Lectures in Monetary Economics

Lecture 3

The New Keynesian model: The baseline version

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The New Keynesian model: The baseline version

Main departures from RBC model:

- Nominal rigidities (price and-or wage)
- Imperfect competition

Structure of the model

The household

$$U(c_t, h_t) = \frac{1}{1-\gamma} C_t^{1-\gamma} - \frac{\nu_t}{1+\sigma} h_t^{1+\sigma} \tag{1}$$

A bundle of differentiated products, C_i

$$C_t = \left(\int_0^1 C_{it}^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta}{\theta-1}} , \theta > 1 \tag{2}$$

$$C_{it} = \left(\frac{P_{it}}{P_t} \right)^{-\theta} C_t$$

The general price level is

$$P_t = \left(\int_0^1 P_{it}^{1-\theta} \right)^{\frac{1}{1-\theta}} \tag{3}$$

The budget constraint

$$B_t + P_t C_t = R_{t-1} B_{t-1} + W_t h_t + \Pi_t \tag{4}$$

The savings-consumption decision (Euler or IS equation)

$$C_t^{-\gamma} = \beta E_t R_t \frac{P_t}{P_{t+1}} C_{t+1}^{-\gamma} \tag{5}$$

The supply of labor

$$\frac{\nu_t h^\sigma}{C_t^{-\gamma}} = \frac{W_t}{P_t} \tag{6}$$

Set $\nu_t = 1$. Taking logs, equation 6 is

$$w_t - p_t = \sigma h_t + \gamma c_t$$

A log-linear approximation of (5) around the deterministic, zero inflation steady state gives (note $c_t = y_t$)

$$\widehat{y}_t^S = E_t \widehat{y}_{t+1}^S - \frac{1}{\gamma}(\widehat{R}_t - E_t \pi_{t+1}) \tag{7}$$

$\widehat{y}_t^S$ is a measure of the output gap (deviation from steady state). The policy relevant gap ought to involve the gap between actual and efficient output!

The firms:
Production

$$Y_{it} = A_t h_{it}^{1-\alpha} \tag{8}$$

If prices were *flexible* in each and every period, firm *i* would choose its price as a constant markup over nominal marginal cost (Ψ_{it}):

$$P_{it} = \frac{\theta}{\theta - 1}\Psi_{it} \Rightarrow \frac{\Psi_{it}}{P_{it}} = \frac{\theta - 1}{\theta} \tag{9}$$

That is, the firm adjusts its price in order to maintain a constant real marginal cost.

In general, the optimal price of the firm involves three elements

- The price setting mechanism (stochastic or determininistic, time or state dependent etc)
- The behavior of markups
- The behavior of the nominal marginal cost

Price setting (Time vs state dependent)

Time dependent

Calvo: A constant probability, q, in each and every period of being allowed to set its price optimally.

Taylor: Periodic price resetting.

The optimal price, P^* maximizes the present discounted value of expected profits, $\prod$.

$$\prod = E_t \sum_{\tau=0}^{\infty} (1-q)^{\tau} \Lambda_{t+\tau} (P^*_{it} Y_{i,t+\tau} - P_{t+\tau} TRC_{i,t+\tau})$$

subject to $Y_{i,t+\tau} = (\frac{P^*_{i,t}}{P_{t+\tau}})^{-\theta} Y_t$

where TRC = total real cost $(= W_t * h_t / P_t)$ and $\Lambda_{t+\tau}$ is the current price of a claim to 1 dollar τ periods later.

