Lectures in Monetary Economics
Lecture 2

The RBC model

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The RBC model

- The RBC constitutes the methodological foundation of the NK model.
- It is a micro-founded DSGE model with rational agents, flexible prices and competitive markets.
- It has good empirical properties (in terms of the match between the model implied and empirical pdf of the data).

A simple, 2 period example

Consumption-savings choice

Utility:

$$u(C_1) + \beta u(C_2) \tag{1}$$

Budget constraint:

$$\begin{aligned} P_1 Y_1 &= P_1 C_1 + B && (2) \\ P_2 Y_2 + RB &= P_2 C_2 && (3) \end{aligned}$$

$B > 0$ means lending in the first period.

$$P_2 C_2 = P_2 Y_2 + R(P_1 Y_1 - P_1 C_1) \tag{4}$$

Euler equation (or dynamic IS curve)

$$u_{c1} = \beta R u_{c2} \frac{P_1}{P_2} = \beta R u_{c2} \frac{1}{\pi} \Rightarrow 1 = \beta r \frac{u_{c2}}{u_{c1}} \tag{5}$$

The Euler equation plays a critical role in the monetary transmission mechanism:
An increase in the real interest rate decreases current spending (consumption).

Work decision

Utility

$$u(C_1, h_1) + \beta u(C_2, h_2) \tag{6}$$

The supply of labor

$$-\frac{u_{h1}}{u_{c1}} = \frac{W_1}{P_1} = w_1 \tag{7}$$

The marginal rate of substitution between consumption and leisure equals the real wage.

The demand for labor

$$\frac{dY_1}{dh_1} = MPL_1 = \frac{W_1}{P_1} = w_1 \tag{8}$$

Combine demand and supply of labor

$$-\frac{u_{h1}}{u_{c1}} = MPL_1 \tag{9}$$

This equation will prove very useful for understanding optimal monetary policy in the NK model.

A multi-period version of the RBC model with uncertainty and a representative agent.

$$V = E_0[\sum_{t=0}^{\infty} \beta^t U(C_t, h_t)] \tag{10}$$

Flow budget constraint:

$$P_t C_t + B_t = R_{t-1} B_{t-1} + W_t h_t + \Pi_t \tag{11}$$

$$U(C_t, h_t) = \frac{1}{1-\gamma} C_t^{1-\gamma} - \frac{N_t}{1+\sigma} h_t^{1+\sigma} \tag{12}$$

$$Y_t = A_t h_t^{1-\alpha} \tag{13}$$

N_t is a preference and A_t a technology (productivity) shock.

The general equilibrium solution of the model for $\{Y, C, h, w = W/P, r = R/\pi\}$ is obtained by solving

$$
\begin{aligned}
\frac{N_t h_t^{\sigma}}{C_t^{-\gamma}} &= w_t \\
C_t^{-\gamma} &= \beta E_t r_t C_{t+1}^{-\gamma} \\
w_t &= (1-\alpha) A_t h_t^{-\alpha} \\
Y_t &= A_t h_t^{1-\alpha} \\
Y_t &= C_t \\
A_{t+1} &= \overline{A}^{1-\rho_a} A_t^{\rho_a} \epsilon_{a,t+1} \\
N_{t+1} &= \overline{N}^{1-\rho_\nu} N_t^{\rho_\nu} \epsilon_{g,t+1}
\end{aligned}
\tag{14}
$$

$b_t \equiv B_t/P_t = 0$

Nonlinear, dynamic, stochastic equations that can only be solved analytically in special cases.

In practice we solve an approximate version of the system, typically a log-linear approximation around the steady state.

The steady state can be "easily" derived by setting $A_t = \overline{A}, N_t = \overline{N}, C_t = C_{t+1} = \overline{C} \ldots$

The log-linearized system around the steady state takes the form

$$
\begin{aligned}
\hat{w}_t &= \sigma \hat{h}_t + \gamma \hat{c}_t + \hat{\nu}_t \\
0 &= \gamma E_t \hat{c}_{t+1} - \gamma \hat{c}_t + E_t \hat{r}_t \\
\hat{w}_t &= \hat{a}_t - \alpha \hat{h}_t \\
\hat{y}_t &= \hat{a}_t + (1-\alpha) \hat{h}_t \\
\hat{y}_t &= \hat{c}_t \\
\hat{a}_{t+1} &= \rho_a \hat{a}_t + \hat{\epsilon}_{a,t+1} \\
\hat{\nu}_{t+1} &= \rho_\nu \hat{\nu}_t + \hat{\epsilon}_{\nu,t+1}
\end{aligned}
\qquad (15)
$$

where for variable x we define $\hat{x} = \frac{x - x^*}{x^*} \approx \log x - \log x^*$ as the percentage deviation of x from its steady state value, x^*.

