

**PART I****PROBLEM 1****NOMINAL INCOME TARGETING****1. A feedback policy**

It is standard practice for central banks to engage in macroeconomic stabilization. Let us consider a bank which has chosen to carry this objective out by targeting the growth of nominal GDP and by manipulating a short term interest rate (say, the Treasury Bill rate). Let  $GDP(t)$  = the -annualized- growth rate of nominal GDP in period  $t$  and  $di(t) = i(t) - i(t-1)$  where  $i(t)$  is the annual rate on T-Bills in period  $t$  (note that an annualized rate is simply the growth rate during the quarter multiplied by 4). We will assume that changes in interest rates have two components: one that is influenced by the central bank and another that is exogenous.

Run the following regression that relates the growth rate of GDP to its past values as well as the past values of changes in the short term interest rate (do it for the period 1970:1-1987:4)

$$(1) \quad GDP(t) = a_0 + a_1 * GDP(t-1) + a_2 * GDP(t-2) + a_3 * di(t-2) + e(t)$$

where  $e(t)$  is the residual of the regression. Note that  $e(t)$  captures the effects coming from other variables which have not been included in the regression, such as fiscal policy, the terms of trade, world economic conditions and so on. Equation (1) postulates that changes in short term interest rates affect nominal income with a lag of two quarters.

Store the estimated coefficients of the regression. Let  $i(t) = i_{ex}(t) + i_{en}(t)$ , where  $i_{ex}(t)$  is the exogenous component of the interest rate and  $i_{en}(t)$  is the component that is set by the central bank (think of  $i(t)$  as the long term interest rate which depends both on the short term interest rate that is controlled by the central bank and on other factors). The exogenous component will be taken to be the same independent of policy. Now suppose that the Central Bank follows the following feedback rule:

$$(2) \quad d[i_{en}(t)] = 0.2 * [GDP(t-1) - 0.06]$$

so the change in interest rate is simply

$$(2a) \quad di(t) = d[i_{ex}(t)] + d[i_{en}(t)] = d[i_{ex}(t)] + 0.2 * [GDP(t-1) - 0.06]$$

This policy rule says that the Central Bank increases the short term interest rate when GDP growth during the **previous** quarter was higher than some target level. We assume that the target rate of GDP growth is 0.06, that is, 6% per year (recall that we are working with annualized variables). The interest rate is decreased when growth falls short of 0.06. More specifically, the interest rate is changed by 0.002 percentage points (20 basis points) for each 1% deviation from the target ( $0.002 = 0.2 * (0.07 - 0.06)$ ). Obviously, in the absence of policy interventions, the change in the interest rate is simply

$$(2b) \quad di(t) = d[i_{ex}(t)]$$

Now calculate the path of GDP from 1988:1 to 1994:4 with successive values for  $e(t)$  and  $i_{ex}(t)$  using:

a) equation (1) without the interest rate rule

b) equation (1) together with the interest rule (2) and (2a).

The objective is to find out whether the assumed interest rate policy succeeds in reducing the volatility of

nominal GDP.

#### A. NO POLICY

Proceed as follows:

Start by calculating the value of GDP in 1988:1

$$\text{GDP}(88:1) = a_0 + a_1 * \text{GDP}(87:4) + a_2 * \text{GDP}(87:3) + a_3 * \text{di}(87:3) + e(t) = \\ a_0 + a_1 * 0.0679 + a_2 * 0.0362 + a_3 * [0.1341 - 0.1487] + 0.024$$

Note that

$$\text{GDP}(87:4) = 0.0679 \quad \text{GDP}(87:3) = 0.0362 \\ \text{di}(87:3) = i(87:3) - i(87:2) = 0.1341 - 0.1487 \quad e(88:1) = -0.0158$$

(note that  $\text{di}(t) = d[\text{iex}(t)]$ )

Now calculate GDP(88:2) using exactly the same procedure

$$\text{GDP}(88:2) = a_0 + a_1 * \text{GDP}(88:1) + a_2 * 0.0679 + a_3 * (0.1493 - 0.1341) - 0.0131$$

Continue until you have exhausted all the numbers for  $e(t)$  and  $i(t)$  that are given in the spreadsheet. Then plot the series for GDP from 88:1 to 94:4. Compute the variance of the GDP series around 0.06 (that is, subtract 0.06 from the GDP numbers you calculated and then calculate the variance). The plot and the variance will give you an idea of how much the economy would fluctuate around the government's favorite nominal growth rate of 6% per year **in the absence** of an explicit central bank policy of targeting nominal income growth..