The choice of optimal price satisfies

$$E_t \sum_{\tau=0}^{\infty} (1-q)^{\tau} \Lambda_{t+\tau} (P_{it}^{*} - M * P_{t+\tau} \psi_{i,t+\tau}) Y_{i,t+k} = 0$$

where M = is the steady state markup $(=\theta/(\theta-1))$ and ψ is real marginal cost.
Log-linearizing around the zero inflation steady state and solving for $p_{it}^{*} = logP_{it}^{*}$ gives

$$\begin{aligned} p_{it}^{*} &= (1-(1-q)\beta) E_t \sum_{\tau=0}^{\infty} (\beta(1-q))^{\tau} (p_{t+\tau} + \psi_{i,t+\tau}) \\ &= (1-(1-q)\beta)(p_t + \psi_{i,t}) + \beta(1-q) E_t p_{i,t+1}^{*} \qquad (10) \end{aligned}$$

Optimal price is a weighed average of expected, nominal, marginal costs.

The Phillips curve
p^*_{it} is the same for all price setting firms.
The aggregate price level evolves according to

$$p_t = qp^*_t + (1-q)p_{t-1}$$

Combining 10 and 10 leads to a Phillips curve

$$\pi_t = \beta E_t \pi_{t+1} + \frac{q(1-\beta(1-q))}{1-q}\hat{\psi}_{i,t} = \beta E_t \pi_{t+1} + \Upsilon \hat{\psi}_{i,t} \quad (11)$$

To get the economy wide Phillips curve we need to replace the firm specific real marginal cost, ψ_{it} with the average economy real marginal cost, ψ.

$$\psi_i = \psi - \frac{\alpha\theta}{1-\alpha}(p_i - p)$$

The Phillips curve

$$\pi_t = \beta E_t \pi_{t+1} + \frac{q(1-\beta(1-q))}{1-q}\frac{1-\alpha}{1-\alpha+\alpha\theta}\hat{\psi}_{i,t} = \beta E_t \pi_{t+1} + \Upsilon\Theta\hat{\psi}_t \quad (12)$$

- Inflation is purely forward looking.
- Inflation fluctuations arise exclusively from fluctuations in real marginal costs (or, equivalently, fluctuations in the average markup in the economy)! No reference to monetary variables.

Marginal cost in unobservable. Re-write the Phillips curve in terms of "output gap". A relevant measure is output, y, relative to its "natural" level, y^N (the level that would obtain if prices were perfectly flexible).

$$\hat{\psi}_t = (\gamma + \frac{\alpha + \sigma}{1 - \alpha})(y - y^N) = \chi \widehat{y}_t^N$$

$$\pi_t = \beta E_t \pi_{t+1} + \Upsilon \Theta \chi \widehat{y}_t^N = \beta E_t \pi_{t+1} + \kappa \widehat{y}_t^N \quad (13)$$

$\widehat{y}_t^N = log Y_t - log \overline{Y}_t^N$

This formulation of the output gap is "demanding" because it relies on the satisfaction of the FOC of the household (equation (6)) in order to transform marginal cost into output.

$$\pi_t = \beta E_t \pi_{t+1} + \Upsilon \Theta \chi \widehat{y}_t^N = \beta E_t \pi_{t+1} + \kappa \widehat{y}_t^N \qquad (14)$$

$\Upsilon = \frac{q(1-\beta(1-q))}{1-q}$
$\Theta = \frac{1-\alpha}{1-\alpha+\alpha\theta}$
$\chi = (\gamma + \frac{\alpha+\sigma}{1-\alpha})$

- Θ relates the marginal cost of firm i to the average economy wide marginal cost
- χ relates the average economy wide marginal cost to the output gap
- Υ depends on the frequency of price adjustment

Solving the NK model

The model consists of the IS and AS equations

$$\widehat{y}_t^N = E_t \widehat{y}_{t+1}^N - \frac{1}{\gamma}(\widehat{R}_t - E_t \pi_{t+1}) \qquad (15)$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \widehat{y}_t^N \qquad (16)$$

where N indicates deviation from the natural (flexible price) level.

- There are two equations in three unknown $\{\pi_t, \widehat{y}_t^N, R_t\}$.
- We need a third equation that describes how R is determined, perhaps as a function of inflation and output (the monetary policy equation).
- The properties of the solution depend critically on the policy equation (more later).