In state space form

$$A_0 E_t x_{t+1} = A_1 x_t + B_0 e_{t+1} \tag{16}$$

When A_0 is invertible,

$$\begin{aligned} E_t x_{t+1} &= A_0^{-1} A_1 x_t + A_0^{-1} B_0 e_{t+1} \Rightarrow \\ E_t x_{t+1} &= A x_t + B e_{t+1} \end{aligned} \tag{17}$$

The Blanchard-Khan (1980) method: Partition the state variables of the system into backward (s) and forward looking (z) variables.

$$\begin{bmatrix} s_{t+1} \\ E_t z_{t+1} \end{bmatrix} = A \begin{bmatrix} s_t \\ z_t \end{bmatrix} + B e_{t+1} \tag{18}$$

The properties of the solution The Blanchard-Khan criterion :

n = the number of eigenvalues of A that lie outside the unit circle
f = number of the forward looking variables.

- If n=f there exists a unique rational expectations solution to the system
- If n< f the system has multiple solutions[1] (indeterminacy).
- If If n> f then the system has no solution (all dynamic paths are explosive, violating the transversality condition).

[1] In this case one needs to use alternative methods to solve the system, for instance, Sims, 2000.

If A_0 is not invertible then the system

$$A_0 E_t x_{t+1} = A_1 x_t + B_0 e_{t+1}$$

can be solved using the QZ decomposition:

$\exists Q, \Lambda, Z, \Omega \; s.t. Q'\Lambda Z' = A_0, Q'\Omega Z' = A_1$, Λ, Ω upper triangular (see Klein, 2002, Sims, 2000).

The software of choice: Dynare

Basic structure of Dynare

// **declarations**

var $x, y, \ldots;$

varexo $ea, ev, \ldots;$

parameters $alpha, beta, \ldots;$

// **parameter values**

$alp = ; bet = ; \ldots$

// **model equations**

model;

$exp(v) * exp(c * gam) * exp(h * sig) - exp(w) = 0;$ // consumption

$(1 - alp) * exp(a) * exp(h * (-alp)) - exp(w) = 0;$ // work

$\ldots$

end;

//**steady state solution**

initval;

$c = log(..); h = log(..); \ldots$

end;

steady;

check;

// **stochastic structure** shocks;

$varea = ..; varev =; end;$

//**simulations**

stoch_simul(dr_algo=0,periods=1000, irf=20, nocorr, nofunctions, order=1) c y h w;

POLICY AND TRANSITION FUNCTIONS:

	c	y	h	w	r
Constant	-0.12	-0.12	-0.18	-0.36	0.01
a(-1)	0.81	0.81	-0.20	1.02	-0.06
v(-1)	-0.26	-0.26	-0.40	0.14	0.01
ea	0.86	0.86	-0.21	1.07	-0.06
ev	-0.27	-0.27	-0.43	0.15	0.02

MOMENTS OF SIMULATED VARIABLES:

VARIABLE	STD. DEV.	AUTOCOR
c	0.028145	0.9568
y	0.028145	0.9568
h	0.007386	0.9578
w	0.035377	0.9571
r	0.002111	0.9568

Empirical evaluation of the simple model.

It has some decent properties: Procyclical wages, consumption and employment.

But without investment it cannot match the most important stylized facts.