Note: The easiest way for calculating the sequence of numbers is by using a spreadsheet. Input the coefficients and formula in cells (see accompanying spreadsheet, example.xls).

#### B) POLICY

Now we will repeat the analysis using the interest rate rule. With the policy given by (2), the equation for GDP takes the form (substitute (2) and (2a) into (1))

$$(1') \quad \text{GDP}(t) = a_0 + a_1 * \text{GDP}(t-1) + a_2 * \text{GDP}(t-2) + a_3 * [d(\text{iex}(t-2))] + 0.2 * [\text{GDP}(t-3) - 0.06] + e(t)$$

Start by calculating the value of GDP(88:1) using (1')

$$\text{GDP}(88:1) = a_0 + a_1 * 0.0679 + a_2 * 0.0362 + a_3 * [(0.1341 - 0.1487) + 0.2 * (0.0726 - 0.06)] - 0.0158$$

where

$$\text{GDP}(87:4) = 0.0679 \quad \text{GDP}(87:3) = 0.0362 \quad \text{GDP}(87:2) = 0.0726 \quad e(88:1) = -0.0158 \\ d(\text{iex}(t-2)) = i(87:3) - i(87:2) = 0.1341 - 0.1487$$

Now calculate the value of GDP(88:2) using the updated version of (1')

$$\text{GDP}(88:2) = a_0 + a_1 * \text{GDP}(88:1) + a_2 * 0.0679 + a_3 * [(0.1493 - 0.1341) + 0.2 * (0.1362 - 0.06)] - 0.0131$$

(note that  $-0.0131 = e(88:2)$ )

Again continue until you have exhausted the numbers for  $e(t)$  given below. Then plot the series and calculate the variance around 0.06. Which series looks smoother and which has a lower variance? Does nominal income targeting with an interest rate instrument succeed in smoothing GDP around the target value?

### Policy aggressiveness. Is it a good idea?

Now assume that the central bank follows a more aggressive interest rate policy, that is the reaction coefficient in equation (2) is 0.4 (rather than 0.2). Redo the analysis. Does the more aggressive policy stance translate into even more stable output? What are the implications for the volatility of interest rates? What kind of lessons do you draw from this?

## 2. A forward rule (optional)

In practice, many central banks use FORWARD LOOKING policy reaction rules. That is, they try to predict how things will look like in the future and then act in a pre-emptive fashion. Let us describe how this might be accomplished. Because things are somewhat more complicated now we will work with the following simplified version of (1),

$$(1.1) \quad \text{GDP}(t) = q_0 + q_1 * \text{GDP}(t-1) + q_2 * d_i(t-1) + e(t)$$

Suppose now that the interest rate is changed according to the deviation of PREDICTED GDP growth from the target level in the absence of policy. A rule like the following might accomplish this

$$(3) \quad d[ien(t)] = 0.2 * [\text{GDP}^{\wedge}(t) - 0.06]$$

where  $\text{GDP}^{\wedge}(t)$  is the predicted (fitted) value from equation (1.1). In this case the equation for GDP under the forward policy rule (equation (1')) is given by

$$(1.1') \quad \text{GDP}(t) = q_0 + q_1 * \text{GDP}(t-1) + q_2 * [d(iex(t-1)) + (0.2 * [\text{GDP}^{\wedge}(t-1) - 0.06])] + e(t)$$

You can again calculate the path of GDP growth feeding values for  $e(t)$  from the sheet below and then plot the resulting values and also calculate the variance around 0.06. Does the forward looking interest rate rule produce values of GDP closer to target than the feedback rule?