1)s_1$$
$$c_3 \leq e_3 + (1 + r_2)s_2 + (1 + r_{12})s_{12}$$

where *e* is how much coconuts RC's coconut tree will have in period t, and *s* are the different contracts RC can take. If s is positive RC is lending to the bank and if s is negative RC is borrowing from the bank.
Notice that RC can borrow or lend infinite amounts of coconuts with fixed interest rate.

a) Write down RC's first-order conditions.
b) Give an economic interpretation to the first-order conditions.
c) Give an equilibrium relationship how r_{12} depends on r_1 and r_2. That is, express $r_{12} = f(r_1, r_2)$ and find $f(r_1, r_2)$
d) What would happen if r_{12} was bigger than $f(r_1, r_2)$?
e) Suppose that $r_1 < r_2$. How is r_{12} related to r_1 and r_2? (Hint: you can use the approximation *log(1+r)=r* for small r)