A more general version of the model with capital

Production

$$Y_t = A_t K_t^{\alpha} h_t^{1-\alpha} \tag{19}$$

The capital stock, K, accumulates according to

$$K_{t+1} = (1-\delta)K_t + I_t \tag{20}$$

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$$
\begin{aligned}
\frac{N_t h_t^{\sigma}}{C_t^{-\gamma}} &= w_t \\
C_t^{-\gamma} &= \beta E_t r_t C_{t+1}^{-\gamma} \\
w_t &= (1-\alpha)\frac{Y_t}{h_t} \\
C_t^{-\gamma} &= \beta E_t C_{t+1}^{-\gamma}(q_{t+1} + (1-\delta)) \\
q_t &= \alpha \frac{Y_t}{K_t} \\
Y_t &= A_t K_t^{\alpha} h_t^{1-\alpha} \\
Y_t &= C_t + I_t + G_t \\
K_{t+1} &= (1-\delta)K_t + I_t \\
A_{t+1} &= \overline{A}^{1-\rho_a} A_t^{\rho_a} \epsilon_{a,t+1} \\
N_{t+1} &= \overline{N}^{1-\rho_\nu} N_t^{\rho_\nu} \epsilon_{\nu,t+1} \\
G_{t+1} &= \overline{G}^{1-\rho_g} G_t^{\rho_g} \epsilon_{g,t+1}
\end{aligned} \tag{21}
$$

TASK: Compute the steady state of this model. Log-linearize around the steady state and then solve the model (or simply input your equations and steady state solution into dynare and let it solve the model). Report the moments, IRFs and variance decomposition. Use the same parameter values as in the model without investment with the addition

δ	$\overline{G/Y}$	ρ_g	Σ_g
0.08	0.2	0.95	0.02

where $\overline{G/Y}$ is the steady state ratio of government spending to GDP. What are the main properties of the model? Any comments?

Main implications of the RBC model

- Supply shocks as the main source of macroeconomic volatility. A single supply shock can account for most of macroeconomic fluctuations.
- Money "neutrality"

Galí's, 1999, criticism of the RBC model:
The RBC model implies that technology shocks lead to procyclical movements in employment, productivity and real wages of the type observed in the data.
But what is the *conditional* effect of supply shocks on employment in the data?
Galí, 1999: In response to a positive technology shock, labor productivity rises more than output while employment shows a persistent decline. Hence, supply shocks cannot be the driving force of macroeconomic fluctuations.

The difficulty of *identifying* technology shocks in the data.
Galí's identification scheme: Only technology shocks can have a permanent effects on the level of labor productivity (identification based on Blanchard and Quah, 1989).

$$\begin{bmatrix} \Delta x_t \\ \widehat{n}_t \end{bmatrix} = \begin{bmatrix} C_{11}(L) & C_{12}(L) \\ C_{21}(L) & C_{22}(L) \end{bmatrix} \begin{bmatrix} e_t^p \\ e_t^T \end{bmatrix} \tag{22}$$

$x_t = y_t - n_t$, x_t is the log of labor productivity.
The long term identifying restriction $\sum_j c_{12}(j) = 0$ implies that e_t^p and e_t^T are shocks with and without a permanent effect on labor productivity respectively. The former is taken to represent the technology shock.

Figure: Technology Shocks and Employment

TABLE 1—CORRELATION ESTIMATES: BIVARIATE MODEL

	Unconditional	Conditional	
		Technology	Nontechnology
Panel A: First-differenced labor			
Hours	−0.26**	−0.82**	0.26**
	(0.08)	(0.12)	(0.12)
Employment	−0.02	−0.84**	0.64**
	(0.07)	(0.26)	(0.13)
Panel B: Detrended labor			
Hours	−0.26**	−0.81**	0.35*
	(0.08)	(0.11)	(0.20)
Employment	−0.02	−0.35	0.38
	(0.07)	(0.49)	(0.56)

Notes: Table 1 reports estimates of unconditional and conditional correlations between the growth rates of productivity and labor input (hours or employment) in the United States, using quarterly data for the period 1948:1–1994:4. Standard

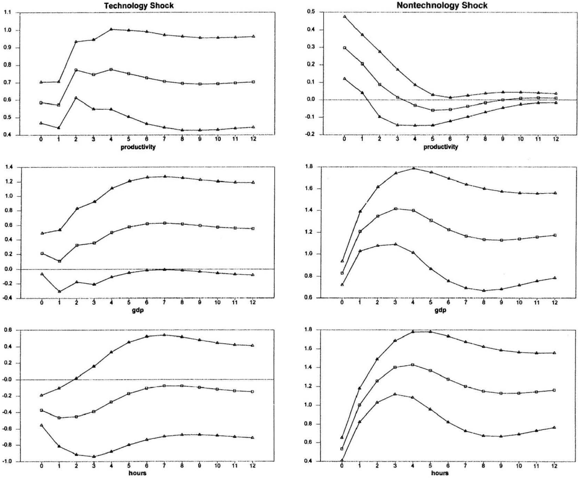


FIGURE 2. ESTIMATED IMPULSE RESPONSES FROM A BIVARIATE MODEL: U.S. DATA, FIRST-DIFFERENCED HOURS (POINT ESTIMATES AND ±2 STANDARD ERROR CONFIDENCE INTERVALS)

Response to the findings of Galí:

- Dispute the ability of the particular identification schemes used to truly identify technology shocks (Chari, Kehoe and McGrattan, 2005). Type of data stationarity, power of long term restrictions, etc.
- Play defense and argue that the new Keynesian model is equally incapable of matching these stylized facts (Dotsey, 1999).
- Suggest plausible, flexible price models that can reproduce these stylized facts. What is needed is models that have either sluggish aggregate demand or some other demand discouraging mechanism (such as low trade elasticities).

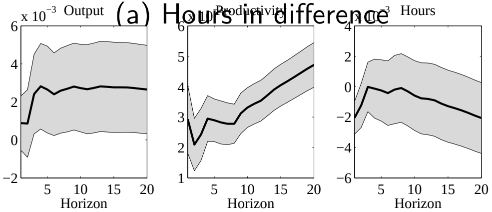
Collard and Dellas (C-D), 2007 EJ.

The role of low trade elasticities.

An RBC model of an open economy with low trade elasticities and sluggish capital adjustment can produce the correct patterns.

Figure: C-D, 2007 EJ, Impulse Response to a Technology Shock: Data

(a) Hours in difference



(b) Linearly detrended hours

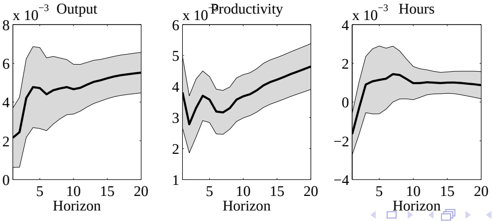


Figure: C-D, 2007 EJ, Impulse Response Function to a 1 s.d. technological shock: Model vs Data

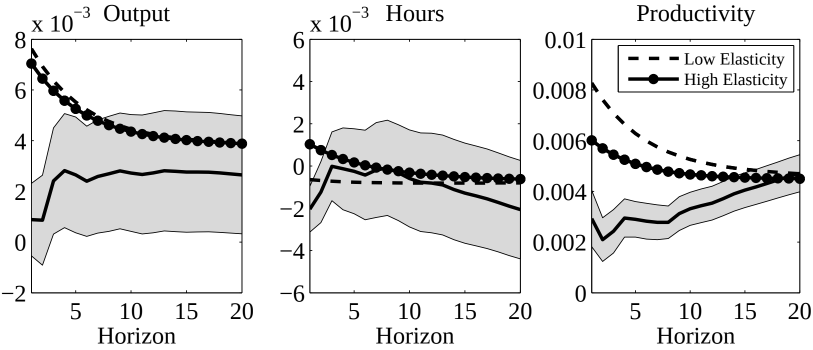


Table: Conditional Correlations

	Corr$(\cdot, \Delta y/h)$			Corr$(\cdot, \Delta y)$		
	Δh	RER	NX	Δh	RER	NX
	Flexible, low elasticity					
All	-0.094	0.110	0.035	0.279	0.081	0.075
Techno.	-0.415	0.153	0.013	-0.340	0.156	0.022
Other	0.029	-0.154	0.150	0.971	-0.152	0.149
	Flexible, high elasticity					
All	0.048	0.060	-0.013	0.436	0.072	-0.008
Techno.	0.042	0.106	-0.093	0.261	0.149	-0.129
Other	0.189	-0.177	0.174	0.914	-0.153	0.151

The upshot: The RBC can meet Galí's challenge

But are there any other reasons to want to abandon the RBC model?

The belief that money is not neutral due to

- either imperfect information problems à la Lucas
- or nominal rigidities (price or wage)

The NK model relies on the latter.

- Empirical evidence on real effects of money (Walsh ch 1.3).
- Empirical evidence on nominal stickiness.
  1. Fundamental difficulty: A constant price does not mean a rigid price! A variable price does not mean a flexible price!
  2. Direct evidence: Bils and Klenow, 2004, (4-6 months) Dhyne et al., 2005, Nakamura and Steinsson, 2007 (8-11 months).
  3. Nominal wage rigidity (Akerlof, 1995)
- Rather limited support for the existence of significant nominal rigidities.